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A Glimpse of a Mathematical Enculturator in Chinese Mathematics Classrooms: An Example from A Shanghai Lesson

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Abstract: This paper has two aims. Firstly, it gives a brief summary of selected accounts of Chinese mathematics classroom teaching based on ongoing analysis of the Learner’s Perspective Study. Secondly, it aims to present a description of an event of a Shanghai mathematics lesson to show how the teacher plays a role as a mathematical “enculturator” who plays an important influential role in the shaping process. The theoretical point of departure is that classroom practice is a process of “mathematical enculturation”. The process is a dynamic, intentional, shaping process, which refers to what happens between the teacher and students within a mathematics classroom.

Keywords: Chinese mathematics classroom, Learner’s Perspective Study, mathematical enculturation

1. Introduction

Within any particular educational system, the possibilities for experimentation and innovation are limited by many considerations. These limitations may be methodological or ethical. They may also be a consequence of the local educators’ capacity to conceive possible alternatives and their own assumptions regarding acceptable classroom practice. These assumptions may show traits of local values and well entrenched practices developed in the history. The Learner’s Perspective Study (LPS) launched in 2000, aims to investigate practices of the classroom in different countries and the meanings attributed to those practices by the teachers and the learners. The project is guided by a belief that we need to learn from each other to get insights into the practices of mathematics classrooms in different countries (Clarke et al., 2006).

With respect to Chinese mathematics teaching, LPS Hong Kong team first collected data from three cities: Hong Kong, Macau and Shanghai. The Beijing team, which joins the project at a later stage, further improves the data set with data from an additional city. Via analysis of the corpus of data either in parts or in whole for different research agenda from multiple perspectives, complementary accounts of mathematics classroom practices are being developed (for example, Mok and Lopez-Real, 2006; Huang et al., 2006; Mok, 2006).

The aims of this paper are twofold. The first is to give a brief summary of selected accounts of the Chinese mathematics teaching based on the analysis of the LPS data in order to make a contribution to the understanding of Chinese mathematics classroom practice. In this preamble, I must make clear that there is inevitable ambiguity in using the phrase of “Chinese mathematics classroom practice” which suggests a specific kind of mathematics classroom practice bearing the label of Chinese. The question for how Chinese learn mathematics has been investigated by many international researchers (e.g., Fan, et al., 2004). As a result of the long history of Chinese culture, the large population, and the vast geographical and cultural variation between different regions within the nation, the rich practice of Chinese pedagogy by itself is a worthy item in the research agenda for mathematics education but not easy to answer in simple ways. Although it is in fact quite impossible to label a kind of practice as ‘Chinese’, reports of different studies and animated reflections suggest some traits of prominent characteristics emerged in these different studies of Chinese teaching. The selected accounts reported in this paper are selected in order to give an abridged representation of the ongoing analysis of a corpus of empirical data in LPS and to enhance the understanding of some features found in Chinese mathematics classroom practice.

This summary also provides a background for the second aim. That is, to present a snapshot of a Shanghai mathematics lesson in which the teacher demonstrated the role of an “enculturator”, based on the theoretical point of departure that classroom practice can be seen as a process of “mathematical enculturation” (Bishop, 1991, 1997). The process is an active, intentional, shaping process, which refers to what happens between the teacher and students within the mathematics lessons of a class. Within the framework, the teacher is seen as a mathematical enculturator who plays a special influential role in the process.

2. A Brief Note on Selected Accounts of Chinese Mathematics Teaching

According to Mok and Lopez-Real (2006), by a comparison of six teachers in Hong Kong and Shanghai, findings show that analysis of a single ‘snapshot’ lesson is unlikely to reveal a lesson “script” in the sense claimed by Stigler and Hiebert (1999). That is, the activities in a lesson are likely to vary between lessons of the same teacher and between different teachers. However, interesting patterns of similarities and differences of the same region emerged in analysis of the two cities in terms of lesson organization (e.g., the use of group work) and teaching approaches (e.g., the use of exploratory activities). The results add an additional evidence to show that study in mathematics is not carried out in a rote-based learning environment. Furthermore, there are two phenomena observed.

The first phenomenon concerns the nature of practicing using problems for the purposes of practicing in the lessons. Teachers often have to provide problems (or tasks) for the purpose of practicing. Some may seem to be repetitive and serve the purpose of familiarization and consolidation. Some may look similar with subtle changes between items. This may in some way support the understanding of the topic in depth. Therefore, both the design of the problems itself and how the teacher uses the problems matter. Huang, et al. (2006), applied the theory of variation of variation. For example, the teachers would vary some aspects such as coefficients in the equations to produce different types of problems for the students to practice. Based on the analysis of the teaching of the particular procedural method of elimination in a system of simultaneous linear equations for two unknowns in Hong Kong, Macau and Shanghai, Huang, et al. show that the teachers in the three cities emphasized practicing with both “explicit variation” and “implicit variation”. If the changes from the prototype of problems are identified visually and concretely, the variation is called explicit, e.g., “$8x-5y=4$ & $3x+5y=18$” and “$5x-4y=12$ & $5x+y=7$”. The variation between the two systems of equations is explicit and easily discerned by the students. The If the changes have to be discerned by learners with abstract and logical analysis, and the prototype can only be obtained after transformation; they are called implicit variation, e.g., “4(x+2)=1-5y” and “ax+by=c”. The analysis of the teaching episodes within the same lesson as well as those in a sequence of lessons shows that the use of these variations is a critical feature of the mathematics lessons found in the three places. These variations are embedded in both the design of the mathematical tasks and the teaching approaches. The analysis explains how such variations possibly enhance the learners’ discernment of the critical features for an object of learning (Marton et al., 2004). It also gives an explanation to the Chinese notion of the inseparable relationship between learning and practice. It further unfolds how practicing extensive problems can go beyond drilling by rote but help building up an interrelated knowledge structure of a mathematical object and developing flexible problem solving abilities.

The second phenomenon concerns a match between the teacher and the students. Mok (2006) analysed the reflections by a Shanghai teacher and his students upon their lessons based on the video of the lessons and the interview data. The findings show that both teachers and students see their lessons in a positive way and there is harmonious match between the expectations of the teacher and students.

These two phenomena describe important characteristics of what happen in the lessons and highlight some features in the culture within the class created as a result of the teacher’s teaching. In addition to these two phenomena, the author in this paper attempts to illustrate a third phenomenon in the lessons, the nature of teacher-student interaction, by zooming into a lesson episode.

3. Methodology: The Learner’s Perspective Study

Since the launching of the Learner’s Perspective Study Project (LPS) in 2000, the LPS team has engaged researchers in the investigation of mathematics classrooms of teachers in Australia, China, the Czech Republic, Germany, Israel, Japan, Korea, the Philippines, Singapore, South Africa, Sweden and the USA. LPS aims to juxtapose the observable practices of the classroom and the meanings attributed to those practices by classroom participants.

Essential features of the research method are (i) the use of three cameras for recording both teacher and student actions; and (ii) the use of the technique of video-stimulated recall in interviews to obtain the teachers’ and students’ personal views of the lessons. Two students were interviewed after each lesson. Each teacher participated in three video-stimulated interviews and completed two substantial questionnaires before and after videotaping, as well as a shorter questionnaire after each videotaped lesson. Three competent teachers from different schools recommended by local researchers were chosen and their eighth grade lessons were recorded for a minimum of ten consecutive lessons for each class/teacher.

The event described in this paper was taken from a grade-7 mathematics lesson by a teacher in Shanghai, China. As a result of the matching of curriculum topics, the students were one grade level lower than the students in other countries in LPS. The teacher was very competent with 17 years of teaching experience and was awarded the title “Senior Lecturer in Secondary School” by the Shanghai Senior Academic Title Appraisal Group in 1997.

4. A Theoretical Framework: The Process of Mathematical Enculturation

According to Bishop (1991), there are two perspectives on mathematical enculturation, namely, that of the curriculum and the other of the process. The analysis presented in this paper puts focus on the process. Mathematical enculturation is an interpersonal process and therefore it is an interactive process between people. Via this process, the concepts, meanings and values of mathematics and its learning will be developed and eventually belong to the students. The process directly refers to what happens between the teacher and students the mathematics classroom in the reports in this study. The process is an active one as a result of the teacher's intentional planning. It takes significant account of the social context, i.e., the interaction between the teacher and the students and between the students, hence, is not a result of transmission. Moreover, the process is apprehended within a knowledge frame concerning specific mathematical knowledge, e.g., simultaneous equations in two unknowns in this lesson.

Two points are recapitulated here to explain the idea further. Firstly, although both the teacher's and the learners' ideas may be changing in the process, it is the learners' ideas which are intended to be shaped. Therefore, the teacher is expected to have a more influential role. The teacher has the authority to design the activities and is expected to justify for the choice and designs. The second aspect is the intentional aspect. What the teacher wants to happen in the lesson is influenced by the teacher's personal understanding of the subject and pedagogy. Such intention will influence how the teacher will communicate the mathematical ideas with the students. For this, the focus of attention is on the communication and sharing of mathematical ideas. The mathematical culture as a result of the shaping process requires the teacher to act as the "mathematical enculturator".

5. A Glimpse of the Mathematical Enculturator in the Lesson

An episode from a grade-7 lesson (SH2-L03) was chosen below to help to see the asymmetrical teacher's role in the interaction. The teacher obviously did a lot in shaping the activity and the class discourse in order to guide the students to see what he wanted the students to understand about the mathematical object "a system of linear equations in two unknowns". In the episode, the analysis captured the following features:

The teacher gave strong guidance before the students started off their own discussion (line 1).

The teacher asked the questions which encouraged the students' reflection upon the mathematics (lines 4, 6), exploratory attempts (line 14) and different opinions (line 18).

The teacher affirmed answers both orally and in written form (line 12), by whole class (lines 9,11).

The teacher showed a demand and demonstrated the accuracy of expressing ideas mathematically (lines 9 to 22).

The students expressed their own answers and explanations under the teacher's invitation (lines 5, 7, 13, 15, 19).

The Problem

"Guess it: How many chickens and rabbits are there? There are x rabbits and y chickens in a cage. There are altogether twelve heads, and forty legs. How many rabbits and chickens are there in the cage?"

The problem was very similar to the problem yesterday which was on two independent equations. Therefore the students were very familiar with the context. Without any difficulty, a student immediately suggested to write down two equations: x+y=12 and 4x+2y=40. Next, the teacher asked the class to simplify 4x+2y=40 to 2x+y=20. Then, the teacher asked the students to find the number of chickens and rabbits.

1. T: Good, so how many chickens and rabbits are there? …Let me give you some pairs of numbers. [Speaking while writing on the board] The first pair: x equals two, y equals four; the second pair: x equals four, y equals twelve; the third pair: x equals eight, y equals four, there are many pairs like that, …for now, discuss with your classmates how many chickens and rabbits there are? [Students discussing and the teacher walking around between desks.]

2. T: (…)

3. [Some students discussed quietly with their classmates, as they were working at their desks.]

4. T: Okay, stop now. How many chickens and rabbits are there in the question? Why? [The teacher resumed the attention of the whole class. The setting changed back to whole class discussion. Three students put up their hands]

5. Capella: There are eight rabbits, four chickens.

6. T: Why not ten and two, ten plus two is twelve?

7. Capella: Because when you substitute x equals two, y equals ten into two x plus y equals twenty (…)

8. T: Oh, it may not be suitable to substitute two and ten into the second equation, so is it suitable to substitute four and twelve into the second equation?

9. (Class): No.

10. T: Oh, no suitable, how about eight and four?

11. (Class): Suitable.

12. T: For they can satisfy both the first and second equations, in this question, the values of the pair of numbers have to satisfy the first equation and also the second equation. In mathematics, we use a pair of big brackets to join them together. [Writing on the board while speaking] For that, we can have a [showing a slide]…in mathematics, we call this a system of linear equations [Writing on the board]…a set of equations formed by the combination of linear equations is called a system of linear equations. So, according to the characteristics of the system of linear equations, what kind of system is it?... The second girl in the row. [The teacher invited the girl to answer the question.]

13. Carry: A system of linear equations in two unknowns.

14. T: A system of linear equations in two unknowns. She said that this equation is [Writing on the board] a system of linear equations in two unknowns. Sit down, so students, what is a system of linear equations in two unknowns, …she thinks that it is a system of linear equations in two unknowns, so what is system of linear equations in two unknown? Can you tell me? Try, that's alright even if you get it wrong, okay, this student is good today, you. [A student raised her hand.]

15. Clean: There are two unknowns, and the power of the unknown (…) is one…we call this linear equations in two unknowns…

16. T: A system of linear equations, um.

17. Clean: A system of linear equations in two unknowns.

18. T: He said that equations that have two unknowns, and the power of the unknowns is one is called a system of linear equations in two unknowns. You seem to have very different opinions.

19. Cell: Two or above (…)

20. T: Two or above, um.

21. Cell: Equation of linear equations in two unknowns (…).

22. T: A system formed by linear equations in two unknowns, good, let me write it down [Writing on the board], that is a pair of linear equations in two unknowns, right? That is also linear equation in two unknowns, he says there should be two or above, so let me write one more [Writing on the board.]

6. Discussion

The teacher in this paper is the same teacher analysed in Mok (2006). The analysis captured by Mok (2006) compared the teacher's reflection of the lesson with the students' reflection on the lesson. It captures an important aspect of the lesson and is recapitulated here to supplement the episode discussed in this paper. Mok argues that the teacher takes up a strong influential role in his lessons, which can be in some sense a feature of 'teacher-dominance'. The adverse effect of the teacher-dominance is significantly reduced by the teacher's clear philosophy for learning and expert pedagogical skills. The teacher demonstrated a very clear understanding of the mathematical topic at a very detailed level. In this episode there was a clear purpose in each of the teacher's questions and comments. The teacher gave opportunities for student discussion but also controls their activity by his choice of tasks. By doing this, the teacher managed very well to guide his students to see the same under his guidance. For example, the teacher's questions and suggestions in the episode help the students be aware the fact that a solution needed to satisfy both equations instead of one. The analysis of the students' interviews by Mok (2006) shows that the students obviously liked their mathematics lessons and their teacher very much. In this lesson, the students followed the teacher's instruction very readily. They were always engaged in the activity. For example, in this lesson although the time for their own trial was short, they readily shared their answers upon the teacher's request. Their focus was mostly on the content of the lesson and the teacher's action. This forms a nice match between the teacher's expectation and the students' expectation. Concerning the choice of problems, the teacher intentionally chose the chick-and-rabbit problem which had the same context as the one in the last lesson. The problem in the last lesson had required the students to construct the equation whereas the problem in this lesson asked the students to solve the pair of equations instead. The variation was explicit and subtle. Consequently, the students had a chance to practice what they had learned in the last lesson and also learned about solutions for a system of equations. This new knowledge was thus presented in a very smooth pace.

In the episode shown, we can see that the teacher gives the students opportunities for discussion and guided exploration although the activities are limited by the nature of his design. The beginning of the episode (line 1 to 11) was close to the Socratic style. Till line 14, the teacher's question invited limited possible exploration based on what they might have observed about equations while he clearly indicated that the students were invited to tell what they thought about the conceptual properties. This is a kind of true invitation as the teacher actually invited more than one student (Clean and Cell) to express their different opinions (lines 14 and 18). This kind of very directive focused reflection on a specific mathematical object happened often in this teacher’s lessons. While welcoming his students to express their mathematical ideas in their own words, he makes an obvious demand in expressing ideas accurately in terms of content and language. His correction of Clean’s answer noting the difference between “linear equations in two unknowns” and “a system of linear equations in two unknowns” is an example (lines 15-17). Many of these features are essential in shaping the students’ understanding and appreciation of the mathematical objects as well as the culture of mathematics learning. Therefore, this style of teaching can be well justified as a kind of enculturation.

Recently, the educators in China affirm the need for further development in both the content of curriculum and pedagogy (Zhang and Dai, 2004). The teacher in this account is the same teacher reported in “Teacher-dominating lessons in Shanghai – An insiders’ story” by Mok (2006). He is well aware of the fact that his teaching style is not the same as the traditional model of teaching, which placed emphasis on practice and students imitating the teacher’s work. He mentioned in the interview:

> “My way of teaching is unlike the conceptual way of teaching traditionally, I let the students to experience as far as possible…. So you have to let them figure it out, there is no need to require them to recite every word. … allows the students to sort out and apprehend according to what they have experienced.” (Mok, 2006, p.89).

He perceives his own model promoting students’ understanding by their own apprehension. This is a kind of product of the teacher’s own understanding of non-traditional models. “Mathematics teachers are passing on values, habits and customs as well as knowledge and skills. They are inducting their students into the culture of mathematics.” (Bishop, 1997). All teachers eventually have to develop and evaluate their own personalized pedagogy. Introducing the evaluation of the teacher’s role from the perspective of an enculturator may give a new window for teachers’ reflection in their professional development.

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