

“Quaderni di Ricerca in Didattica (Matematica)”, Supplemento n.4 al n. 19, 2009.
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The Impact of Written Reflections in a Geometry Course for Preservice Elementary Teachers

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Abstract

In this concurrent mixed-methods study, we analyze preservice elementary teachers' written reflections and demonstrate how written reflections influence their learning of geometry. Our findings suggest participants performed better on tasks when they participated in written reflections and preservice teachers, who wrote reflections at the beginning of the semester, produced stronger reflections. In their reflections, we expected our participants to write about what they learned in a discovery based geometry lesson, however our prospective teachers also discussed classroom culture and teaching in mathematics. Our results indicate incorporating reflections into the mathematics classroom increases participants' achievement on related tasks, allows preservice teachers an opportunity to reflect on the learning and teaching of mathematics, and serves as a further assessment of student understanding.

Introduction

In order to teach mathematics conceptually, one needs a deep understanding of the content. Research shows elementary and secondary preservice teachers in the United States have strong procedural skills but are limited in their conceptual understanding of the underlying mathematical principles involved with the calculations (Ball, 1990; Simon, 1993). Although some prospective teachers recognize their inadequacies and acknowledge their inability to answer conceptual questions (Ball), mathematics educators must strive to help deepen preservice elementary teachers' conceptual understanding of mathematics. This endeavor may be cultivated with the use of reflections (Wheatley, 1992).

Sigel (1981) defines reflecting on mathematics as the act of distancing oneself from the action of doing mathematics. For instance, mathematical reflections occur via informal questioning, frequently seen in classroom discourse, or through formal interviews between teachers and their students. Explanations to student work can serve as an informal written method of reflection, whereas, formal written reflections can be acquired through the use of journals, portfolios, and directed forms of written reflections. Wheatley (1992) encourages the use of reflections with the completion of any task related to learning mathematics. Wheatley asserts students engaged in problem solving situations where reflection is a vital part of the classroom norms are better able to decipher challenging problems than students who do not actively reflect.

Writing in the mathematics classroom can serve as a tool for reflecting and for understanding concepts more deeply. The National Council of Teachers of Mathematics ([NCTM], 2000) advocates written communication as an essential element for transmitting one's mathematical thinking to the teacher and to other students. Furthermore, written reflections allow a teacher to “monitor a student's capacity to analyze situations, frame and solve problems, and make sense of mathematical concepts and procedures” (NCTM, p. 19). Although reflecting is widely used in the mathematics classroom (Costa & Kallick, 2000; Griffin, 2003; Sparks-Langer & Colton, 1991; Yost, Sentner, & Forlenza-Bailey, 2000), there is little quantitative research to establish its effectiveness on student understanding. The purpose of our concurrent mixed-methods study

is to add to the knowledge base of reflections. In this study, we measure the relationship between written reflections and achievement on content related to the reflections. At the same time, we investigate the nature of the participants' written reflections. Our research questions are:

1. Do written reflections in a geometry course designed for prospective elementary teachers affect their performance on content related to the reflections? i.e. the null hypotheses are:
 - a. Preservice elementary teachers who reflect on activities will perform as well on related warm-up exercises as participants who do not reflect on activities.
 - b. Preservice elementary teachers who write strong reflections will perform as well on related warm-up exercises as participants who do not write strong reflections.
 - c. Preservice elementary teachers who reflect on activities later in the semester will write comparable reflections to those written by participants who reflect earlier in the semester.
2. What experiences, intentions, and perceptions do preservice elementary teachers' share through written reflections to guided questions pertaining to geometry lessons?

Theoretical Framework

Constructivism. We provide a brief discussion on constructivism since the curriculum used in our research advocates constructivist learning through collaboration. Constructivism is a learning theory where learners construct their own understanding by creating mental structures and connecting prior knowledge through personal experiences. In the 20th century, two forms of constructivism evolved: psychological constructivism based on the work of Piaget (von Glasersfeld, 1984) and social constructivism attributed to Vygotsky (Richardson, 1997). Social constructivism is rooted in the idea that one learns through social interaction. This philosophy motivated a fundamental change in the way mathematics is taught at all levels of education especially courses designed for preservice teachers. Social interaction has influenced the mathematics curriculum for prospective elementary teachers as well as the presentation of content (Seaman, Szydlik, Szydlik, & Beam, 2005). In our study, we incorporate social interaction with preservice elementary teachers with the intent of stressing conceptual understanding, reasoning, and mathematical communication in their future classrooms. Through novel learning environments, preservice teachers begin to realize the benefits of different teaching models and the importance of subjective experiences (von Glasersfeld, 1996). We hoped to witness comments of this nature in our preservice elementary teachers' reflections.

Cooperative Groups. In a social learning environment, student interactions with the material, their peers, and the teacher are integral in developing conceptual understanding. Successful cooperative groups increase student activity, on-task verbal interactions between students, opportunities to receive help from multiple sources, and positive attitudes toward cooperative learning (Leiken & Zaslavsky, 1997). Teachers, such as ourselves, who support autonomous learning are more apt to listen to students share their thoughts and feelings about the content and process of learning. These teachers also encourage students to reflect on their competence and interest in learning the material (Boekaerts & Minnaert, 2006). We expect group members to learn the material and to ensure other group members learn the material. Although the driving force of cooperative learning is group interaction, teachers assess students on an individual basis (Johnson & Johnson, 1991). This was the structure of our course. Through written reflections, we explore how a group-learning environment affects the classroom culture and the learning of mathematics.

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Reflections. Typically, teachers introduce reflections through questioning techniques designed to enhance class discussion. Ideally, this type of reflection is an opportunity to examine one's actions in solving mathematical problems as the object of reflection rather than focusing on the computational mathematics component itself. Including reflective activities in the classroom allows students to consider other problem solving methods and to evaluate the efficacy of their thinking (Costa & Kallick, 2000). It also provides the teacher with a means to formulate learning activities suited to the students in the classroom based on their previous knowledge.

LaBoskey (as cited in Griffin, 2003, p. 209) categorizes reflective thinkers into three categories: concrete, alert and pedagogical thinkers. *Concrete thinkers* are procedural in nature, ask “how” questions, and allow their emotions and attitudes to interfere with their learning. They tend to be unaware of their need for conceptual understanding and resist teaching techniques, which differ from their own learning experiences. An *alert thinker* tends to ask “why” questions and is flexible with different teaching methods. A *pedagogical thinker* is self-aware and concerned with the student and the student's learning. He/She is knowledgeable about pedagogy and content.

Many programs designed for preservice teachers require reflection. Much like expert teachers, prospective teachers begin to think and articulate their thoughts about classroom experiences (Sparks-Langer & Colton, 1991). Eventually, the learner grows from a “concrete thinker to an alert thinker” who uses reflection as a way to become more alert about teaching and learning (Griffin, 2003, p.338). This requires much practice (Yost, Sentner, & Forlenza-Bailey, 2000). By instilling these ways of thinking, preservice teachers can think beyond “discrete information and right answers,” which limit the creative aspects of problem solvers (p. 46). In our research, we explore the nature of preservice elementary teachers' reflections and investigate if written reflections influence preservice elementary teachers' understanding of geometry content.

Methods

Our sample consisted of 55 undergraduate prospective elementary teachers at a mid-size, doctoral granting institution in the western United States. Preservice elementary teachers from two sections of a discovery-based geometry course designed specifically for prospective elementary teachers participated in this study. The same instructor taught both sections, which met two days per week for 75-minutes. The morning section (Class 1) consisted of 28 female preservice teachers. The afternoon section (Class 2) comprised of 26 females and 1 male.

We used the text, *Geometric Structures: An Inquiry-based Textbook for Prospective Elementary Teachers*, (Aichele & Wolfe, 2008). The authors designed the text to encourage cooperative discovery learning, whole class discussions, writing, problem solving, and the use of manipulatives. The primary emphasis of the course is to develop preservice elementary teachers' spatial reasoning. Topics focused on properties of two- and three-dimensional shapes, measurements, constructions, and transformations.

We utilized a crossover quasi-experimental research design (Montgomery, 2005). For each participant, we collected data through a pre-test, 7 written reflections, 14 warm-up exercises, 4 quizzes, 2 tests, and a final comprehensive exam. The instructor administered the pretest in order to determine if there were statistically significant differences between the two classes' prior knowledge of course material. It included sample material encompassing the entire course. Participants completed two quizzes, each worth 20 points prior to each test. The final was worth 100 points, and tests were each worth 150 points. All exams contained procedural and conceptual questions. The first test assessed participants' understanding of polygons and angle relationships, quadrilaterals and their definitions, symmetry, and transformations. The second test

focused on the Pythagorean Theorem, and perimeter and area of polygonal shapes, sectors and circles.

Reflections. Based on the toss of a coin, Class 1 completed reflection activities prior to Test 1, and Class 2 completed reflection activities after Test 1. The instructor provided the reflection assignment (total of seven before each exam) in written form to the participants with each homework assignment. The reflection assignment required the participants to write about the purpose of the lesson, mathematical ideas they learned and used, how they resolved their struggles, and any remaining questions related to the activities. The reflection assignment along with other homework was due the following class period. The reflections, written outside of class, gave preservice teachers an opportunity to describe their understanding of the lessons through a medium other than homework and quizzes. The reflections made up ten percent of the course grade based on completion of the assignment. Assessment of the reflections was the basis for classroom discussions and helped correct the participants' misconceptions.

Warm-up Exercises. Participants completed warm-up exercises related to the previous lesson at the beginning of the class period, but the warm-up exercises did not contribute to the course grade. The instructor allocated two to three minutes for each warm-up activity, which she projected on an overhead screen. The instructor did not answer any questions prior to the warm-up. Upon completing the warm-up, the instructor collected and scored the warm-ups using scores of correct, partially correct, and incorrect while the participants discussed the warm-up solution and other questions from the previous lesson. Depending on the warm-up results, the instructor either allowed the preservice teachers to continue answering each other's questions or elaborated on the previous lesson. As a whole, the class summarized the important concepts from the previous lesson. The remaining class time was devoted to working on the new lesson. The instructor used the warm-up problems to assess participants' understanding of geometry concepts. Most of the warm-up problems measured preservice teachers' ability to transfer knowledge ascertained from the prior lesson to a novel problem. Additionally, they provided the instructor with prompt feedback regarding the prospective teachers' understanding of the lesson.

Results

Our results indicate our preservice teachers performed better on the warm-up activities when they participated in the reflection activities. We also found the class who wrote reflections earlier in the semester tended to write stronger reflections – these participants displayed more evidence of alert and/or pedagogical thinkers than Class 2. Through an analysis of the reflections, we found some of our prospective teachers embraced the opportunity to express their stance on learning and teaching mathematics.

Quantitative Results

Pre-test, Tests and Quizzes. An independent 2-sample *t*-test conducted on the pretest scores, indicated no statistically significant differences between the two classes ($p = 0.89$). The mean scores for Class 1 and Class 2 were 12.61 and 12.44 (out of 20 points) respectively. Table 1 is a summary of in-class assessments in the order administered to the students; participants from Class 1 had a higher mean on all of these assessments. However, independent sample *t*-tests suggested there were no statistically significant differences between the two classes on these assessments with the exception of quiz 4 ($p = .02$).

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Table 1. Statistical Results for Quizzes, Tests, and Final

	Class	<i>N</i>	<i>M</i>	<i>SD</i>	<i>t</i>	<i>df</i>	<i>p</i> -value	Mean Difference
Quiz 1	1	27	16.07	3.15	0.136	52	0.89	0.11
	2	27	15.96	2.85				
Quiz 2	1	26	16.39	2.84	0.776	50	0.44	0.65
	2	26	15.73	3.22				
Test 1	1	28	121.89	15.08	1.475	53	0.15	7.00
	2	27	114.89	19.89				
Quiz 3	1	28	16.25	2.05	1.237	52	0.22	0.67
	2	26	15.58	1.94				
Quiz 4	1	28	14.89	3.99	2.258	51	0.02	2.49
	2	25	12.40	4.04				
Test 2	1	28	122.82	14.24	0.998	53	0.32	4.93
	2	27	117.89	21.76				
Final	1	28	73.39	12.31	0.501	53	0.62	1.65
	2	27	71.74	12.11				

Warm-Up Exercises. In order to analyze the effectiveness of the warm-up exercises, we dichotomized the scores into correct and incorrect. We coded scores that were originally partially correct as incorrect and created a cross-tabulation table, as described by Agresti (1996), in order to conduct a Chi-squared test for independence between participant’s performance on the warm-ups and the use of reflections. To determine during which treatment phase a preservice teacher performed better, we compared the proportion of successes on the warm-ups for the two phases of the study. We used proportions of successes to account for the few missing data values. For example, if a participant completed five of the seven warm-ups in the first phase and four of the five were correct then he/she was successful 80% of the time during the first phase. If the same participant completed six of the seven warm-ups in the second phase and had correct responses to four of the warm-ups, then he/she had a 67% success rate during the second phase. We classified these participants as those who did better during the first phase. Participants who performed equally well in both phases are not included in the table (Agresti). The data shown in Table 2 implies participants’ performance on the warm-ups was higher during the reflection phase of the study. For example, the achievement of 68% of the participants in Class 1 and 65% of the participants in Class 2 was higher on the warm-up problems during the reflection treatment. A Chi-square test indicated there was a significant association, ($p = .02$) between participating in reflection activities and preservice teachers’ performance on the warm-up problems.

Table 2. Cross-Tabulation Table for Warm-up Performance

		Treatment phase during which participant did better		
		First	Second	Total
Class 1	Count	17	8	25
	% within section	68%	32%	100%
Class 2	Count	8	15	23
	% within section	35%	65%	100%

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The Reflections. Two researchers assessed the reflections using a 0-5 scale; 0-1 represented a weak reflection, 2-3 represented a medium reflection, and 4-5 represented a strong reflection. We categorized reflections as strong if they incorporated aspects of a pedagogical thinker, medium reflections instilled ideas of an alert thinker, and weak reflections showed evidence of a concrete thinker. The sum of the scores for the seven reflections was the preservice teachers' reflection score. The correlation between the two reflection scores awarded by the researchers was $r = 0.91$.

We did not find a statistically significant association between the strength of the reflection and performance on the warm-up exercises. However, we did find the period (early or late in the semester) in which the participants reflected associated with their performance on the reflections. A loglinear model predicts the odds that a participant's reflection category is $j + 1$ instead of j ($j = 1, 2, 3$ where 1 = weak, 2 = medium, and 3 = strong) increases by a multiplicative factor of 2.2 when the participant is in Class 1 rather than Class 2. Considering the two extreme scores of 1 (weak) to 3 (strong), the estimated odds that a reflection is classified as strong instead of weak is 4.7 times higher for Class 1 than Class 2.

Qualitative Results

The participants' written reflections served as the qualitative data of our study. During the transcription phase, we conducted a microanalysis and created over 200 code words. These code words consisted of reoccurring themes that emerged from the data, such as describing future teaching methods, struggling with mathematical ideas, using various strategies to solve problems, over thinking questions, working in groups, and enjoying fun lessons. In the second phase, we scrutinized the code words more closely and placed them under the umbrella terms (Strauss and Corbin, 1998): teaching, self-awareness, articulation, mathematics learning, understanding concepts, and classroom culture. We coded many of the reflections together in order to achieve a consensus for code word definitions. This helped us maintain reliability, allowed for triangulation, (Patton, 2002) and served as a means to discuss discrepancies.

We found the reflections provided insight into the preservice teachers' thoughts about the class and about their future teaching. Through axial coding (Strauss and Corbin, 1998), our original six umbrella terms collapsed into three themes: *Learning Mathematics*, *Classroom Culture*, and *Teaching Mathematics*. Our preservice teachers described the mathematics learned in class, how the classroom culture helped them cope with any struggles, and teaching techniques useful for their future teaching careers. In the following, we discuss each of these themes.

Learning Mathematics. Through the reflections, the instructor deciphered the preservice teachers' level of understanding of the lesson. The preservice teachers described the mathematical learning in various ways. Similar to LaBoskey's (as cited in Griffin, 2003, p. 209) work, our participants' thoughts were primarily articulated through *pedagogical*, *alert*, and *concrete* voices. Although all classroom activities promoted conceptual understanding, a majority of the preservice teachers described what they learned through mathematical facts or procedures. In an analysis of all reflections, 31 different participants described mathematical ideas conceptually. This is in stark contrast to mathematics depicted via procedures and facts, by 54 participants. Thus, participants fluctuated in their reflections sometimes they expressed their thoughts through a concrete/alert voice and sometimes through an alert/pedagogical voice.

Most prospective teachers cited prior knowledge from their K-12 experiences or previous classroom activities that helped them understand the new material. Some participants were able to transfer their prior knowledge into the new lesson. Seeing relationships between concepts and how they are utilized in application settings can serve as a tool for teachers to relate material to students in meaningful ways. In fact, 40% of our preservice teachers remarked on their new

found connections between the classroom problems and the real world.

Classroom Culture. The curriculum advocated constructivist learning through discovery learning, group work, whole class discussions, and manipulatives and thus influenced the culture of the classroom. Our participants recognized and acknowledged this classroom culture in their reflections. Their realization of the classroom culture manifested in remarks related to four ideas: persistence, lack of anxiety, peer assistance, and value of manipulatives.

In analyzing the reflections, we found some preservice teachers did not hesitate to make continued effort with the lessons, even when they struggled. In reflecting on their struggles, the prospective teachers shared how they transferred the discovery aspect of the classroom culture to their personal study environment. Although our prospective teachers all struggled with elements of the material, their reflections did not exhibit feelings of anxiety. On the contrary, they were confident their difficulties would be resolved with the help of their group members or the instructor and through classroom discussions. It is noteworthy that our participants were more likely to mention seeking assistance from their group members than the instructor. Group work and peer assistance were effective means for “cementing” ideas, resolving struggles, and providing different perspectives to the material. Group members also helped one another with using manipulatives. Some prospective teachers were unfamiliar with a few manipulatives, but recognized their value as learning and teaching tools.

The process of learning under a constructivist model goes hand in hand with some frustration. This was no exception for our preservice teachers. They found it difficult to communicate their own understanding, use manipulatives, recall terminology, engage in discovery learning, implement multiple strategies, and comprehend their peers’ understanding. Some participants appreciated multiple approaches to solving problems, while others were unsure of when to implement different strategies. It appeared new strategies led to new conflicts, which the preservice teachers did not appreciate.

Teaching. Even though the class was a mathematics content course, 53% of our preservice teachers reflected on their future teaching. This is impressive given the participants were not prompted to discuss their future teaching of the material in their reflections. Over half of our preservice teachers stated the activities were practical and were excited about the idea of using this material in the future, but had reservations about their future teaching. Some participants felt unsure of their ability to implement manipulatives into their future teaching. Others were not convinced they should teach conceptually, rather they believed in teaching procedurally or through “fun and educational” activities. Still others commented on the importance of teaching conceptually and posed questions like, “What would a good lesson using these concepts in a first grade class look like?” These comments elicit classroom discussions about pedagogy. On several occasions, the participants practiced teaching the material and shared their problem solving strategies with their classmates. Some found this experience uncomfortable, but beneficial.

Discussion

Our research supports findings, which promote reflections as a means to increase student achievement, but through a quantitative approach of investigation. Our qualitative lens of inquiry captures a glimpse of the preservice teachers’ understanding of mathematics, how they reacted to the classroom culture, and for some how/what they thought about teaching mathematics. Our findings advocate the use of written reflections to:

1. improve achievement in mathematics,
2. engage preservice teachers in inquiry based reflections prior to a methods courses, and
3. assess participants’ understanding of the content in a discovery learning environment.

Improve achievement in mathematics. Our results suggest written reflections in the mathematics classroom are statistically related to higher participant performance on warm-up exercises. This supports Wheatley’s (1992) qualitative findings indicating the use of reflections results in greater mathematical achievement. In their reflections, some participants claimed that after completing the warm-up exercise, their understanding of the material “clicked.” Thus the warm-ups may have helped solidify concepts, which in turn may have made the quiz and test material more accessible. This may explain the lack of statistically significant differences between the scores of other assessments. The fact preservice elementary teachers are conscientious and grade motivated is another plausible explanation of these results.

The evidence of learning demonstrated in the warm-ups while reflecting, regardless of the reflection period, suggests instructors can incorporate reflection activities at any time. On the other hand, our results show preservice teachers who reflected at the beginning of the semester tended to write stronger reflections. This may be because reflecting was part of the culture from the onset of the class. Thus, in order for students to produce meaningful reflections instructors need to instill the use of reflections early in the semester.

Engage preservice teachers in inquiry based reflections prior to a methods courses. It is essential for prospective teachers to begin reflecting on their teaching and learning of mathematics prior to their methods courses (Mewborn, 1999; Yost, Sentner, & Forlenza-Bailey, 2000). Mewborn stresses the value of giving preservice teachers “opportunities to talk with classroom teachers, teacher educators, and their peers to gain additional perspectives on the problems of classroom practice” (p. 338). In their reflections, our preservice teachers frequently referenced their peers rather than the teacher, as a source for resolving struggles. Participants also commented or asked questions related to pedagogy although the instructor did not solicit it. Furthermore, the instructor was open to such conversations and allocated time for this discourse. Given our participants took it upon themselves to pose pedagogical questions illustrates their readiness to reflect on their teaching and learning prior to their methods courses. The fact some participants reflected on their future teaching of mathematics while engaged in their own learning of mathematics illustrates the natural integration of simultaneously reflecting on learning and teaching.

Although the instructor designed the reflections to increase achievement, they seemed to help participants make connections to prior knowledge and to communicate their understanding or lack of understanding of the topics. They also served as a mechanism to receive feedback from the instructor. Through the reflections, our preservice teachers had an opportunity to ask questions, describe their struggles, and pose any concerns in a safe environment. It was also an opportunity for them to acknowledge that problems could and would be resolved with the help of others. They recognized the importance of cooperative learning in a discovery classroom setting.

Assess participants’ understanding in a discovery learning environment. Reflections along with warm-up exercises can assist teachers to assess student understanding on a day-to-day basis. In a discovery based course it is easy for students to make false conjectures, misinterpret definitions, inappropriately apply strategies, or simply give up. The reflections and the warm-up exercises helped the instructor uncover such misconceptions early in the learning process. The combination of reflections coupled with the warm-up activity allowed the instructor to identify such misinterpretations, to address misconceptions during class discussions, and to modify lessons for future teachings of the course. We acknowledge that the teacher feedback to the reflections may have affected the preservice teachers’ achievement on the quizzes and exams.

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