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Chinese and Italian students in primary school: a didactical experience in a multicultural class

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Abstract. This work is inserted in a broader research project, conducted within the GRIM of Palermo and essentially dedicated to the analysis of historical-epistemological and didactic obstacles related to same fundamental concepts introduced in primary school (but not only) and referred to algebraic and pre-algebraic thought, in a multicultural prospective. One of the open problems of the "new" globalised school is to interpret the behaviour of students in multicultural classes.
Main objective of the experimental research[1] is to study the cognitive behaviour of pupils (3-10 years old) to the resolution of algebraic and pre-algebraic problems. More specifically, we try to study, through a parallel analysis carried out in different mathematical contexts created ad hoc, how the different types of logical argumentative structure of the problems could engrave on the resolution behaviour of the students (types of solutions, passages of the decisive algorithm and errors). How these registers can influence their process of learning of specific ability related to the treated concepts.
The described activity, has been conducted according to Brousseau's theoretical framework taking into account a multicultural didactic environments, with a parallel analysis and a comparison between Italian and Chinese pupils. The choice to investigate on students of these *cultural references* is specify in the paper with different epistemological and didactical consideration referred to their original cultures. The selected concept that constituted the mathematical context is referred to algebraic thought, and so to a first approach to the concept of variable and unknown, since the Primary school. The collected data have been analyzed in parallel according to a quantitative and qualitative analysis. Quantitative analysis has been conducted using the software CHIC for non parametric statistics.

Key words: algebraic thought, variable, unknown, Chinese writing language, multiculticulturalism, Jiuzhang Suanshu (Nine Chapters).

1. Introduction and research objective

One of the open problems of the "new" globalised school is to interpret the behaviour of students in multicultural classes. The issue of "multi-cultural" classes is in Italy a phenomenon that, even though it is rather new, it is in wide expansion: the integration of foreign students in the Italian classes had, in recent years, an increasing rate and become no more an exception but, on the contrary, an inevitable reality. «*The present situation requires therefore to consider and to reorganize an idea of education in balance with the new needs and resources, in order to strengthen the trend of differences integration, the change and mutual adaptation, an open trial correlated with identities recognition and acceptation and with incorporated knowledge*» (Canevaro, 1983). In order to acquire this goal, several national and international researches have discovered and tested concrete learning environments able to enhance meta-reflection on some contents proposed in the classroom and promote motivation, meta-cognitive and cooperative attitudes that are often absent in "traditional teaching". From this point of view, the differences and analogies that could be detected in the class activities can be developed in didactical sources that could enrich the whole class. In these relationships the teacher has to play the essential role of a "*knowledge mediator*" trough different knowledge, different reassuming schemes…different culture references.

The request to try "alternative" teaching strategies, with student coming from different countries, to the usual routine in all education levels starting from kindergarten, is stressed by several researches in mathematics education (D'Amore, 2000, 2001) that underline the poor cognitive effectiveness of standard teaching methods and propose different methods and applications able to involve *all students* within their own knowledge, stressed to their own culture. *But how we can set these "alternative" teaching strategies?* This is a very big problem for the researches. In the specific case of Mathematics, a greater attention was

1 This didactical experience described in this paper is a part of the thesis work of Benedetto Di Paola, conducted on the same research subject and treated in a more specific manner. The thesis is availed on the web: http://math.unipa.it/~grim/Thesis_PhD_Di%20Paola_09.pdf

paid to the problems of the didactics in multicultural classes and these themes were included into several school programs and described in many papers.
It becomes evident, in this context, that the starting point of any didactical activity is to specify and highlight all the "moments" that could characterise cultural models of integration: pupil's previous knowledge, his/her motivations, his/her expectations and abilities, his/her personal and intellectual characteristics; all that constitutes the necessary prerequisites of every correct pedagogic intervention. (Garcia Hoz-Guerriero-Di Nuovo-Zanniello 1997, 238-239).
This is, even if in a first approximation, the aim of a ampler research project conducted within the GRIM of Palermo on the binomial *mathematics-culture*; project in which this paper is inserted as a possible reflection on probable differences and similarities on reasoning schemes and practical/procedural behaviour of different cultural students inserted in Italian context. Specifically about Chinese students as very distant culture from the Italian/Western one.

In the paper the selected concept that constituted the theoretical/experimental mathematical specific context is referred to algebraic thought, and so to a first approach to the concept of variable and unknown since the Primary school.
According to us, this kind of research could be significant for the research in Didactic of Mathematics because it could be able to inform and to "discuss" with teacher and researches about several didactical problematic referred to situation of multiculturalism in class and in particular on the presence of Chinese students on different cultural context, as the Italian one. Didactical obstacles and teaching/learning "difficulties" related to historical-epistemological and didactic references of the students, references that con be linked to the original culture of the students and showed in explicit or implicit way in school activities.

In this sense, this paper is a part of the PhD thesis work of Benedetto Di Paola; thesis conducted on the same research subject and treated in a more specific manner.

2. Theoretical framework and research questions

What we analyze when we discuses about of the binomials *mathematics-culture*, fundamental relation for this work and for a large tradition of research in Didactic of Mathematics, is not only the presentation of specific techniques through witch a certain group of people treat the mathematical knowledge; but a critic discussion of possible correlations between the cultural context on witch these people live and the treated mathematical concepts that are elaborated and obtained. This is the approach that we are following, even if in a first approximation, in our work. As we said in the first paragraph, we are interesting in the analysis of the reasoning schemes adopted by the student in particular algebraic problematic situation, strictly interpreted in relation to particular aspects of students *culture* and their logical-linguistic argumentation.
For what we said before, and with the aim to discuss the treated problematic, the "research tools" that we decide to use are referred to:
1. the role of the history of mathematics as an instrument of observation and analysis of multicultural learning/teaching situations: the case of the Nine Chapter as canon for the construction of mathematics (1st Cent. B.C.-1st Cent. A.D.);
2. the role of natural language in the development of mathematics in the history of thought. (Spagnolo, 1996, 2002);

The first reference linked to the *historic and historic-epistemological analysis of mathematical thinking*, could be, according to us, useful to study the different patterns of reasoning (to deduce, to conjecture, to demonstrate) in the European and Chinese cultures. This kind of analysis is conducted with the typical argumentations of history and epistemology and it is the basic reference for all the work. We underline that the choice to study the Chinese mathematical thought is due to the fact that the Chinese culture, as regards Natural Language, Philosophy, Logic etc., is the most distant from the western culture and so to analyze the reasoning schemes used by the Chinese student in the resolution of a mathematical problem could allows us to reflect on the differences of argumentation adopted in the two countries and, above all, it could allows us to reflect on our own cultural reference system, the Western one.
In this specific case we are discussing to the principal mathematical reference in Chinese education: the *Nine Chapters of Mathematical Art*.
Even if very briefly, we will discuss it in the section 2.1. of this paper.

The second reference is instead linked to the initial student' acquisition of the concept of variable (in a pre-algebraic sense) and so a possible interpretation of the reasoning scheme referred to this content for the development of the algebraic thought.
This aspect, according to us, it is specifically stressed to the structure of the written Chinese language and the phase of its *acquisition*. According to us, the structure of the Chinese written language has an algebraic form and in this sense can be very significant for studying the acquisition of the algebraic thought by the Chinese students from the first school years.
The problem regarding the sense of variable could be in fact, according to us, connected with some particular aspect of the structure of the ideograms in the written Chinese language and the logical-argumentative schemes adopted by the Chinese students in the class to solve sample problem regarding the idea of variable and parameter. We will present these our considerations in the section 2.2. of this paper.

In this articulated research context, we defined three macro research questions that allowed us to construct and characterize a complex experimental work on Chinese and Italian student attending Italian school from Elementary level to Upper secondary one in Palermo. As we said, the experimental work that we discuss in this paper is a part of this research.

- *Do the Italian and Chinese students, in the resolution of particular problems, put in evidence different resolution strategies reported to the effect of their origin culture (Natural Language, logical-argumentative schemes, algorithms, etc…)?*
- *Is it possible to underline these differences in the acquisition of the algebraic and pre-algebraic thought?*
- *Can the study of such differences help the understanding of the phenomenon of teaching/learning in multicultural situation?*

To be able to interpret the comparative study between the Chinese thought and the Italian one in situations of teaching/learning in multicultural perspective, we refer to the studies of J. G. Gheverghese (1987) and U. D'Ambrosio (2002). For the observations regarding the languages and particularly the differences between the Chinese language and the Indo-European languages we refer to the following works: Chemla (2001), Needman (1981) and Granet (1988).
The principal theoretical reference for the methodology of the experimental work is the Brousseau`s *theory of the Situations* (Brousseau 1998) that was very significant to analyze the specific phase of verbalization and socialization of cognitive styles in such multicultural milieu.
To put in evidence the socialization of the cognitive styles (*phase of validation* of the a-didactic situation) became then the carrying element for the comprehension of the phenomenon. Accepted the principle by which the education comes to be "realized" around the student, considering then the social and physical milieu in which he lives, the Didactics of the Mathematics has to build on the different experiences offered by the contemporary presence in class of different cultures, each one with the own mathematical inheritance, with the own mathematical knowledge, to a cultural exchange and a mutual enrichment.

The topic we deal with allows also a series of transversal theoretical reflections that need to a consider as broad as possible frame of reference that considers not only the motivation/emotional side of the didactical activities but also the role of such a didactical methodology, centred on playing activities. (Piaget, 1976, Brousseau, 1998). It could "disrupts" that pre-arranged context expected and feared by the pupil in which he carries out mathematics.

2.1 The Nine Chapters on Mathematical Art (*Jiuzhang suanshu*), typical Chinese reasoning schemes and possible East Asian Identity in Mathematics Education.

As we said in the previous paragraph, the first approach to the research was brought ahead trough a historical-epistemological analysis of a possible comparison of different representative texts of the two cultures: the *Nine Chapters of Mathematical Art* (as the principal reference for the Mathematics in Chinese education) and the "*Elements*" of Euclid (as regard the Western mathematics).
This study allowed us to underline, in a first approximation, a different historical approach to a mathematical thought (according to the binomials *mathematics-culture*) as regards logical structure of the proposed problematic situation, types of solution strategies for these, different type of argumentation, justification and generalization of their solutions detected in the two books etc.

According to our hypothesis we try, so, to verify in a experimental way the piagetian thesis of convergence between the historical development of the mathematical thought and the individual development one (Garcia and Piaget, 1989) of students. Particular attention in this sense was turned to the comparison of the logical-argumentative references as regard the natural language and Mathematics.

The *Jiuzhang suanshu* or *Nine Chapters on the Mathematical Art* is a practical handbook of mathematics consisting of 246 everyday problems of engineering, surveying, trade, and taxation. It played a fundamental role in the development of mathematics in China, not dissimilar to the role of Euclid's *Elements* in the western mathematics. It is so principal reference for mathematics in Chinese education, a canon both for the construction of mathematics (1st Cent. B.C.-1st Cent. A.D.) and for the teaching/learning of the same in the various historic periods. Among the most notable is the commentary of Liu Hui (263 A.D.) presented in the collection of the Mathematical Canons of the Tang Dynasty (618-907 A.D.).

The key concept that organizes the description of the *Jiuzhang suanshu* is the concept of "*class*" or "*category*" (*lei*) that plays a fundamental role in the commentaries. The elements that we find relevant to understand the specificity of the book and so of the related culture are so: the *problems* and so the typology of the problematic situation putted on the book and judged relevant for the Chinese culture for the time of the book, the *modus operandi* described in the book (the "procedure" (*shu*), the algorithmic in the term sense intended by Chemla (2004, 2007) that are useful to classify, understand and so describe the categories), the *calculus instruments*, the *demonstrations* (in the Chinese sense of term), the *epistemological values*.

The structure of the book is gradually articulated from the simple given of a problem (*wen*) related to a particular category, to solve it, "generalizing" step by step, through an *analogical* reasoning, through a *variable mutation*[2], the proposed situation and defining hence a general solution strictly connected to the proposed contest in a holistic vision (Nisbet, 2001). So, it is through a work on the procedure that is possible to define the situation classes. The solution process is an abductive process where deduction and induction are together in a unique reasoning scheme. The perfection is defined in terms of simplicity and generality through a global vision of the problematic.

As Nisbet declare "*The social worlds of East and West today reflect to a substantial degree their origin in Chinese and Greek culture, respectively…the social differences influence cognitive processes…we might expect to find cognitive differences among contemporary peoples that parallel those found in ancient times*.

Some of the differences that Nisbet puts in evidence are:

-the relationship between the field and the object, and the perceived relations among events;

-the organization into categories and covering rules, instead of organizing in terms of similarities and relationships (typical for the Chinese culture);

-apparent contradictions, Westerners resolve the situation by deciding which of the two propositions is correct, whereas Easterners are inclined to find some truth in both propositions. Westerners thus emphasize non-contradiction, whereas Easterners value the "Middle Way.".

2.2 Observations on the Chinese written language

From the mathematical point of view, as we said before, our research is aimed to study the initial student' acquisition of the different grades of the concept of variable (in a pre-algebraic sense) and so an interpretation of their reasoning scheme referred to these for the acquisition of algebraic thought.

Many national and international researches discuss the algebraic performances of Chinese students as better than the Western ones in many aspect. As they declare, the Chinese students appear very good student solving algebraic problem, applying variable, parameter and finally equations. It seem that they solve algebraic or pre-algebraic exercises in a very quickly and *mechanic* way, memorizing the algorithmic of solution and proving to generalize it step by step.

[2]According with the Chinese philosophy in which nothing is clear divided in white and black, neither the colors interpreting the circle *Ying* e *Yang*. "The oriental dialectic welcome the possible contradiction inside on a logic reasoning since only through these the verity is known. (Nisbett, 2003). The fundamental principles that regulate the oriental dialectic are so verifiable on:

a) *Principle of mutation*: the reality is a process subjected to a constant mutation;

b) *Principle of contradiction*: since the mutation is constant, the contradiction is also constant;

c) *Holistic principle*: since all the thing varying continually and it is always in contradiction, nothing, in the human life as in nature, is possible to understand independently.

According to us and our experimental studies they have a parallel control of the semantic and syntactic view of various posed mathematical problems/exercises (and in particular on algebraic problems, defined in relation to the school grades and the age of the students), a significant way to "read" a problematic situation that can be seen better than the Western students.
The question is *Why*?

This aspect, according to us, it is specifically linked to the structure of the written Chinese language and on its phase of acquisition (from the first school years for the students that study directly in China and with the help of the parents for the Chinese guys that live and study in Italy).
According to this hypothesis, we tried to study this aspect in a experimental manner, observing the behaviour of Chinese and Italian Elementary school students in front of a sample "algebraic problem in which this aspect could be visible.

Before to discuss the didactical experience and the results that we obtained, it is significant to discuss briefly some aspect of the Chinese written language in relation to its algebraic form as regards the internal rules of composition, that are stressed to the idea of variable and parameter. In this sense we referred to research works of Chemla (2001), Needman (1981) and Granet (1988).

In Chinese, the construction of the ideograms are classified in different categories or "*meta-rules*" of composition. The ideogram presented, in the Chinese writing, is one of the composition rules of the fundamental characters. Needham reports the classification in six classes and he discuss them in this way:
a) *Hsiang hsing*, lett. «Forms of imagines» (pictograms): *tree* 木; *sun* 日;*moon* 月; *mountain* 山;*horse* 马; *bird* 鸟; crow乌 (*it like* 鸟 *"bird", but missing the dot in the head; the eye is invisible because a crow's eye is black like the feathers*);
b) Chih shih, lett. «Indicators of situation» (indirect symbols);
c) *Hui i*, lett. «Union of ideas» (composition by association or logic composition). 80% of the ideograms are of the associative kind (Needman, 1981). They represent a sort of mental equations as semantic combinations of two or more characters that are composed by association. We could find different examples for this:
- 男[nán] *man*= 田 (*tián*) "field" + 力 (*lì*) "strength.
Such equations constitute a semiconscious mental foundation for whoever is acquiring familiarity with the language." (Needham, 1981, pp. 35-36, vol. I).
- 好[hǎo] *good* = 女(*nǚ*) "woman" and 子(*zǐ*) "child";
The two components combine to represent the meanings "good" and "like";
- 林 [*lín*] (森林 *sēnlín*) forest = *tree* 木 + *tree* 木 (plus 木). Two 木 (*mù*) trees side by side.
- 休 [*xiū*] *stop, rest* = 亻(人 *rén*) + 木 (*mù*) *tree*. *A person stopping to rest under a tree.*
d) Chuan chu, lett. «Transferable sense» (symbols that is possible to interpret reciprocally).
e) Hsing sheng, lett. «Language or sound». These characters are defined in a determinate general manner: the radical is associated to a phonetic sign to indicate the category on which we have to find the meaning of the word. So a lot of words with the same sound are written without confusion. (Needham, 1981, p. 38).
-园 [yuán] *garden* = 口 (*wéi*) "surround", suggesting a garden fence, and (full form:) 袁 *yuán* phonetic or (simple form:) 元 *yuán* phonetic;
-袁 *yuán* or (simple form:) 元 *yuán* phonetic, and 辶(辵 *chuò*) "go" (to go far) = "*far*"
f) Chia chieh, lett. «Loan» (caratteri fonetici in prestito). The formation is much similar to the precedent case but the way to construct the character is different.

The idea of a variable (as *thing that varying*) and a parametric system inside the composition of many characters could be defined through some simple examples:
- *Gǔ* (古) = as unitary ideograms "*old*" composed by "*ten*" and "*mouth*" (in reference to the Chinese philosophy That which has passed through 十 ten 口 mouths, i.e. a tradition dating back ten generations) strictly licked to other different characters licked with it by a semantic or phonetic units:
-固 = *to harden* (annoyed and hardened), with the radical 31: 口
-枯 = *to fade* (annoyed and done harden) "From 木 (*mù*) "*tree*" and 古 *gǔ* ("*old*") phonetic.
-古 "*old*" also it is suggested the meaning, "*withered*"
-故 = *reason*, *cause* (aged, dried him and fixed him) with the radical 66
-姑 = *mother-in-law* (elderly woman "*dried him*") with the radical 66 to the left.

-箇 = *solid thing and hardened*,
-(個 = *old men*).

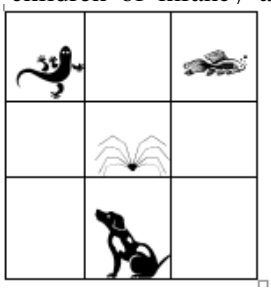
- 田 (*tián*) "*field*". That it is linked to other 138 different characters:
-里 [*lǐ*] "*village*": *From 田 (tián) 'field' and 土 (tǔ) 'earth'. "Village of 25 or 50 families; place of residence; (the length of the side of the said village:) length measure of about 600 meters" Since the adoption of the metric system, a lǐ is exactly 500 meters. The word lǐ meaning 'inside' in its full form is written with 里 and 衣 (yī) 'clothing', either 裏, split with 亠 on top and the rest below, or 裡, with 衤(衣) on the left side. The simple form is just 里 without 衣* (Karlgren, 2002)
-果 [*guǒ*] "*fruit*": *field + tree*

These characteristics of the written language put in evidence an internal research to a use of a common strategy to define "different characters" in which the *radicals* part assume a role of a parameter (in a mathematical sense of the terms) and it can vehicle the meaning or the sound of the character. It seems to us a sort of research of a possible *fundamental algorithmic* to construct different "words" and so to read and to write these in a continuous parallelism, in a continuous relationship, between "serial thought" and "global thought" on each single character.
The ideogram is formed therefore from two meaningful parts that give a new meaning, but at the same time one of the parts also has phonetic value and it communicates the sound.

This observation could give use the possibility to underline that the reasoning pattern inducted from the written natural language could brings naturally, unconsciously, the Chinese people to use (in different contest) some pre-algebraic reasoning schemes that, as we said before, gives them the possibility to find a first approach to the sense of variable inside the written Chinese language.

3. Methodology and first results

In this paper we refer to a particular game experimented with Chinese children of infancy and elementary school, *Sudoku/Magic box* opportunely simplified.
The game is referred to the *box/matrix* showed in the figure beside.
We proposed it in the classroom with other five different images of animals on the cards and a series of rules for the composition/solution of it:
1.each animal cannot be in the same line or column of its enemy (we presented the enemy animals);
2. each animal has to appear in the square only once;
3. each student has to insert in the box, all of the nine possible different animals showed in the image cards;
4. the solution has to be only one.



This is one of the possible games for a first approximation research, conducted in a multicultural milieu with Chinese students and also pupils from other countries, into the relationships between the "serial thought" and the "global thought" in the reading and understanding of the problem and the use of the "variable" (as position and single animal of the box) to solve the problem.
We involved in the experiment about 95 children (13 Chinese students) aged 3-10; the age range was chosen to investigate in the broadest possible way the different behaviours and different verbalizations of the pupils in this situation.
The experimentation was divided in two phases:
1. situation/game with children of the infancy School "Ferrara" of Palermo and of the of the Primary School "Costa G." of Palermo, first two years, to observe through quantitative and qualitative analysis (classroom experiences videotaped), the behaviours enacted by the students and the different playing strategies and the recurrent reasoning of Italian and Chinese students;
2. semi-structured interview (videotaped) to two foreign (Chinese) students, inserted in the Sicilian scholastic context at the Elementary school, regarding the same situation/game.

As we previously said, the game was chosen and adapted according to a series of critical reflections and researches previously carried out within the GRIM on the same topic. To structure the game, we

considered particular linguistic aspects that characterise the structures of Chinese written language, referring particularly to tabular representation of the phonetic characters (the matrix as mathematical reference (Needham 1981, 46)), the possible interpretation of an ideogram as union of single elements or unitary character (global/serial vision), possible use of reasoning schemes typical of Chinese students on this type of problematic situation (Cai, 1999, 2007).
The selected data, are analyzed both quantitatively, through the analysis of the protocol, and qualitatively with single case studies. For the quantitative analysis we used the software for inferential statistic Chic (Classification Hiérarchique Implicative et Cohésitive).
According to the results, the game of *Sudoku/Magic Box* seems confirm, even though it is a first approximation, the results previously discussed in other research works: compared to Italian students, Chinese pupils present a different kind of reasoning in the following items: problem reading data, data organization, "type" of language used to describe the solution…
We can therefore consider the situation/game as a first good instrument of investigation. The theoretical framework of Brousseau's theory, as regard the methodology of investigation and the key concept of observation, was significant for the aim of this study and central for the particular setting of the class. In this sense, the collected data relative to the Chinese students, seem to confirm a concrete, pragmatic behaviour, already highlighted in the works of Chemla (2001) and Spagnolo (2002); behaviours strictly bound to procedural thinking, to algorithm through which students use each single case (each animal proposed in the game) not only as simple procedural description (each case as a particular problem) but also as a representative of all the possible series, connected through the construction of an algorithm; typical reasoning of construction adopted in the written Chinese language and, as we said before, discussing on the *Nine Chapters of Mathematical Art*, as typical *modus operandi* of the Chinese culture.
This kind of strategy is evident comparing videotapes with the data analysis; it does not appear analysing strategies adopted by Italians student.

As regards this aspect of our research there are other interesting aspects regarding the way to "read" the *box/matrix* and discuss it in its "solution".
The most evident difference between Chinese and autochthonous argumentation is that Chinese students seem to use mainly a pragmatic way of reasoning. During the game it often happens that they try in deed to show the truthfulness of a particular assertion with a sketch or a particular "operation". The Italian students instead used to justify the adopted strategy, a kind of "local reasoning", with "theoretical" reference to the scheme of the situation. The chosen "theoretical" references, result to be more and more formally rigorous during the game.
From the analysis of the strategies adopted by the students (Italians and Chinese) in the resolution of the assignment, it is also possible to underline the ability of the Chinese children to read and therefore to interpret the *box/matrix* proposed in a holistic way, with a global vision of it. They show therefore attention to the particularity of each single case, each single card image presented in the game, reading the table, in a unique vision.
They underlined immediately, as first step of the game, what was important for the solution of the game, the essential elements of the situation that were the data meaningful for the problem.

Examples of question to Chinese students were:
…We have one "non influence" animals that we can consider only at the end;
…We have animals that can be posed only in one part of the box.

If the Italian children prefer a strategy based on attempts and errors, looking for first step, the single relationships among the various image cards (animals) in the game and working on the box for lines or columns and only after in parallel, through lines and columns; the Chinese student, maybe only because, as we just said, of the relationships that is possible to find between this kind of situation/game and some of the linguistic aspect of the Chinese written language (as we introduced previously we can refer to the type of writing and reading of an ideogram), underlines a more uninhibited attitude, working immediately in parallel on lines and columns and reading so the box in an unitary way. Seems to us that the algorithmic that it is possible to find is linked to the way to "generalizing" step by step through an *analogical* reasoning and through a *variable mutation*; fundamental algorithmic of the composition of the written Chinese characters and one of the most important algorithmic of the *Nine Chapters of Mathematical Art*.

In this sense, another interesting considerations can be output from the videotaped classroom experience, in particular the interview to the two Chinese students. From this further qualitative analysis, evidently comes out how, in the two culture (Italian and Chinese), the meaning of the term "To think for

cases" is interpreted. "To think for cases" as mechanic isolated acceptation and "To think for cases" as *variable mutation* finalized to a possible formalization and generalization of a algorithmic In this sense the proposed a-didactical situation could allow critical and more careful reflections on the possible correlation between Chinese language, entirely "abstract" and with an "algebraic nature" (in the mathematical meaning) with a complex syntax, Chinese thought and mathematical reasoning (logical-argumentative problematic) adopted by students in class to solve a mathematical problem. The analyzed situation/problem could be considerate, as a starting point for future more specific and deepest researches in this context. Particular attention in this sense will be turned, in the future developments of the research to the depended analysis of the *algebraic nature* of the written Chinese language and the correlation that it could have in the main study of the difficulties showed by the student (Western and Chinese) of different grades in the acquisition of the algebraic thought: from "To think for cases" as attempt and error to "To think for cases" as *variable mutation,* as tool to generalize (in sense of "*capability of noticing something general in the particular*", Love, 1986; Mason, 1996) that is an interesting aspect as usual procedural scheme for writing and reading Chinese written characters with the meta-rules described before. Further experimental investigations will be referred to a more ample set of students. Students of Chinese ethnic groups inserted in the scholastic context of Palermo and also Chinese students inserted in their native school context.

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