

**Reference to Doctoral Thesis of Aldo Scimone:
*Mental Representations of Pupils About
an Open Historical Problem: Goldbach's Conjecture
The Improvement of Mathematical Education
from a Historical Viewpoint***

The Thesis of Aldo Scimone consists of following parts:

- Introduction
- Chapter One
 - History of Goldbach's conjecture
 - Bibliography
- Chapter Two
 - The first experimentation
 - The second experimentation
 - Bibliography

In the **Introduction** the author pointed out that the pedagogical value of open problems and conjectures is in teaching mathematics remarkable, especially for the educational methodology of problem-solving. So the aim of this Thesis is to analyze the educational value of mathematical conjectures to improve some abilities of pupils facing unsolved questions. For his experiments the author chooses the famous Goldbach's conjecture, which states: *Every even number greater than 2 can be expressed as the sum of two prime numbers.* For this choice the author has several important reasons. At the end of the Introduction the author presents a list of open problems from number theory that could be used for classroom experiments.

The first part of Chapter One is **History of Goldbach's conjecture**. Here is described in detail the history of the mentioned famous conjecture, beginning with the Goldbach's letter to Euler from June 7, 1742, Euler's reply to Goldbach, then Cantor's table, Brun's and Vinogradov's results, and finally the result of Chen Jing Run, who proved that every sufficiently large even number is the sum of a prime and a product of at most two primes. The second part of Chapter One represents a rich bibliography to the Goldbach's problem.

There are two experimentations described in Chapter Two.

The first experimentation covers a sample of 88 pupils of a high school in Palermo (in the third and fourth classroom). The students worked in pairs – for the part relating to interviews, and individually – for the production of solution protocols related to the proposed conjecture. First, the author made an analysis *a priori* and then two interviews. The data obtained have been analyzed by statistical methods (factorial analysis), using a program for PC. The results of the experiments have been illustrated by creating several graphs (implicative graph, hierarchic tree, similarity tree and the factorial analysis by SSPSS). According to their approach to the problem, using the obtained data, the author has made three main categories of the given pupils. (see p. 45). Partial conclusions of the first experimentation are on page 48 (see Some final observations).

The second experimentation has been made in some classrooms of primary, middle and high schools in Piazza Armerina. At primary school the pupils were given two questions:

How can you obtain the first 30 even numbers by putting together prime numbers of the table you have just made? (for the 1th phase)

Can you derive the even numbers obtained by summing always and only two primes? If it is so, can you state this is always the case for an even number? (for the 2th phase)

At middle and high schools he asked the following question:

The following assertions is always true? It always possible to resolve an even number into the sum of two primes? Let you argue your assertions.

Analogously, as in the first experimentation, the author has made an analysis *a priori* and then the qualitative one, and, using obtained data, quantitative analysis of the results of all experimentations were done. The results of analyses mentioned have been illustrated in some graphs.

The final conclusions are shown at page 70. The author writes that the second experimentation has pointed out a characteristic behaviour of pupils at all degrees of the observed schools, while facing Goldbach's conjecture. There are, of course, differences between pupils of the schools mentioned. The author described given differences in detail.

From what was said above, it is clear that Mr. Aldo Scimone is able to organize and realize a significant experimentation in the theory of teaching mathematics. As Mr. Scimone writes at the end of his work, the results of the Doctoral Thesis imply several questions on the behaviour of pupils versus "demonstrative process" which can give rise new experimentations in order to comprise even better metacognitive process in the learning phase of pupils.

In my opinion, the Thesis by Aldo Scimone fulfils all the conditions for gaining the PhD degree; therefore, it is recommended.

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