Ass. prof. RNDr. Vladislav Rosa, PhD., Department of Mathematics Education, Faculty of Mathematics, Physics and Informatics of the Comenius University, Bratislava

Opponent report

Mental Representations of Pupils Open Historical Problem: Goldbach's Conjecture. The improvement of Mathematical education from a Historical Veiwpoint

Applicant: **Aldo Scimone**
Supervisor: **prof. Filippo Spagnolo**

The Thesis consists of three parts and appendixes. In the introduction the author expresses an idea that history of mathematics problems is an alternative way of the description of history of mathematics. He gives a stress on general pedagogical value of open problems, hypotheses and assumptions in mathematics teaching, particularly in methodology of problem solving. He forms the main goal of the PhD Thesis – analysis of educational value of mathematical hypotheses for the improvement of abilities of pupils confronted with unsolved tasks through observing of the consistent transformation pupils' attempts from argumentation to proofs. He presents a lot of open problems and hypotheses from Theory of numbers as sources or impulses of internal development not only this mathematics discipline, but also mathematics itself. Through the simple argumentation and fascinating process of substantiation he justifies a choice of the historical Goldbach´s conjecture: "*Every even number greater than 2 can be represented as the sum of two primes.*"

The first chapter is addicted to the history of Goldbach´s conjecture. It attractively links the Goldbach career together with his secular written and also personal contact with many important mathematicians, especially with Euler. By means of samples of mutual correspondence and a high number of works dealing with different problem modifications he created an appealing complex view the most important contributions to the solution of Goldbach´s conjecture practically from its declaration up to present. Bibliography related to this chapter shows a considerably high number of 95 works.

The second chapter describes experiments realized on different levels. In the first experiment, which was realized with 88 pupils attending the third and fourth year of study (16-17 years) of secondary school, the author used the method of individual and matched activity. Pupils working individually were expected, within two hours, to answer the following question:

a) *Using the enclosed table of primes, the following even numbers can be written as a sum of two primes (in an alone or in a manner more)? 248; 356; 1278; 3896*
b) *If you have answered the previous question, are you able to prove that it occurs for every even number?*

The pupils working in couples were expected, within an hour, to answer this question (in a written form and only if they have agreed):

Is it always true that every even natural number greater than 2 is a sum of two prime numbers? Let argue about the demonstrative processes motivating them.

In both cases the procedure was acoustically recorded and the transcript of those records with comments is presented in this part of the chapter.

The second experiment was carried out in three levels: pupils from the primary school (6-10 years), pupils from lower secondary school (11-15 years) and pupils from higher secondary school. The experiment was carried out on the lowest level in two phases: In the first phase the pupils could answer this question:

How can you obtain the first 30 even numbers by putting together prime numbers of the table you have just made?

In the second phase, the pupils created small groups and tried to answer the following question:

Can you derive the even numbers obtained by summing always and only two primes? If it is so, can you state this is always the case for an even number?

The pupils from lower secondary school solved the following problem within 100 minutes:

Is the following statement always true? "Can an even number be resolved into a sum of prime numbers?" Argue your claims.

The procedure had four phases:

c) discussion about the task in couples (10 min.)
d) individual written description of a chosen solving strategy (30 min.)
e) dividing of the class into two groups discussing the task (30 min.)
f) proof of a strategic processing given by the competitive groups (30 min.)

Pupils from higher secondary school solved the same problem like the pupils from lower secondary school in the same way and within the same time limit.

Individual works were analyzed (a-priori analysis), the identification of parameters was carried out and those were subsequently used as a basis for the characteristics of pupils' answers. It enabled to do a quantitative analysis of the answers, to establish an implicative graph (graph functionality), hierarchical diagram, diagram of similarities and also factor data analysis. The analyses, graphs and diagrams (or trees) are, together with conclusions, part of the evaluation of each experiment. In this chapter the author also lists an unusually high number of used sources – 80 works, 28 of those are publications of his supervisor!

The theme of the PhD Thesis is actual and attractive. Also just running international monitoring, that measure a level of mathematical knowledge and which have an enormous domestic and international response in many countries of the world (PISA), positively show the significant influence of motivation on approached performance of pupils. The chosen way – the method of investigative processing of the history of the chosen topic in the first chapter and the method of description of the experiments, chosen styles and types of analyses in the second chapter - shows the high level of author's theoretical and application preparation for scientific work. Apart from the high technical level of the work there is also a brilliant (see pragmatic, understandable, transparent) style of the PhD Thesis that enhances its reading attractiveness.

From the concept point of view there could be two reservations related to the introduced Thesis:

1) theoretical part of the Thesis – the first chapter, without didactical evaluation of the content which faces to the possible utilization of the topic in order to improve

mathematical culture (e. c. future teachers during pre-gradual preparation, teachers of mathematics from praxes during their in-service training, pupils of last years of higher secondary schools in creation of the content of chosen mathematical subjects) fades out partially self-contentedly;

2) practical – experimental part of the Thesis, despite technically excellent and no mistakes containing processing with the utilization of high-tech PC tools of data evaluation and analysis, has a lack of concretely form working hypothesis which should the realized experiments confirm or disprove.

The above mention reservations on no account diminish a high technical level of the Thesis. The turned conclusions outgoing from the realized analysis plastically demonstrate characteristic approaches pupils of primary and secondary schools to problem tasks. These knowledge are original (new) and in some cases rather surprising. They confirm the necessity and importance of further experiments that would lead to better recognition of meta-cognitive processes, which are decisive for successful mathematics teaching and also performances of pupils.

I consider the introduced PhD Thesis as a contribution to the theory of mathematics teaching and at the same time as an impulse for its further development. Its quality highly fulfill the requirements listed in the Decree No. 131/1997 of the Law Code of the Ministry of Education of the Slovak Republic about postgraduate study. Therefore I recommend
to accept the Thesis and after its successful defence to grant the applicant the academic degree PhD.

Bratislava, October 31st 2002.