COMENIUS UNIVERSITY BRATISLAVA

FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS

Department of Algebra, Geometry and Didactics of Mathematics

Academic discipline: 11-17-9 The theory of teaching mathematics

Teaching mathematics to non-sighted students: with specialization in solid geometry

Doctoral thesis

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Contents

1 Introduction

Can you imagine that you are not able to see, even for a moment? Hard idea? Yes. The vision is central to our biological and socio-cultural being. The faculty of vision is our most important source of information about the world. The largest part of the cerebrum is involved in vision and in the visual control of movement, the perception and the elaboration of words, and the form and color of objects (Adams and Victor, 1993). As for the socio-cultural aspect, it is almost a commonplace to state that we live in a world where information is transmitted mostly in visual wrappings, and technologies support and encourage communication, which is essentially visual.

Nowadays, we notice use of mathematics in lot of disciplines, the serious mathematical grounding is necessary not only for prospective mathematicians, but it begins to be popular also at humane sciences as sociology, psychology, linguistics or philology. We are also witnesses to rapid expansion of information technologies that require new technicians all the time, whose education is based on mathematics as well. Mathematics, as a human and cultural creation dealing with objects and entities quite different from the physical phenomena (like planets or blood cells), relies heavily (possibly much more than mathematicians would be willing to admit) on visualization in its different forms and at different levels, far beyond the obviously visual field of geometry, and spatial visualization (Arcavi, 1999).

If we follow the mentioned facts, we cannot wonder about the attendance of visually impaired people who would like to engage in study of mathematics. Thus, it is needed to create acceptable conditions for studying and deal with problems, which visually impaired people are encountering, as far as everyone should have the same right of education, regardless of disability.

The changes in society, inflow of liberty and humanism, caused the integration of handicapped people (Italy, 1985; Slovakia, 1993) have became an actual problem and one can partly speak about it as fashion trend that is carrying its advantage and limitations.

These facts, as well as author’s experience with working with visually impaired pupils have inspired us to pay more attention to study of mathematics of visually impaired people.

We have specified the following **aims**:

- to find out the attitude of visually impaired people towards mathematics by interviewing them;
- to detect their ability to solve mathematical problems and consequently to compare the approaches and strategies of their solving obtained from both non-sighted and sighted people;
- to describe the actual situation of teaching mathematics to visually impaired students in Slovakia;
- to focus on geometry and observe how visually impaired students are able to manage it;
- to propose possible solutions/tools for teaching space geometry that will be determined not only for visually impaired pupils and their teachers but also for teachers who are teaching integrated students at common schools;
- to highlight an assessment of Theory of didactical situation in specific milieu.

The realization of determined aims were carried out in 2 phases:

Theoretical phase focused on study of human eye and its behavior and study of history of reading codes for the visually impaired. We have devoted to the personality of visually impaired child, its development, using of other senses and to the communication with visually impaired child during the education. We have mapped teaching of mathematics to visually impaired students on each level, in the sense of presentation of an overview of actual situation in Slovak schools. We have spoken also about the problem of Braille notation, in the concrete of problems concerning mathematics. Particularly we dealt with Slovak norm and its limitations for notation of mathematics. Hence, we studied requirements for the suitable notation and semantics of mathematical languages in the sense of possible universal Braille notation creation for mathematics.

Finally, as the last topic, but not the least, was to get familiar with Activity theory as a potential tool for describing and evaluating our realized experiment. We concerned about the newest knowledge of Didactics of mathematics including the Theory of didactical situations as the suitable tool for describing the teaching process in point of view of integrated visually impaired student.

Experimental phase consisted of conversation, observation, work with visually impaired students and their teachers and of pre-experiment. That all has been the base for formulation of research hypotheses that were verified experimentally (New experiment, chapter 6) and consequently evaluated; all in the accordance with three phases of an experiment – preparation, realization and evaluation.

H1: The sighted and non-sighted pupils perceive the space and its objects in different ways. The point of view on the space geometry of visually impaired people is point of perception and it is dynamic. The point of view on the space geometry of sighted people is static.

H2: Based on the senses the non-sighted pupils are able to differentiate and name basic geometric figures and solids.

H3: When exploring new room and objects in it, the non-sighted are using several senses; sense of touch, smell and ear; while sighted rely only on sight.

H4: The non-sighted pupils will describe objects in the space (shape and position) better and more exact as sighted pupils.

H5: The non-sighted pupils have better imagination about position of the objects in the space as sighted pupils and so they build more precise scale model of the room, even if they build it on the basis of given audio record.

2 General overview

2.1 How does the vision work? [1]

In order to be familiar with the process of vision we present the way it works and consequently, some of the diseases that may lead to the blindness, which is interesting for us in the sense of the sight losing history. So, we can speak about imagination of visually impaired people that is connected with the time period when they lost the sight.

The process of vision takes more than 65% of all pathways to our brain to work. It takes up more brainpower than any other one thing that we do with our brain at a given time. This is because vision is not really a single, solitary happening, but a whole bunch of things happening at the same time. Vision is so complicated it involves 20 visual abilities. It is far more than just seeing objects clearly, but also involves processes such as how we move our eyes together, how we focus, how we achieve depth perception, how we perceive the world around us, how we process, store and recall information, etc. This is why we say vision is a dynamic process. Vision is actually developed like walking and talking. It is learned over time from birth by our experiences and how we react and solve problems. The visual skills we learn early provide the foundation for later visual complexities. Any weak link in the visual process can affect the outcome, especially if the visual system is under stress.

The human eye, elegant in its detail and design, represents a gateway to the vision. The eye processes light and takes mental "snapshots" of images, which are then developed in the brain. In order to create vision, all parts of the eye must work together as a team. The eyeball, or **globe**, is spherical in shape and about 2,5 cm across. It houses many structures that work together to facilitate sight. The eye is comprised of layers and internal structures; each of them performs distinct functions. The outside layer of the eye is comprised largely of a tough, white, protective tissue called the **sclera**. The sclera helps maintain the shape of the eyeball. At the front of the eye is an equally tough but

[1] This chapter was processed according to Internet sources presented in Bibliography as The Human eye and Eye diseases.

clear structure called the **cornea**, which is responsible for letting light into the eye and bending it. Going from outside to inside, the next layer of the eye is the **choroid**, which carries the blood supply necessary to nourish the eye's internal structures. Finally, there is the layer called the **retina**, lining the inside of the eye, which is sensitive to light and receives stimulation to its specialized cells.

In order for vision to take place, a sequence of processes must occur involving the structures within the eye and the brain:

The first part of this chain is that light rays must travel through the eye to ultimately focus on the retina. There are a number of structures involved in the bending or refracting of light so that it focuses properly. Light first passes through the clear cornea at the front of the eye, and then through a watery substance (very much like thickened water) called the **aqueous humor**, which fills the small chambers located behind the cornea. As light continues on its pathway it passes through the **iris** (named for the Roman goddess of the rainbow). Iris is a beautifully colored and textured ring-shaped muscle that that gives the eye its color. The hole in the center of the iris, called the **pupil**, dilates and constricts to control the amount of light entering the eye. By contracting or relaxing, the iris can change the size of the pupil to compensate for changing lighting conditions. It changes the size of the pupil from very small (about 2 mm) to large (about 8 mm). The next structure light will penetrate is the **lens**, another a bit of clear, stiff, jelly-like tissue, shaped like a large lentil (about 10 mm in diameter) that is attached to muscles which contract or relax to change the shape of the lens. The lens can squeeze tight into a ball or be stretched flat, allowing us to shift our focus between near and far objects in response to the need for clarity. Once through the pupil and lens, the light then passes through the larger posterior (back) portion of the eye that is filled with a clear liquid called the **vitreous humor**. From there, the light will come to the **retina**.

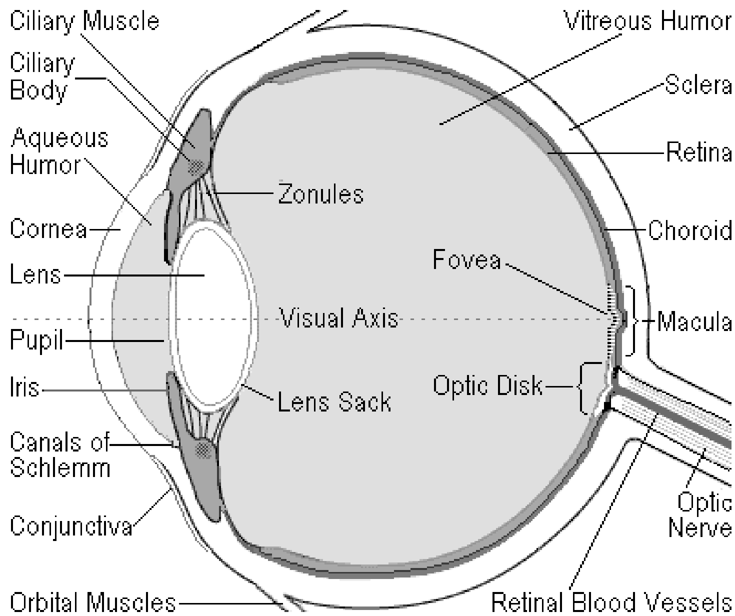


Fig.2.1: The anatomy of the eye

The retina is responsible for converting light into neural signals that can be relayed to the brain. The retina consists of a team of five types of cells whose role it is to collect light, extract basic information about color, form, and motion, and pass the pre-processed image on to centers in the brain. These cell types are ***photoreceptors***, ***bipolar cells***, ***horizontal cells***, ***amacrine cells*** and ***ganglion cells***. They are arranged within the retina in three layers, from the back to the front.

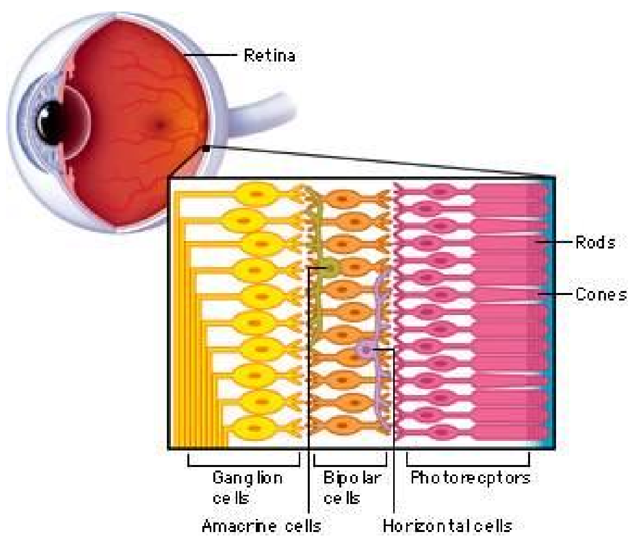


Fig.2.2: Layers of the retina

Photoreceptors convert light signals into neural impulses that are relayed to a variety of other cells types in the retina for processing. The ganglion cells at the front of the retina are the final relay station in the eye, and they pass signals into the brain via the optic nerve. Photoreceptors are divided into two subtypes, *rod* and *cone cells*, named for their shape. Rod cells (numbering about 100 million) are very sensitive to changes in contrast even at low light levels, but consequently are imprecise in detecting position (due to light scatter) and insensitive to color. Rods are generally located in the periphery of the retina and used for night vision. Cone cells (about 7 million in number) are high-precision cells that are specialized to detect red, green, or blue light. They are generally located in the center of the retina in a region of high spatial acuity called the **fovea**.

Even if all of the structures of the eye work perfectly, what we know as vision cannot happen without the brain's interpretation of the electrical impulses sent by the retina. The **optic nerves** within each eye meet in the front part of the head at a point called the **optic chiasm**, which functions like a cloverleaf on a highway. All the fibers from the left half of each retina turn towards the right side of the brain and vice versa. The end result of this crossing is that the left half of the brain looks at the right visual world, and the right half of the brain looks at the left visual world. This all works out because the right side of our brain controls the left side of our bodies and vice versa.

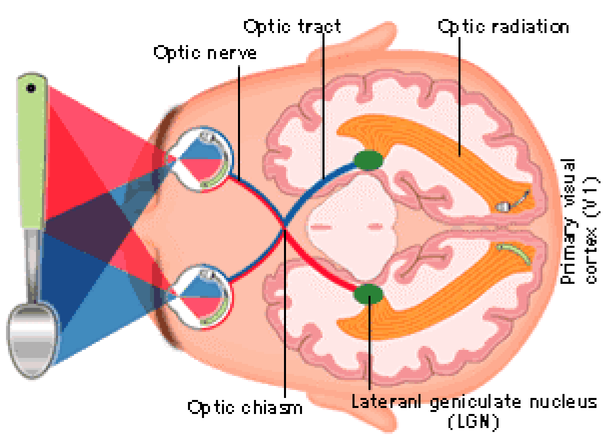


Fig.2.3: The visual pathway

137 million photoreceptors send the information via the optic nerves that transport electrical impulses to the brain where they are interpreted in the **primary**

visual cortex (V1). On the way to primary visual cortex, fibers of optic nerves enter a nucleus in the center of the brain called the **thalamus**. The thalamus acts as a central depot for information coming into and going out of the cortex, and it has centers specialized for different types of information. The center that deals with vision is called the **lateral geniculate nucleus** (LGN), a layered structure with cells that respond to form, motion, and color. Fibers from the optic nerve enter the LGN, where streams of information about the visual image are further separated and then sent on to the primary visual cortex. Primary visual cortex is responsible for creating the basis of a three-dimensional map of visual space, and extracting features about the form and orientation of objects. Once basic processing has occurred in V1, the visual signal enters the secondary visual cortex, which surrounds V1. Secondary visual cortex is principally responsible for perceiving color and the relationships between form and color. The location of every centrum that manages our vision in the brain we can see in figure bellow.

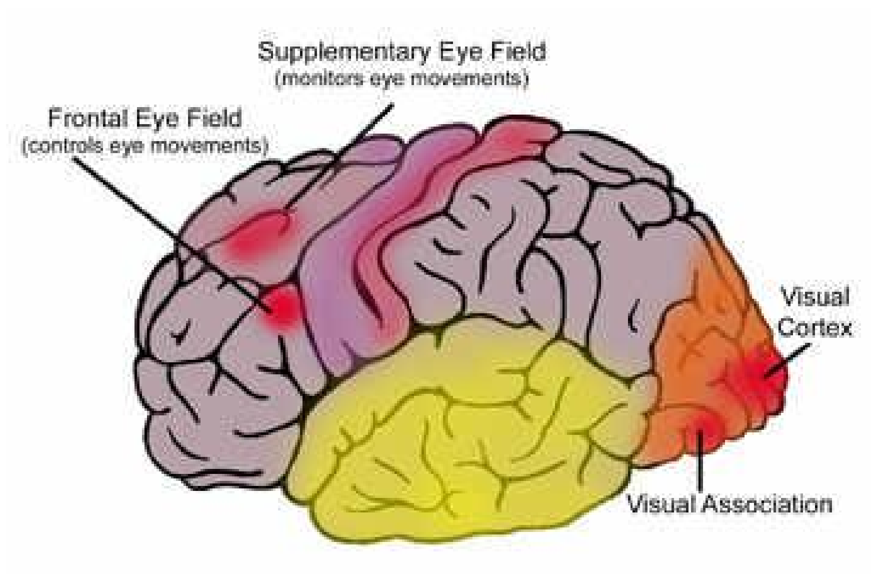


Fig.2.4: Eye’s field in the brain

2.2 Diseases of the eye

When all parts of the visual system are working, the eyes can move together, can adapt to light and dark, perceive color and accurately evaluate an object's location in space. They are sensitive to differences in contrast, and can also provide detail vision, which is measured as visual acuity. By convention, we know "normal" visual acuity to be reported as 20/20. As the bottom number of this expression gets higher, it tells us that the vision is poorer than "normal." For example, the start of the range known as "legal blindness" is represented by the visual acuity finding of 20/200. One way to understand the meaning of this finding is that the eye being tested sees at 6,10 meters what the "normal" eye would see at 60,96 meters. People whose vision is in the category of "legal blindness" may still be able to use vision to do some of the things they need to do.

All eyes are not the same, nor are they all perfect. Some eyeballs are too long or have too much focusing power, causing the person to be **myopic** (nearsighted). Others are too short or have too little focusing power, and the result is **hyperopia** (farsightedness). Some eyeballs may have uneven curvature, called **astigmatism**. Options for correcting these "mechanical" problems are standard eyeglasses, contact lenses or refractive surgery. Other problems may be caused by disease or injury, and are not correctable by conventional means. People whose vision is irreversibly impaired due to diseases such as macular degeneration, glaucoma, cataract, diabetic retinopathy and others can be helped by vision rehabilitation.

Cataract

- Opacity or cloudiness of the crystalline lens, which may prevent a clear image from forming on the retina, often resulting in blurry vision, poor night vision, or sensitivity to light.
- Although commonly a result of aging (senile cataract), some people are born with them (congenital cataract). Trauma, medication, and long-term inflammation can also cause cataracts.

Glaucoma

- It is a group of diseases associated with increased intraocular pressure in the eye, because of too much aqueous humor present in the eye. The pressure damages the optic nerve and retinal nerve fibres, which communicates vision from the eye to the brain.

- Age, family history, diabetes, those who are extremely nearsighted, and people of African descent are all highest risk factors for glaucoma.

Macular Degeneration

- It is a medical condition where the light sensing cells in the macula malfunction and over time cease to work. There are two basic types of the disease: Standard Macular Degeneration and Age Related Macular Degeneration. Macular degeneration that is not age related is most commonly caused by an inherited condition. In macular degeneration the final form results in missing or blurred vision in the central, reading part of vision. The outer, peripheral part of the vision remains intact.

Xerophthalmia

- Greek for dry eyes, is a medical condition in which the eye fails to produce tears. It may be caused by a deficiency in Vitamin A and is sometimes used to describe that lack, although there may be other causes. It can eventually lead to blindness.

Trachoma

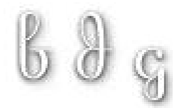
- Trachoma is an infection of the eyes that may result in blindness after repeated re-infections. It occurs in places where people live in overcrowded conditions with limited access to water and health care. Trachoma spreads easily from person to person and is frequently passed from child to child and from child to mother within the family. Infection usually first occurs in childhood but people do not became blind until adulthood.

2.3 History of reading codes for the visually impaired[2]

The education system of the visually impaired began in 1784, when Valentin Haüy opened the world's first school for the visually impaired, the Royal Institution for Blind Children, at 68 Rue Saint-Victor in Paris. That time it had twenty-four pupils; school accepted only students of either noble birth or great intelligence. The school taught several practical trades: weaving, knitting, spinning, shoemaking, basketry and rope making, as well as basic academic subjects. Within two years, the Academy of Music would sponsor benefit concerts for the school while Haüy kept the royal funds flowing by taking the children to Versailles to entertain the king at Christmas with demonstrations of reading, arithmetic, and using tactile maps.

School had also its own print shop run by the students. The first method of printing books for them was a system of characters resembling the Latin alphabet - the Roman Line Letter Type. Valentin Haüy discovered this method accidentally, while watching the process of the ordinary press. He observed that sheets fresh from the press and printed only on one side showed the letters in rather sharp relief, and he at once set about enlarging the characters for the fingers, and having them printed the reverse of the usual type, so that they would read from left to right on the sheet. He reasoned that, since the characters could be felt, the only thing needed was to enlarge them so that the visually impaired could distinguish them by touch. Accordingly, in his first experiment, he simply had the types reversed and made larger, with the result that the letters read from left to right on the sheet. He did not ask what kind of characters could be most easily read with the fingers, and this was his initial mistake. He laid down the fundamental principle that we must establish all possible contacts between the non-sighted and the sighted, and he pushed his idea to the extent of insisting that the letters of their alphabets should be similar in appearance, forgetting that it is not really the eye nor the finger that reads, but the brain.

[2] This chapter was processed according to Internet sources that are related to Louis Braille and they are presented in Bibliography.

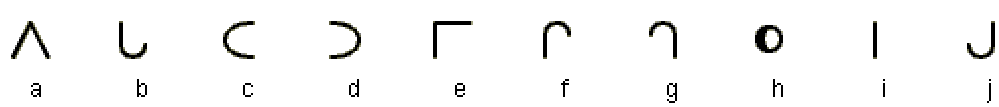


Valentin Haüy's embossed print letters were widely spaced and used surprisingly ornate fonts.

Haüy's method was spread rapidly from Paris to Great Britain, Germany, Austria and America. It was hailed as a path to deliverance for the visually impaired; but the rejoicing gave way to disappointment when it was discovered that from one-third to one-half of the visually impaired in the schools could not decipher Haüy's Line Letter.

The chief defect of his method was that he used curved forms, which the visually impaired reader finds extremely difficult. Size was his first consideration, not shape. He did not know that the more elaborate a raised letter is, the less easy it is for the non-sighted to recognize, or that the finger detects sharp angles much more quickly than curves, or that points like the period are perceived very clearly.

Countless modifications of Haüy's Line Letter were attempted in France, England and other countries with the object of discovering a more legible type; but none of them was successful, as is shown by the rapidity with which they were tested and thrown aside. Only one linear type has survived to this day - the angular Moon Type, invented by an Englishman, William Moon. This is a very large and distinct print adapted to the fingers of the visually impaired adult, who need something to practice their touch on before they learn Braille. Some of the Moon letters resemble the letters of the Latin alphabet; others are simplified letters or other shapes. The Moon alphabet is easier to learn than Braille, particularly for people who loose their sight in later life.



some letters of Moon type

So obvious was the failure of these early systems that in 1832 the Scottish Art Society offered a gold medal for the most practical method of embossing for the sightless. Fifteen typographic systems made their appearance, in which angular forms predominated, and there was one which somewhat resembled the dot system of our day. In spite of the fact that points are distinguished more readily than lines, the jury of awards decided upon the Alston form of line type.

Alston type (1830's)

It requires a philosophic spirit to understand this apparently foolish disregard of the most workable way to overcome the handicap of blindness. The jury had a sincere desire to keep the non-sighted and the seeing as close together as might be in their reading and writing and in all the activities of life. Besides, little was known about the sense of touch in those days. Educators and inventors were under the delusion that the loss of vision renders the other senses far keener and more alert. They supposed that what looked good to the eye would with modifications be equally acceptable to the fingers. Among the many who advanced theories concerning the visually impaired people, Diderot alone pointed out that while they may acquire the same amount of knowledge as the seeing, their processes of acquiring it would probably be quite different. He wrote his famous essay on the non-sighted about the year 1749; but his wise words fell upon barren soil. Those who took an interest in the handicapped were governed by tradition and custom.

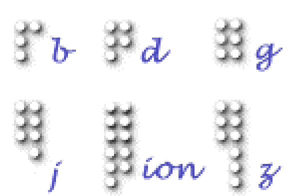
Meanwhile, on 4th January 1809 at Coupvray, near Paris, would be born Louis Braille, the fourth child of a saddle maker. In 1812 at the age of 3, Louis injured his eye in an accident while playing with his father's tools. Over the next year, the infection spread to the other eye, and Louis Braille lost all of his vision.

When Louis was nine years old, his father entered into correspondence with the Minister of the Interior regarding curriculum and whether it might be beneficial for Louis to attend the Institution Royales des Jeunes Aveugles in Paris. After lengthy consideration, the Minister nominated Louis for attendance to the school. At school, Louis applied himself to his studies and was an accomplished student.

Haüy, now an old man, had not been inside the school in years. Losing control of the school in the aftermath of the revolution, he had been forced to flee France. Before his departure, he rescued one of his most promising students, Rémi Fournier, from the chaos at the Quinze Vingts. Together they spent over a decade in virtual exile working with visually impaired students in other European countries, including Russia. Schools for the visually impaired were an idea that time had definitely come, with Liverpool (1791), Vienna (1804), Berlin and St. Petersburg (1806), Amsterdam (1808), Dresden (1809), Zurich (1810), and Copenhagen (1811) appearing in rapid succession using many of Haüy's ideas and methods.

Large influence on Louis Braille's future had Charles Barbier de la Serre. Barbier fled the Revolution by spending some time in the United States as a land-surveyor in Indian territory and returned to France by 1808, where he joined Napoleon's army and published a table for quick writing followed a year later by a book describing how to write several copies of a message at once.

The French army under Napoleon had been defeated for the last time at Waterloo in 1815, but before that, they had nearly conquered Europe and were considered even by their enemies to be the best army in the world. In his own war experiences, Barbier had seen all the troops in a forward gun post annihilated when they betrayed their position by lighting a single lamp to read a message. A tactile system for sending and receiving messages could be useful not only at night, but in maintaining communications during combat with its unique terrors for artillery crews. Louis was about 12 years old when Charles Barbier brought his writing system, called "sonography" to the school. Louis immediately saw the potential, as well as the problems with the system. The Barbier system was based on phonetic soundings and 12-dot cell (6 high and 2 wide, arranged in a rectangle).



Some of Captain Barbier's original sonographic code.

The character thus obtained was large, unwieldy and more than a fingertip can cover, though capable of an almost unlimited number of combinations. There were no punctuation marks, numbers or musical signs, and there were lots of abbreviations, because the cells stood for sounds instead of letters.

Louis decided to work experimenting with the code on his own. In October, 1824, Louis, now 15, unveiled his new alphabet right after the start of school. He had found sixty-three ways to use a six-dot cell. He cut Barbier's character to two and thus produced his well-known 3 by 2. At 17, Louis became the first visually impaired apprentice teacher at the school. He taught algebra, grammar, music, and geography. Despite his busy schedule, he kept tinkering with the code. By 1828, he had found a way to copy music in his new code. In 1829, at age 20, he published *Method of Writing Words, Music, and Plain Songs by Means of Dots, for Use by the Blind and Arranged for Them*, his first complete book about his new system.

After some next slight modification it reached its present form in 1834, and is the system, which has since borne his name. We do not find, however, nor does it appear, that Louis Braille, in arranging his system, paid attention to any other considerations than one, namely the methodical arrangement of the letters of the alphabet.

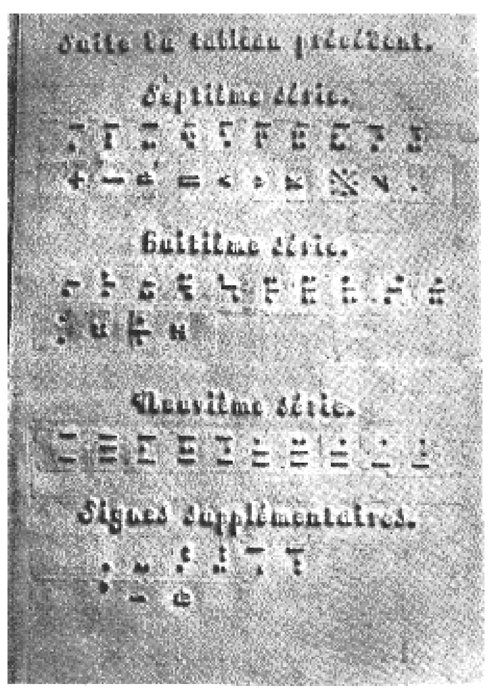


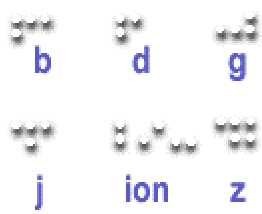
Fig.2.5: Braille tablet from1829

The Braille system was used at the school but is was not so easy to apply it everywhere. 2 years after Louis's death, in 1854, France adopted Braille as its official communications system for visually impaired people. The Braille system spread to Switzerland soon after but encountered tremendous resistance in England, Germany and America, and often for the same reason: Braille's seeming opacity to the sighted because of its lack of resemblance to print (Actual Slovak Braille alphabet is attached in Appendix 4, Italian Braille alphabet in Appendix 5).

About 1859 or 1860 the Braille system was introduced to America and was taught with some success at the St. Louis School for the Blind. In 1868, the British and Foreign Blind Association came into existence and having brought Braille into this country, gave to it a powerful impetus by printing and disseminating books in that type.

It will be seen that both in England and France there was, even at so late a date as 1878, considerable diversity of opinion as to claims of Braille as the best method of reading and writing for the visually impaired.

In America the same thing occurred. William Bell Wait, superintendent of the New York Institute for the Blind, introduced a now almost forgotten code called "New York Point" in 1868.



New York Point (1868)

"War of the dots" divided the schools of the visually impaired into two camps for until the issue was finally settled around 1916. One that used NY Point and another that advocated American Braille. Both sides lost the war because the British stood by the Braille code it was using. Ultimately, the wealth of code already available in the British Empire and the desire for a unified English language code lead to the acceptance of the Braille code we use today.

2.4 Braille notation of mathematics

The study of Braille notation in mathematics was one of the topics of the master degree thesis (Kohanová, 2003). In following we briefly describe the main principles.

Nowadays, there are two systems of Braille codes used over the world, traditional 6-dot Braille and then 8-dot Braille. While 6-dot Braille can intrinsically represent only 64 combinations of raised dots (unique characters), 8-dot Braille gives you possibility to represent 256 characters, which is better. As far as it still was not enough, so called "tools" like prefixes or "switches" are being used.

Using of these special "tools" Braille can support a much larger set of characters, because they are changing the meaning of the following character. This comes at some cost, which means that the basic character that can be represented in Braille can have different meaning in different contexts. For example: dot 1 ⠁ can be interpreted as "a", "A", "1", etc (depending on the prefix cell). This ambiguity causes difficulties with reading and writing Braille.

Unfortunately, this is not an only problem. Each country has its own Braille standard (some countries have more than one) based on their national alphabet. And here we can meet some other difficulties due to the miscellaneous representations of national characters (letters with accent or other diacritical marks). Mathematical notation for the visually impaired is really national specific. In addition, most of the mathematical symbols in Braille may consist of several Braille cells[3]. Consequently, mathematical Braille notation is rather extensive.

Since there does not exist an universal Braille code for mathematical notation which is used in whole world, it is clear that for visually impaired students it is not easy to read, leave alone to study, foreign technical materials even they speak not only native language.

As the last, but not least, we have to mention one thing, which is known in general, namely reading and writing mathematics is fundamentally different from reading and writing plain text. While Braille is adequate for the representation of text, it is not up to the task of representing mathematics. The two basic reasons for this are:

[3] One Braille cell consists of 6 raised dots organized in three rows and two columns.

Linearity

Text is linear in nature while for example mathematical equations are two-dimensional. (e.g. quadratic equation: $ax^2 + by + c = 0$)

Character Set

Text can be generally represented in a somewhat limited number of characters, which normally include upper and lowercase letters, 10 digits, various punctuation marks and a small set of special characters. Equations on the other hand can contain all of the normal text characters plus a large number of special characters.

2.5 Slovak Braille code

In Slovak Republic we are using the Slovak Braille code dated from April 19th 1996. It contains general rules for writing literary text and some subject related rules. The rules for notation of mathematical text contain 77 symbols, namely the characters for

symbols (9)
all types of brackets (7)
indices (5)
relational operators (16)
powers and roots (3)
fractions (5)
sets (7)
sums and products (2)
mathematical analysis (3)
mathematical logic (8)
geometry (12)

It is evident that this amount does not fit the bill. If we focus upon elementary level one might reckon that the number of characters for notation of mathematical symbols is sufficient. On the other hand, it is not completely satisfactory for the secondary level where the scale of mathematical knowledge increases very sharp. For example, the Slovak norm does not say how to write following symbols:

reverse implication "$\Leftarrow$"
minus sign of Theory of sets "$\setminus$"
combination of functions "$\circ$"
is identical "$\equiv$"
it is not identical "$\not\equiv$"
does not divide "$\nmid$"
it is not parallel "$\nparallel$"

It includes only 3 symbols of mathematical analysis, concretely integral, derivation and infinity. If we consider all the possible symbols, which might appear by sequences and series, convergence, limits, derivations and integrals, it is obvious that 3 symbols are not enough. Moreover, it would be also helpful to have contracted notation for minimum, maximum, goniometric functions and variation, combinations and permutations, as the other foreign norms have.

It also does not contain rules for notations of vectors, logarithms, matrices, determinants, etc. There is only general rule, saying to follow black print notation. Applying this rule, we can write *vector a* as character a with symbol arrow as the index above the character.

a , / 3 o :	- represents vector a in Braille, where
a	- represents the letter a (b1)
, /	- represents the beginning of index above (b6,34)
3 o	- represents the arrow (b25,135)
:	- represents the end of index (b156)

On the other hand, it could be written as bold letter a. There are not any rules for writing bold, italics or underlined text in Slovak Braille code.

If there are several ways how to write down the same symbol in black print and the set of Braille symbols is very limited, it would be highly advisable to enrich Slovak Braille code for mathematics applying rules precisely describing the missing areas of mathematical notation. If we will do so, visually impaired student does not have to introduce the way of notation of these problematical symbols by him/her self anymore. It is maybe very easy readable and comprehensible for him/her (when s/he invented the missing way of notation), but probably not for the others, they might not to understand it.

2.6 The language of mathematics and Braille notation

There were few experiments trying to build up a universal Braille notation for mathematics. By such an activity one can consider two different points of view. First, requirements for the suited notation (Schweikhardt, 1998); second, semantics of mathematical language (Spagnolo, 2003). Herein we present both.

Requirements for the suited notation

The requirements for the suited notation of mathematics for visually impaired consist of four theorems:

Theorem 1: A mathematical notation for the visually impaired people must be readable by the finger.

The notation must help the reader to comprehend the complex terms. The sighted are able to understand type of many mathematical objects at a glance. Examples may be roots, fractions, absolute value or vectors. A visually impaired reader, however, has to touch one character after the other to fill a puzzle, which finally makes clear what the content of the expression is. One could build the mathematical notation for the visually impaired for example upon how expressions are spoken. So it should allow concluding the type of the object already from the first character.

Theorem 2: The number of characters in a mathematical expression should be as low as possible.

The comprehension of complex terms increases with the compactness and clearness of the notation. That is the reason for the variety of mathematical symbols. A mathematical notation for the visually impaired should follow the same principles and allow the same degree of abstraction as the notation of the sighted.

Theorem 3: Tactile symbols should be understandable intuitively.

Many mathematical symbols include a visual component which helps to understand object and its meaning. For example the arrows in conjunction with limits or vectors are very helpful. Also some semantic and syntactic symmetries should especially be kept in notation for the visually impaired (e.g. ( ), < >, or { }).

Theorem 4: Integrated learning and working of non-sighted and sighted students and colleagues should be supported.

In the latest time there is a tendency to integrate visually impaired students into the class of sighted students. Hence, the situation requires that the notation of visually impaired should be very easy transformable into the form that is comprehensible for the sighted and so it allows their communication.

Semantics of mathematical languages

As regard to semantics of mathematical languages it is necessary to reckon with following reliable statements.

- Sometimes, in mathematical communication both of concerned (the one who transfer, the one who receive) are not very clear conscious of mathematical language they are communicating in.
- The creating of important semantic categories shouldn't be accelerating, but these categories should be built patiently on multiplicity of experience.
- One should consider that even recognizing of grammatical rules of the

natural language (its logical analysis) is for the students very complicated and long time taking process. Secondly, one should keep in mind also its epistemological severity.

- The analogy of access to the adoption of various mathematical languages and their epistemological analysis is surely applicable and helpful.

2.7 Teaching of visually impaired

2.7.1 Characteristic of visually impaired pupil

One of the most important conditions of successful pedagogical activity on visually impaired student is knowledge of her/his personality and understanding of her/his abilities, as well as the respect for her/his handicap.

Differentiation of visually impaired pupils

According to ophthalmologic criteria we differentiate the set of visually impaired students into: *unseeing*,
partially sighted,
short sighted and
binocularly wrong.

Unseeing pupil has defect of both eyes and no visual sensations. In this category belong also pupils who are at least able to distinguish light and darkness. Among unseeing and practically blinds is also the pupil who cannot use the sight as leading analyzer by space orientation and at education.

Partially sighted pupil (pupil with rests of the sight) belongs to the group of visually impaired pupils and it is actually boundary case between blindness and purblindness. Partially sighted pupil can use the rests of the sight by some of the actions, but the visual analyzer also doesn’t have the leading position at education.

Purblind pupils falls into markedly heterogeneously group that has common sign of reduced ability of visual perception. This ability exhibits mainly as decrease of the speed and accuracy of visual perception and as quick tiredness by visual work. The ideas of purblind pupil are often not exact, not clear or mispresented. Despite mentioned facts the sight is leading analyzer at education.

Binocularly wrong pupil belongs into group that consists of amblyopic and squint pupils. In case of this pupil it doesn't come to creation and fusion of equivalent images of perceived objects that causes the problems of perceiving the space. The leading analyzer is the sight.

The World Health Organization classifies the visual impairment as shown in the Table 2.1:

Category	**Grade**	**Criteria** (based on visual acuity [visual field] in the better eye)
Normal vision		
Normal vision	0	20/25 or better
Near-normal vision	0	20/30 to 20/60
Low vision		
Moderate visual impairment	1	20/70 to 20/160
Severe visual impairment	2	20/200 to 20/400
Blindness		
Profound visual impairment	3	20/500 to 20/1000 or visual field less than 10°
Near-total visual impairment	4	worse than 1/1000 or visual field less than 5°
Total visual impairment	5	no light perception

Tab.2.1: Classification of visual impairment

Evolution and teaching of visually impaired child

The evolution of visually impaired child is running the same as the evolution of sighted children basically. Loss of the sight that has very important role in the life of human brings some evolutionary curiosity. It is related to the fact that after loss of the sight the system of reception and recognition of reality is rebuilt. Lot of attributes of objects and phenomena have visual character that can't be perceived by visually impaired child (e.g. light, colours). There are difficulties also with spatial orientation, position, direction, distance, movement of objects, etc. All these problems originate because the sight is distant analyzer, which means, it enables to identify spatial units and their distribution remote. The *hearing* is also distant analyzer but it does not enable identification most of the spatial attributes. In this case the hearing is substituted for the *touch* (contact analyzer). On the other hand, there are many obstacles by haptic sensation, for example: physical attributes of object (high temperature), chemical attributes (combustible material), position of the object (distance, move) and psychogenic obstacles (fear of accident, bad feeling at contact with object). In these cases also typhlotechnical instruments are used (e.g. indicators, detectors).

The process of formation of sensual experience of visually impaired persons is retarded. With the help of teacher and special pedagogical instruments child adopts system of knowledge and step by step develops ability to use aural, kinetic, cutaneous and others analyzers. So the sensual base is build that makes possible to develop more complicated psychic processes - perception, imagination, memory, thought and speech.

In the cognitive field there are limited possibilities of remote perception, more proportion of partial perception and complications with processes of analysis and synthesis, retarded tempo and inaccuracy of perception.

As a consequence of loss of sight occurs also problems in emotional sphere, for example feelings of inferiority, overestimation of resources, fade from community of sighted. Hence might arise troubles by process of socialization and integration.

For sighted students are concepts related to spatial objects and optical effects mostly clear, because they can base on their visual experience. When visually impaired student forms concepts like the see, the light, the perspective, s/he has to base on very consistent description of the teacher or on information mediated by sighted. Very significant are processes of formal logical thought (comparing, sorting, and meditation according to analogy) that could be accelerated to the level of cognitive processes of sighted and eventually also equal.

The new object or event is perceived in different way by visually impaired than by sighted. The reason of it is the specifics of haptic sensation and also the fact, that s/he focuses attention to the other attributes of object (s/he is perceiving more irrelevant attributes) than sighted person. The job of the teacher is to decoy her/his attention from irrelevant things and advertise the fundamental characters.

In order not to stagnate in process of learning it is important to deepen and repeat everything regularly. The memory represents very strong factor here, because of specialty of evolution of cognitive processes. Thus memory of visually impaired is on advanced level than sighted, steady activity of memory helps them to store and recreate situations, objects and their details that sighted persons do not receive so often (Jesenský, 1973). Secondly, the way of thinking in analogies is very intense and strongly supported by memory. Therefore, extraordinary attention is paid to development of the memory.

2.7.2 Communication with visually impaired child

Communication with visually impaired person is in some points of view problematic. Main source of conflicts are two ultra opinions:

1. Visually impaired human is person who is helpless and depending on help of other people
2. Visually impaired human is common citizen without any limitation.

We have to be conscious of necessity to communicate with visually impaired as with human of equality before position, however limited because of her/his ocular handicap.

Major communicative senses are hearing and sight. For the visually impaired that are hearing and touch, which are compensating the loss of the sight. The man obtains by sight till 85 % of information. Thus, natural consequence of ocular handicap is information deficit. The hearing (distant analyzer) is for the visually impaired sense of information and orientation, it enables him/her not only orientation in the space - in the environment but also in the time - in the story. Visually impaired person perceives by hearing surrounding world and people, whose voices and sounds are characterizing ambient space and actual social climate or story's situation. The touch (contact analyzer) compensates the sight in the field of graphical communication. Haptic sensation represents non-verbal expression of information that is sensible by touch - models, relief and other typhlographic pictures.

Verbal communication through speech is for the visually impaired sensible in complex range. Markedly s/he feels handicap in non-verbal communication, s/he notes particular demonstration e.g. of cadence, power and coloration of voice; proxemics. By touch s/he takes in caress, hand on the shoulder, etc. However, s/he does not register visual demonstrations as looks, mimicry of the face, gestures, moves and poise of the body, which are considerable accessories of verbal communication.

There are three participants of communication at education:

1. visually impaired student
2. the teacher
3. other students

Considering visually impaired student we can contemplate about two kinds of communication:

1) between visually impaired student and the teacher
2) between visually impaired student and her/his schoolmates.

The first case is more dominant and it runs in various forms. Oral communication is irrecoverable at education; however, this fact does not valid in mathematics. Vice like, mathematics requires exactness, definiteness, totality and comprehensibility of presentation. It is very arduous only by oral communication (e.g. when modifying expression or by geometrical construction) and so it is supported by graphical way - text or picture. This connection is typical for mathematics; because of insufficient style of expression some students rather prefer notation or picture. If we are talking about graphical communication, we mean communication supported by for example relief's picture, typhlographic images and plane or space models (construction kit, cubes, skewers, paper).

In following we present some of the general rules for the teacher who works with integrated visually impaired student. The teacher exercises principles of behaviour and education:

1. the principle of prevention of visual defectiveness, i.e. adjustments of behavioural and educational conditions in order to prevent negative consequences of visual impairment;
2. the principle of correction of visual defectiveness, i.e. to go over, moderate or modify negative consequences of visual impairment;
3. the principle of integration of visually impaired students, i.e. to employ such educational measures suitable to make easier the fusion of visually impaired student with the community of sighted students in accord with individual frame of the integrated student;
4. the principle of reeducation of the sight, i.e. the systematic and multilateral usage of the rest of visual components in order to further develop the sight but without its injure;
5. the principle of compensation of the sight, i.e. to substitute absented visual information by other analyzers and by thinking at the process of education.

During the process of education teacher should also respect possible characteristics, expressions of behaviour of the student that are consequences of visual impairment, in particular:

- the absence, incompleteness or misrepresentation of visual perception;
- the absence, incompleteness or misrepresentation of visual images;
- worse quality of analytic-synthetic action that is related to worse skill of distinction;
- reduced factor of concentration;
- disorders of perceiving the space;
- disorders or insufficient level of reading and writing;
- disorders of visual-motoric coordination;
- disorders of colored vision;
- limitations of continuous visual work;
- limitations of movements and physical labor;
- inappropriate emotional reactions;
- inadequate facial gestures and body language;
- disruption of social relations;
- slower working rate;
- relatively fast coming the feeling of tiredness.

2.7.3 Overview of actual situation

The overview of actual situation of teaching mathematics in Slovak schools has been also the part of our thesis of degree. We focused on each level of education: primary, secondary and university level.

Primary schools

Most of the Slovak visually impaired children (non-sighted and partially sighted) attend special primary schools. At the lessons of mathematics they use Braille books with tactile pictures, to make notes they use electronic notebooks and for calculations mechanical typewriter. The disadvantage of typewriter is in the first place since it takes too long to get result of calculus, so pupils try to calculate in their minds and second: notation of the calculations is too verbiage and pupil is lost after a while.

Teaching of mathematics on the elementary level means first of all helping children to use and organize their experiences, which they gain from actions and interactions with the world around them. In the opinion of some authors (Csocsan, 2002) the main goal of mathematical education is to develop an awareness of numbers and coping with different relations and dimensions. The most frequent mathematical problems of non-sighted pupils are as follows:

- generalizing – finding the similarities in different activities in everyday life;
- translating activities and actions into mathematical language;
- lack of the flexibility in problem solving and in calculations;
- translating and transferring three-dimensional objects into two-dimensional iconic forms [Example: The non-sighted pupil cannot understand a geometrical drawing of a cube from a perspective view because of her/his lack of visual experiences. S/he also has difficulties in enlarging and minimizing two-dimensional forms.].

Secondary schools

There are special high schools for visually impaired students, but mostly oriented on music, some handicrafts, etc. If a student wants to come into contact with mathematics then s/he needs to attend "normal" high school. As we know, mathematics is a subject, which is important for studying not only natural sciences but it also begins to be popular at humane sciences. The direct consequence of this mathematical requirement almost everywhere causes that also more and more visually impaired students today start their education in mainstream schools, which is place, where they can study mathematics.

Because teachers of these schools are not special educated in this field they often have to use the ”trial and error” method to find out the best way of teaching their visually impaired students who are the only integrated among sighted students. Visually impaired students encounter also with lack of textbooks and study material and limited Braille notation for mathematics.

On the other side, visually impaired students of this level of education mostly do not have problems with calculations; they already know all basic mathematical operations. However, scale of mathematical knowledge increases here very sharp in all fields: algebra, analysis, and geometry. Hence, they will have to overcome a lot of other new challenges, especially with Braille notation of all new symbols. After study of system of Braille notation in several European countries we can state that more or less each of the mentioned norms suffers from lack of the rules for notation of mathematical text. Therefore, the major part of visually impaired students has created their own particular mathematical language that is adapted to their conditions and requirements. But this forms new problems, because these languages do not have to be comprehensible for people who are visually impaired students communicating with.

University

Comparing to secondary school, there is quite a different situation in math for a visually impaired student at universities. The student is supposed to have skills necessary to study - make notes during lectures, read scientific text, perform complex calculations, communicate with teachers and other students in written form, etc. There is much more independent work required.

If s/he graduated at special school/class and used only Braille notation and spoken language for before mentioned purposes, s/he will have to overcome a lot of new challenges.

There are very limited sources of scientific literature in accessible form for a visually impaired student. Therefore he should be able to read different mathematical notations.

Another way of delivering mathematical expressions in accessible, written form is electronic text document on personal/portable computer or special note taken for the visually impaired users. This sort of document usually contains linear mathematical notation with expressions built up of ASCII characters.

Visually impaired student can access this type of notation in two ways. S/he can use refreshable Braille display and read line by line corresponding Braille cells (groups of 6 or 8 raised dots) by touching or listening to synthetic voice, which reads each written ASCII symbol for him/her. The second method is more difficult for reading complex mathematical expressions, although could be, however quicker for longer text with simple mathematical expressions. The ideal is combination of both methods, when student can choose appropriate method depending on current situation (what is s/he reading, writing or calculating). This way the student is able to take notes at the lesson, calculate or pass exams without any problem. Actually, it is not true in some cases. It is startling, as we find out, that some teachers who are not very familiar with computers refuse their usage as writing tool at exam, so the visually impaired student had to pass exam verbally where his sighted schoolmates answered in writing.

Some solutions, originally dedicated to electronic publishing of scientific text documents (TeX, LaTeX, AmSTeX, HrTeX, MathML), could be red and written by visually impaired student.

Computer Algebra Systems (CAL Systems) are dedicated at the first place for algebraic calculations, e.g. differentiation or integration; solving of equations. They are also able to perform numerical calculations; visual graphs of functions, curves and 3-dimensional objects. Such software is Derive, MuPAD, MAPLE, MathCad or Mathematica. All of them contain as well a lot of functions of analysis, linear algebra, statistics, numerical analysis, number theory, graphics, etc.

They are also useful for visually impaired students, especially by calculations. It has no sense to urge them to act calculations that are often just very tedious and mechanical. That is why CAL Systems are helpful. If the commands we put into command-line are linear, it means they are fully textual and therefore suitable for visually impaired students. The other advantage is that screen-reader does not have any problem to read linear text on the screen and so, access it to the student. Thus, visually impaired students of Computer Science use CAL Systems for example for calculations during Algebra seminars. It is useful tool for calculations with matrices, which are time consuming and quite complicated. If they understood a principle, such a tool can save a time and a lot of manual work.

2.7.4 Utilization of information technologies - The Lambda system

It has been mentioned above the information technologies might be very helpful for visually impaired students who are studying mathematics, since they have largely improved the educational opportunities. The most significant requirement is for secondary school students to perform mathematical exercises rapidly and efficiently in the same way as their sighted classmates. Teachers who do not have any knowledge of Braille (usually those in integrated schools), are asking for most suitable tools using which they can get involved in learning processes directly and ensure that everything is clear and understandable to the non-sighted student. Later on, at university, it is important to have a mathematical writing system that is powerful, flexible, and compatible with most common format standards, to enable independent scientific and mathematical work to be distributed digitally.

The fact that has to be considered is accessibility. In the last period LAMBDA[4] - Linear Access to Mathematic for Braille Device and Audio-synthesis appears to supply all needed requirements. The LAMBDA project makes the provision for an integrated system based on linear code and a software management system (the editor). The code

[4] The purpose of the project is to produce an efficient system for non-sighted middle school, high school and university students to manage mathematics documents.

(Lambda Math Code) is a direct derivation of MathML[5], which is an XML[6] based language. Optimally used for Braille peripherals and vocal synthesis, it automatically converts data in real time, without the possibility of ambiguities, into an equivalent version of MathML. Both incoming and outgoing data are then converted into the most common forms of mathematical writing (LaTeX, MathType and Mathematica). The editor allows the writing and manipulation of mathematical expressions in a linear way. Few facts about Lambda:

- In our society, the need of writing mathematics texts is current and general only throughout education period, while only a small percentage of the adult people feel this necessity for professional or personal needs. Since this is true also for non-sighted people, it is predictable that a tool to write mathematics, as LAMBDA, should be used the most by young people who are *learning* mathematics.

- LAMBDA is above all, even if not only, a didactics instrument. It is the functional component which implements the strategies devised in order to make easy to read, write and manipulate text and mathematical expressions by means of vocal output and Braille display, in an educational setting.

- It is important to define the didactics requirements needed for a software writing system (but also simply typewriting) compared with Braille traditional ones. In traditional writing (tablet and punch or Braille type) the pupil is directly involved in symbol building up and this promote the full comprehension of the relation between the shape and its meaning. To go through this passage is fundamental to fully learn Braille code, as much as the pen writing represents for sighted

[5] The Mathematical Markup Language is language used for displaying mathematical notation and content, especially on the web. It is a World Wide Web Consortium (W3C) recommended standard, and has been receiving increasing support by mathematical software vendors.

[6] Extensible Markup Language is a W3C initiative that allows information and services to be encoded with meaningful structure and semantics that computers and humans can understand.

children. The change towards mechanic writing systems (as typewriter and PC) requires this phase is totally concluded and the symbol preserves its meaning independently on the system used to produce it. But, in order to do mathematics on the computer, there are other skills necessary and the teachers have to carefully verify. If they are missing, traditional tools are preferable and the passage to new technologies has probably to be put off. With no doubt, whoever is going to use the computer to do mathematics must be able to manage textual documents using the PC.

- All most frequent operations such as, for instance, opening and saving files, selecting a portion of text, deleting, correcting, copying, pasting… are performed according to Windows standard modalities and do not present any training or adaptation-related problems.

- As we start to work with LAMBDA editor, there are few possibilities to input characters and mathematical symbols (Bernareggi, 2006):

    ü *input from the menu*

    (The tag is gathered within the groups that are shown in a pull-down menu, so the user can choose the proper tag browsing the menu items. It is not a quick input strategy, but it is very easy for those users who are not acquainted with the system.)

    ü *input from a list*

    (All the tag names are sorted in lexicographic order in the list. One just digits only few characters of the element name and the list is reduced to few items, easily manageable both through Braille display and voice.)

    ü *input by short-cut keys*

    (This input is available by default for some dozens of common elements, but the list can be increased and customized according to the user needs.)

ü *input through the mouse*

(The selection of mathematical tags by mouse from icon graphic menu is implemented as well, since the mathematical editor is can be used by sighted assistants too.)

ü *input according to the context*

(In order to adhere the structure of the expression, separator or a closing tag can be automatically inferred by the system. Input technique exploits two short-cut keys to input the corresponding separator or the corresponding closing tag.)

- *Exploration strategies*

On many situations, it is important to perceive global information, related to the structure or the relation, to define every single time the most suitable paths and methods to face different issues. For example, having following expression:

$$\frac{x^2-4}{x^2+4x+4}+\frac{4}{x+2}$$

Its linear representation is:

//x^2−4ø x^2+4x+4\\+//4ø x+2\\

It is evident that in reading the linear representation it is more difficult to find specific parts and to quickly understand the relations among the structures making the expression that is an immediate operation for sighted people who use global and bidimensional exploration.

Lambda offers exploration through movement operations, in sense that one can move to the next nominator, denominator or corresponding separator or tag. The second possibility is tag structure of an expression that enables to understand overall structure of expression and to find specific parts.

The most compressed structure of expression mentioned above is:

//ø \\+//ø \\

the other depth level seems as follows:

// (   ) ∤ (   ) \\+//4∤x+2\\

This visualization modality that hides the content of the block by maintaining blank spaces is useful as well to get some information about the size of hidden blocks.

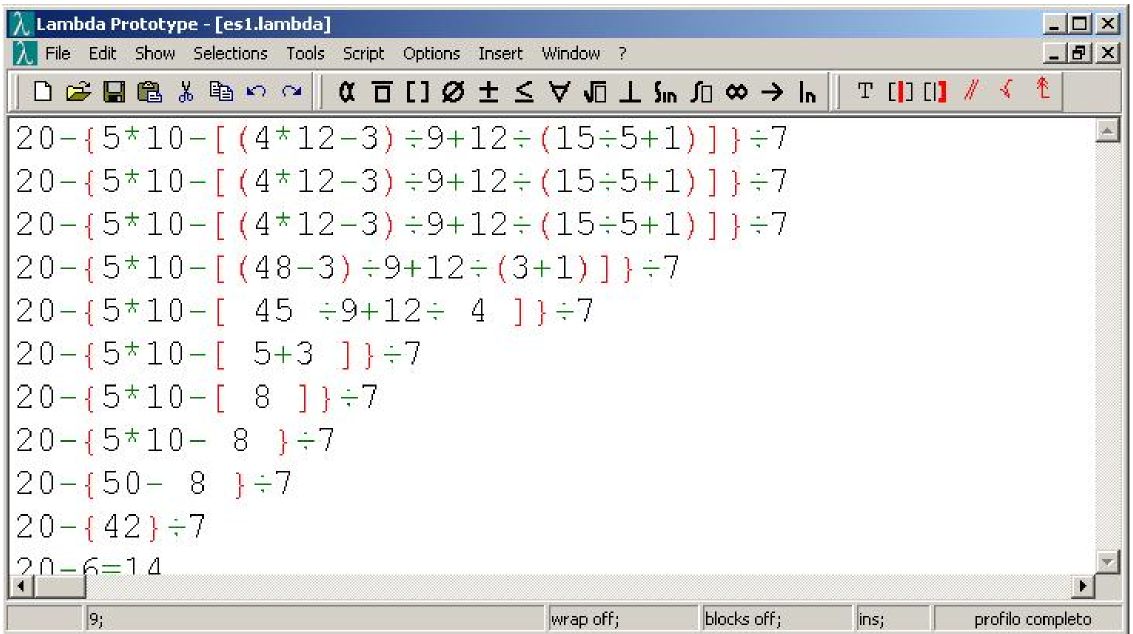


Fig. 2.6: The main window of LAMBDA

- *Error checking*

The problem of unintentional typing errors is very significant for a visually impaired user whose hands writing on the keyboard can check entered texts later on only on the Braille display. Typing error in mathematics (for instance, 3+x instead of 3-x) distorts the meaning of the expression and can only rarely be identified from the context; therefore preventing these errors is crucial.

Other errors originate from the Lambda code syntax, especially the block based structures that must always be correctly closed, by entering the proper marker. Lambda offers a solution:

- the operation of inserting close and any possible intermediate blocks is aided by the program; user gives a generic close (or separation) command and the program inserts the proper marker, based upon the context. Of course, nothing is entered, and an error message appears, if no blocks are open.

- if the user tries to leave a line out without closing all markers, the command is blocked and an error message appears;

- *Block management*

An efficient management of the blocks of linear math writing is one of the chief objectives of the Lambda system. The system recognizes blocks, namely it is capable of understanding which *close* marker is linked to each *open* marker and the inverse. So you are able to delete, edit, copy or shift from a marker to the one it is concatenated to.

Certainly, the objective of LAMBDA is not that of taking the place of the student. Rather, it aims at providing an efficient tool to allow him/her to do the same things as others do, in a different way and, possibly, with analogous efforts.

2.7.5 The experience of working with Lambda system

First we get in contact with Lambda system in December 2005 at international conference in Rome: „I don't see The problem: new prospects to access Mathematics and Scientific studies for Blind students", where the software was presented by its athours and by italian visually impaired pupils who are using and testing it. Later on, by initiative of Support Centre for Visually Impaired Students, Comenius University, Bratislava we offered to run the mathematical club for visually impaired pupils. We started to give lessons in April 2006 together with 2 university teacher-students (specialization: mathematics-informatics) at Elementary school for Visually Impaired in Bratislava. Together there were 5 meetings and 4 pupils who were attending the course. All 4 pupils were students of 9th grade, 2 of them were short sighted, 1 virtually non-sighted and 1 non-sighted.

The main aim of the course was to test the utilization and efficiency of Lambda system at Slovak elementary and secondary schools. In the case that Lambda will prove to be a significant tool for study of visually impaired students, the Support Centre for

Visually Impaired Students plans to provide the Slovak version of Lambda (at the length of the courses we used English version) and find the ways of its practical application at Slovak schools.

Program of single meetings:

1st lesson:

- basic features of the program
- movements by shortcuts
- how to open, edit, save exercise
- structured notation, shortcut F8

2nd lesson:

- priority of the operators
- double copy of the row (CTRL+D)
- working line, checking line
- use of calculator

3rd lesson:

- brackets of all types
- elimination of brackets
- automatic completion of right bracket (CTRL+K)

4th lesson:

- exponentiation of numbers
- exponentiation of compound expressions
- exponentiation of expressions with variables

5th lesson:

- fractions, nominator, denominator
- roots
- solving of simple equations

During these 5 lessons all 4 pupils had learnt to work with Lambda and they considered it as userfriendly and as helping tool to do mathematics. Hence, we decided to translate Lambda menu into slovak language and in academic year 2006/2007 to continue with courses. One of the outputs of these courses will be also methodical guide of using Lambda software for teachers and parents of visually impaired children. It is actually subject of master thesis (dissertation) of one of the above mentioned teacher student, the thesis will be submited in spring 2008.

3 Theoretical framework

3.1 Didactical situation in specific conditions

The trend of education of visually impaired persons in the world is to integrate these students among sighted ones in the common schools. It follows from the idea that in these specific conditions we find different (new) relations in the classroom. In the next we try to describe how is the whole teaching process changed in comparison to classic one. One has to distinguish between communication between teacher (T) and sighted students (SS) at the lesson and between teacher and non-sighted student (NSS), in addition, between non-sighted and sighted students, as mentioned in previous chapter. For example, there has to be specially adapted explanation of new concepts for visually impaired student. Following the Theory of didactical situation introduced by Brousseau (1997)[7], when acquiring new knowledge ("connaissance") in the frame of **didactical situation**[8], the teaching of a new notion consists of setting up its situations and carrying out interactions in which the learner can take part. It is itself, an interaction. This interaction is also largely specific to the knowledge being taught but it takes a form of a-didactical situation[9], necessarily different from the non-didactical forms in which knowledge ("savoir") is used (Brousseau et al., 1999). This result changes the entire approach to mathematics education and the education of teachers. Consequently, we can represent the **didactical system** (triangle) as a system of relationships (didactical contract) between three subsystems: educator (E → teacher: TE, tutor: TU), learner (L → sighted student: SS, non-sighted student: NSS) and knowledge (K), where the non-

[7] Theory of Didactical Situation, which fundamental methodological principle is built upon: "a piece of mathematical knowledge is represented by a "situation" that involves problems that can be solved in an optimal manner using this knowledge". The characteristic situations for pieces of mathematical knowledge can be studied or even modeled within the framework of mathematics itself, which sometimes makes it possible to use computation to predict their evolution.

[8] Situation which enables to obtain new knowledge.

[9] The part of didactical situation that enables to the student to acquire new knowledge own and consequently s/he is able to put it to use in situations which s/he will come across outside any teaching context and in the absence of any intentional directions.

sighted student has particular position in the frame of didactic situation (DS), because of the way that s/he obtains the knowledge.

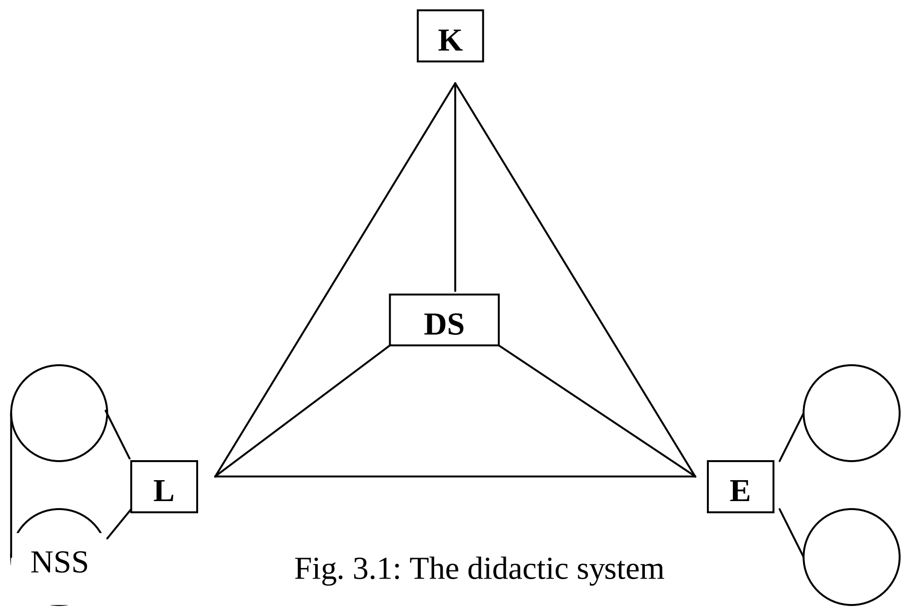


Fig. 3.1: The didactic system

In general, we think the process of acquisition the knowledge is different than the same process of sighted students. In follows, we describe the didactic triangle in the form of sides (Sbaragli, 2004), which are stated by three main vertices, focusing on non-sighted student, taking into account personal experience of interviewed people – teacher of mathematics and non-sighted students:

- *teacher (TE) – knowledge(K)*

  This side is characterized by the verb "*to transpose*", where the main activity is the first part of the didactic transposition from the *scholarly knowledge* to the *knowledge to be taught.* The question is whether the teacher is conscious of necessity of different approach that s/he has to use when transposing the knowledge to the non-sighted student. It consists in extra preparation of a-didactic situation, so the non-sighted student can also participate in activity. When making analysis a-priori, the teacher has to take into account possible difficulties of understanding and questions that may appear at the lesson and so be able to react dynamically and clear, but not at the expense of other sighted students.

- *tutor (TU) – knowledge (K)*

  This side is specific by various ways of transposing the knowledge, which are not used when teaching sighted students. Sometimes the teacher is not conscious of all that might be a problem to understand for non-sighted student, s/he finds it out at the lesson and in analysis a-posteriori. So as a tutor, s/he has to select different view on given subject and try to transpose the scholarly knowledge to the knowledge to be taught in appropriate and customized way, sometimes by using models and tactile pictures.

- *student (NSS) – knowledge (K)*

  This side is expressed by the verb *"to learn"*, where the predominant activity is *the involvement* of the non-sighted student that is characterized by nouns "*motivation – interest - volition*". S/he accepts the devolution[10] and takes personal care of her/his proper knowledge. The knowledge is so constructed by the student and finally institutionalized[11] by the teacher/tutor.

- *teacher (TE) – student (NSS)*

  This side can be represented by the verbs "*to facilitate - to advice - to guide*". When the non-sighted student faces with a-didactic situation, s/he produces her/his knowledge as a personal response to the didactic milieu by keeping the didactic contract. The teacher should allow a harmonization of the different phases of the learning process and

[10] Devolution is the action of the teacher on the student, by which the teacher makes the student to take responsibility for a learning situation or problem, and accept the consequences of this transfer her/himself.

[11] In institutionalization the teacher situates the student's production (the piece of knowledge constructed in adidactical situation) in accordance with the scientific or cultural knowledge socially accepted.

intervene only in cases of possible misconceptions or false beliefs of the student.

- *tutor (TU) –student (NSS)*

  "*Individual consultation*" – it is the name of this side. Consultation between non-sighted student and tutor is held apart from lesson, depending on needs that have occurred at the lesson. Thanks to individual approach there is time and space for deeper understanding of the piece of knowledge and its consistent arrangement into the general knowledge structure.

- *student(NSS) – student (SS)*

  This side can be described by the verbs "*to help – to correct*" and by the noun "*cooperation*" of the non-sighted and sighted student at the lesson. In case of any doubt, incertitude or misunderstanding the non-sighted student can ask for help of her/his sighted neighbour. On the other side, the sighted student (who is good at mathematics) can correct some errors or mistakes if s/he sees them on the screen of her/his neigbour's computer.

It is needed to emphasize that this model works on voluntariness and interest of the teacher/tutor, especially tutor works in her/his free time. So in real it seems like the integration of visually impaired student into common school is fictive, concerning the point of view of studying mathematics. The integration by itself has rather social aspect. Consequently, the question is: Who can be integrated and who not? Sometimes the integration might harm, on the contrary, it might help to some of non-sighted students to express them. The other remarkable thing is the question of limit. Since in Slovakia there is no standard for teaching mathematics to integrated visually impaired students on secondary level, the teacher has to determine requirements on these students by his own, on his subjective opinion.

To obtain new knowledge in the point of view of visually impaired student it is needed also to clearly specify the **didactic contract**[12] in sense what kind of compensating tools it is allow to use (notebook, Braille typewriter, etc.), which are actually becoming the part of **didactic milieu** (DM) and its levels. The mentioned tools are part of **material milieu** (MM) that activates student (S) to activity that leads to acquisition of new knowledge. Concerning the *cognitive element* of material milieu of visually impaired student we may predict that it is different than the cognitive element of sighted student.

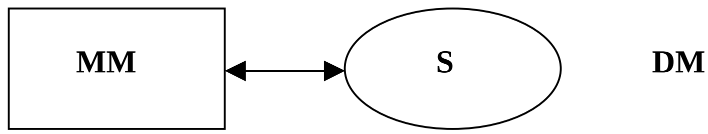


Fig. 3.2: The Didactic milieu

Hence, mathematics is described in terms of *situations* and consists mainly of "dealing with problems" in a wide sense. Teaching and learning mathematics is not considered as teaching and learning mathematical ideas, notions or concepts, but as teaching and learning a *situated human activity* performed in concrete institutions. So the old central questions in mathematics education: "How do students learn mathematics?" and "What can we do to improve their learning?" are substituted by more comprehensive ones: "What are the necessary conditions for a situation to implement the specific mathematical knowledge it defines?" and "How can situations be designed and their development managed in a given educational institution?". (Bosch et al., 2005) Considering the fact that in the classroom is non-sighted student we have to answer given questions also in this sense, which says about the way: "How we shall transmit mathematical knowledge to the non-sighted student?", "What conditions does the

[12] The relationship which determines implicitly what each partner, the teacher and the student, will have the responsibility for managing and be responsible to the other person for, specifically target on mathematical knowledge.

situation need?”. Talking about transmitting and receiving the knowledge we shall mention also the **didactic transposition**[13]**.**

This process, first pointed out by Chevallard (1985), acts on the necessary changes a body of knowledge and its users have to receive in order to be able to be learnt at school. Chevallard introduced a distinction between:

- *original* or *scholarly* mathematical knowledge (“savoir savant”) as it is produced by mathematicians or other producers;
- knowledge *to be taugh*” (“savoir enseigné”) officially prescribed by the curriculum;
- knowledge as it is *actually taught* by teachers in their classrooms
- knowledge as it is *actually learnt* by students.

This process of transposition the knowledge from researcher’s field to the student includes the stages when the:

- producer of the knowledge (researcher) suppresses all reflections, conceals the history of origination and reasons which led to the knowledge and personal influences which guided success, before communicating the knowledge to the mathematical world. We call it *depersonalization, decontextualisation* and *detemporalization.*
- teacher simulates in the classroom a scientific microsociety and creates conditions and situations within the knowledge that will appear as the optimal and discoverable solution to the problems posed. Actually, the teacher extracts knowledge from her/his social or university context and adapts it to the unique context of her/his classroom. The didactic transposition produces here some effects, such as simplification, creation of fake objects or production of totally new ones. It is process of *recontextualization* and *repersonalization.*
- students *redecontextualize* and *redepersonalize* their knowledge, they identify what they have produced with the knowledge which is in current use in the

[13] The process that isolates notions and properties, takes them away from the network of activities which provide their origin, meaning, motivation and use. It transposes them into a classroom context. (Brousseau, 1997)

scientific and cultural community. Their intellectual work is similar to scientific activity that requires some production, formulation, proving and constructing models, concepts and theories.

When talking about didactic situations it is needed to mention the **epistemological obstacles** that were introduced by Bachelard (1938) in his studies on the scientific thought and in the '70s transposed into mathematics by Brousseau. One can analyze the mathematical knowledge from a historical perspective in order to shed some light on the students' processes of construction of knowledge. Some of the faults are not caused by ignorance, incertitude, chance, but by previous knowledge, which was interesting and successful, but now is revealed as false or non-adapted (inappropriate). Obstacle is formed as knowledge of this type with its objects, relations, premises, proofs, unheeded consequence; it is immune to refusal, it tends very easy to accommodate, to take place of error of teaching or of insufficiency of the subject or of intrinsic difficulty of the knowledge. Brousseau claims that there is a logic behind students' mistakes and explains them in terms of a knowledge that suffices to solve some problems fruitfully but fails to appropriately solve others. We classify the origin of the epistemological obstacles, which come up in the didactic system according to Brousseau into:

- **obstacles of ontogenetic origin** that are related to demarcation of the subject (the neurophysiologic as well) at the time of child’s development. They are linked with pupils and their maturity; with development of intelligence and perceptual systems. During the learning process every individual develops skills and competences suitable to their mental age (which is different from the chronological age) that are related to different stages of neuron’s networks. In each individual the scheme of neuron’s links is fixed after birth, some of them have priority in the face of others links because of stimuli. Few used links are becoming passive, so the stimuli are very important for the evolution of the brain. The obstacles of ontogenetic origin can be eliminated under influence of the milieu that is result of these elements: biochemical, sensory, familiar, didactic, etc.

- **obstacles of didactic origin** that depend on didactic transposition. Every teacher chooses strategically a project, a curriculum, a method, personally interpreting the didactic transposition, according to personal, scientific and didactic beliefs. The teacher believes in the choice he made, considers it to be effective and thus proposes it to the class; but what was proven to be effective for some students, may not be effective for all the others. For some others the choice of that particular project may turn out to be a didactic obstacle.

- **obstacles of epistemological origin** depend on the nature of the knowledge itself. They have constructive role in education so one should not to avoid them. The detection of them is possible through a confrontation of the history of mathematics and today's students learning mistakes. Indeed, one of the roles of the didactician is
    - to find the students' recurrent mistakes and to identify the underlying conceptions,
    - to find the obstacles in the history of mathematics and
    - to compare the historical obstacles and the learning ones in order to determine their epistemological character.

3.2 Activity theory

From the number of many theories we have chosen Activity theory, which is suitable for us as the tool for description of realized experiment.

Activity theory originated in the former Soviet Union as a part of the cultural-historical school of psychology founded by Vygotsky, Leontiev and Luria. According to Vygotsky, psychology in the 1920's was dominated by two unsatisfactory orientations, psychoanalysis and reflexology, which was later developed into behaviorism. Reflexology attempted to ban consciousness by reducing all psychological phenomena to a series of stimulus-response chains. So he formulated a completely new theoretical concept to transcend the situation: the concept of artifact-mediated and object-oriented action. In activity theory the unit of analysis is an activity that is being composed of a subject, and an object, mediated by a tool. A subject is a person or a group engaged in an activity. An object is held by the subject and motivates activity giving it a specific direction. Behind the object there always stands a need or a desire, to which [the activity] always answer. The mediation can occur through the use of many different types of tools, material tools as well as mental tools, including culture, ways of thinking, signs and language. So human action has a tripartite structure.

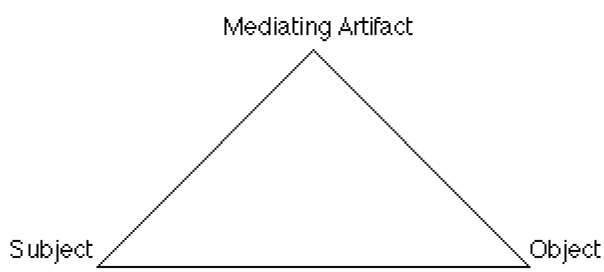


Fig. 3.3: Human action

In Vygotsky's early work there was no recognition of the part played by other human beings and social relations in the triangular model of action. Leontiev extended the theory by adding several features based the need to separate individual action from collective activity. The third hierarchical level, which Leontiev added to the theory of activity, was the level of operations. The uppermost level of collective activity is driven by an object-related motive; the middle level of individual (or group) action is driven by a conscious goal; and the bottom level of automatic operations is driven by the conditions and tools of the action at hand.

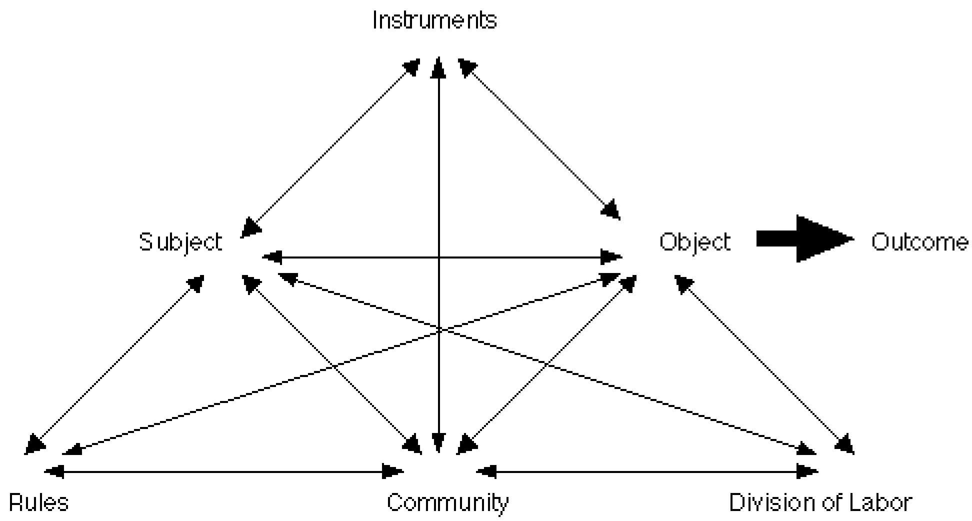


Fig. 3.4: Model of activity system

In this model of an activity system, the subject refers to the individual or group whose point of view is taken in the analysis of the activity. The object (or objective) is the target of the activity within the system. Instruments refer to internal or external mediating artifacts, which help to achieve the outcomes of the activity. The community is comprised of one or more people who share the objective with the subject. Rules regulate actions and interactions within the activity system. The division of labor discusses how tasks are divided horizontally between community members as well as referring to any vertical division of power and status.

Giving an example from school environment we can concretize the model. We may focus on work activity of the teacher. The object of her/his work is the student with her/his level of knowledge. The outcomes include deepening and consolidation of knowledge and learning of new piece of knowledge. The instruments include all typical teacher's tools as blackboard, chalk, other tools in the classroom, as well as the existing knowledge of the student and new concepts related teaching methods. The community consists of other students in the classroom, all teachers in the school and parents. The teacher may share the responsibility with community for the achievement of the object that is called the division of labor. The rules specify the kind of products, knowledge and experiences that will be approved or acceptable, access to tools and artifacts and who is permitted to do which aspects of the activity.

The same activity of teaching looks different if we take the point of view of another subject in the community, for example a parent. The object and outcomes are the same, but the rest changes. The situation also changes when we are talking about activity of learning, where the subject is the student.

The third generation of activity theory represented by M. Cole needed to develop conceptual tools to understand dialogue, multiple perspectives and voices, and networks of interacting activity systems. In this mode of research, the basic model is expanded to include minimally two interacting activity systems.

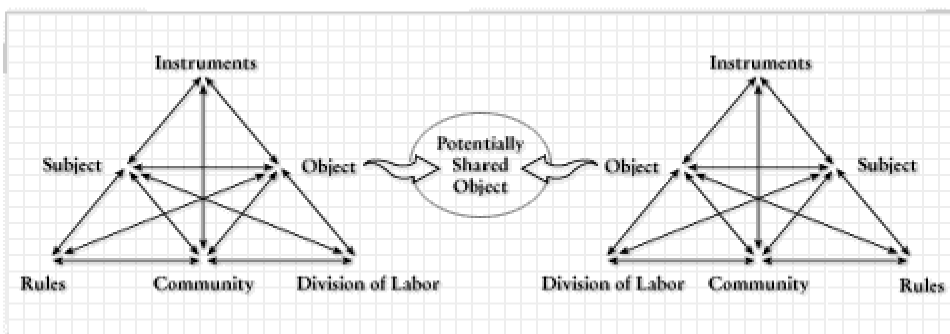


Fig. 3.5: Interacting activity systems

In general we can talk about five principles of Activity theory:

1. **Object-orientedness**

Every activity is directed toward something that objectively exists in the world, which is an object. In Activity theory the notion of object is not limited to the physical, biological, and chemical properties of entities. The properties determined culturally and socially are also objective and they could be studied with objective methods.

2. **Hierarchical structure of activity**

According to Leontiev, interaction between human begins and the world is organized into functionally subordinated three hierarchical levels: activities, actions and operations. An activity is directed at an object, which motivates activity, giving it a specific direction. Activities are composed of goal-directed actions that must be undertaken to fulfill the object. Actions are conscious, and different actions may be undertaken to meet the same goal. Actions are implemented through automatic operations. Operations do not have their own goals; rather they provide an adjustment of actions to current situations. Activity theory holds that the constituents of Activity theory are not fixed but can dynamically change as conditions changes. All levels can move up and down.

3. **Internalization/Externalization**

Activity Theory distinguishes between internal and external activities. It highlights that external activities cannot be understood if they are analyzed separately from internal activities, because they transform into each other. Externalization converts internal activities into external ones and vice versa, internalization is the transformation of external activities into internal ones. Internalization provides a means for people to try potential interactions with reality without performing actual manipulation with real objects (imaginings, mental simulations, considering alternative plans, etc.).

4. **Mediation**

Human activity is mediated by a number of tools, both external and internal. Tools are created and transformed during the development of the activity itself and carry with them a particular culture - historical remains from their development. The mediation is done by artifacts, which broadly define and include instruments, signs, language and machines.

5. **Development**

Activity theory requires that human interaction with reality should be analyzed in the context of development. The basic research method in Activity Theory is not traditional laboratory experiments but the formative experiment which combines active participation with monitoring of the developmental changes of the study participants.

3.2.1 Zone of proximal development

Vygotsky (1978) noted that the possibilities of genuine education depend not so much on the already existing student's knowledge and experience (level of actual development) as on the characteristics that are in the zone of proximal development. The student can gain the potential for knowing with the help of "more knowledgeable other" (MKO). The MKO is somewhat self-explanatory; it refers to someone who has a better understanding or a higher ability level than the learner, with respect to a particular task, process, or concept. The MKO doesn't have to be necessarily teacher or older adult. In fact, the MKO does not have to be a person at all, it could be also some electronic performance support systems, like electronic tutors that are used in educational settings to facilitate and guide students through the learning process. The point of MKOs is that they must have (or be programmed with) more knowledge about the topic being learned than the learner does.

Vygotsky considered the zone of proximal development (ZPD) as the distance between the "actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" .

The lowest threshold is the level of actual development (LAD), which contains the student's actual knowledge, skills and experience. Then follows the zone of proximal development (ZPD), which aims on cognitive change basically connected with the guided development of student's understanding. According to Tchoshanov (2001) there is one more zone after ZPD. It is a new zone – zone that goes beyond the development of understanding. It is a zone of formation of student's in-depth learning. While in ZPD the functions of comparison, reproduction, assimilation, and coping are of primary importance, in a new zone the functions of construction, generation, and creation are most important. This upper threshold of development and cognitive instruction is called a zone of advanced development (ZAD). Whereas the ZPD is the interpsychological dimension where social activity and interpersonal dialog are taking place, ZAD is the intrapsychological dimension where advanced individual activity and intrapersonal dialog is going on. When moving from LAD to ZPD, the guidance is crucial in helping student to do so. We cannot declare the same about student's shift from ZPD to ZAD. In other words, if ZPD is a domain of guided cognitive change (understanding), ZAD is a zone of student's individual (independent) activity. Hence, one might regard ZAD as a domain of higher cognitive accomplishment and creativity that can be attained by student in the process of intense individual studies.

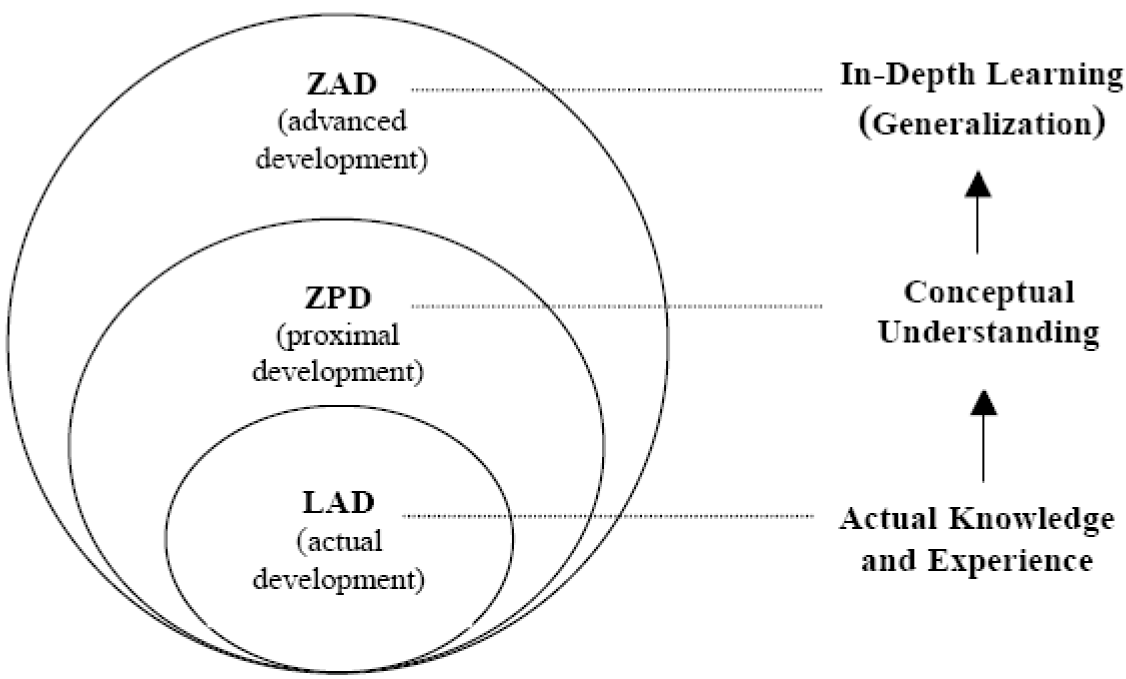


Fig. 3.6: Zones of cognitive development

3.3 Van Hiele's levels

Van Hiele (1986) published a theory in which he classified five levels of understanding spatial concepts through which children move sequentially on their way to geometric thinking. Different numbering systems are found in the literature but the van Hiele's spoke of levels 0 through 4.

Level 0 (Basic Level): Visualization

Students can name and recognize solids as total entities, but do not recognize properties of these solids. Although they may be able to recognize characteristics, they play no part on the recognition and sorting of the shapes.

Level 1: Analysis

Students begin to identify properties of solids and learn to use appropriate vocabulary related to properties, but interrelationships between different shapes and properties cannot be explained. Properties are seen as independent of each other.

Level 2: Informal Deduction

Students are able to recognize relationships between and among properties of shapes or classes of shapes and are able to follow logical arguments using such properties.

Level 3: Deduction

Students can go beyond just identifying characteristics of shapes and are able to construct proofs using postulates or axioms and definitions. The interrelationship and role of undefined terms, axioms, definitions, theorems and formal proof is seen.

Level 4: Rigor

This is the highest level of thought in the van Hiele hierarchy. Students at this level can compare and work in different axiomatic systems. Geometry is seen in the abstract with a high degree of rigor, even without concrete examples.

At each level of geometric thought, the ideas created become the focus or object of thought at the next level as shown in Fig. 3.7 (Van de Walle, 2001).

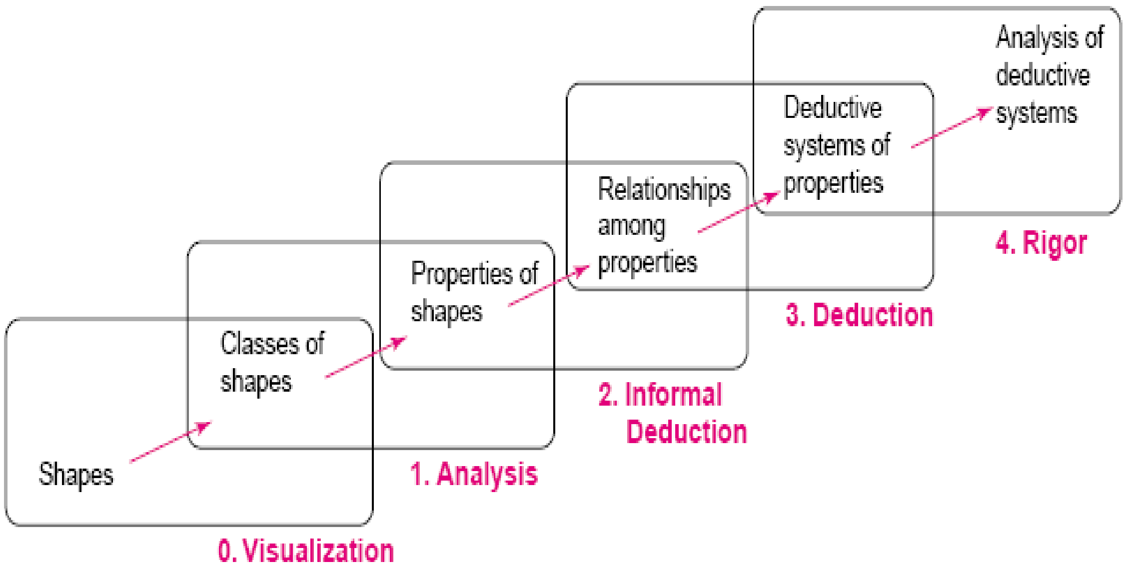


Fig. 3.7: Van Hiele's levels

According to Jirotková (2001) there are three levels of the quality of the mental picture of a perceived solid:

1. the solid is a 'personality' for the pupil,
2. the solid is unknown to the pupil, however, the pupil perceives some relationship between the considered solid and another solid which is a 'personality' for him/her,
3. the solid is entirely new for the pupil

4 Methods used in the research

Among most important and most frequented research's methods belong: didactic tests, experiment, observation, questionnaire, interview, sociometric method, projective method, method of verbal answers, metaanalysis. (Turek, 1998)

Methods that we have used in our research are: questionnaire, interview, experiment, analysis a priori, analysis a posteriori and evaluation through quantitative analysis of research's course and results.

5 Preparation and goals of the pre-experiment

Goals of the pre-experiment

- to find out the attitude of visually impaired people towards mathematics by interview with them
- to acquaint oneself with problems they have/had in conjunct with mathematics at education
- to detect their ability to solve mathematical problems, even at the moment they strictly don't have to/didn't have to deal with mathematics at all
- to compare approaches and procedure of solving problems of non-sighted and sighted people
- to investigate which part of mathematics is the most difficult for visually impaired and why

5.1 First phase

In the frame of Socrates/Comenius project [14] we carried out first phase of the experiment, which was realized in Palermo, Sicily, in March 2004. It consisted of visit of *UNIONE NAZIONALE CIECHI* **-** *UNC* (National Union of Blind) and *INSTITUTO CIECHI* **-** *IC* (Institute for Blind). It enabled us to know the situation of education of visually impaired people in Sicily. Prof. Scaccia, who is director of IC, had introduced us history of institute, its way of running, Italian Braille code and in particular Braille code for mathematics. Last version comes from year 1998.

In order to obtain information which could help us by achievement of our specified goals, we prepared questionnaire which consists of 4 problems: one problem of algebra, one problem of analytic geometry, one business problem and one problem of Euclidean geometry. Its text was as follows:

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[14] Project Socrates/Comenius 2.2.A runs between Universities in Bodoe-Norway, Bratislava-Slovakia and Palermo-Sicily. It provides mobility for future European teachers.

Open problems

Solve the following problems. We don't mind the way of solution, but we take great interest in all used procedures. You are required to explain the strategy and all adopted reasoning you have used.

1. A father is 42 years old and his son is 16. In how many years will the father's age be triple than son's age? How would you interpret the obtained result?
2. Which are the symmetric points of the points P [3,2], Q [-2,3] as regard to the origin O [0,0]? Which are the symmetric points of these points P, Q as regard to the bisector belonging to the 1st and 3rd quadrant?
3. At the end of the season bag costs € 2.50 less. Then its price is lowered in 30 % again, so the bag costs € 28. What was the initial price?
4. Two triangles are given by having two proportional sides and the angle placed between them equal. How are these triangles related? A side belonging to the smaller triangle is 2 cm long; the corresponding side in the bigger triangle is 6 cm. The other side in smaller triangle is 3,2 cm long, the corresponding one is 9,6 cm long. How are the sizes of these triangles related?

5.2 Methodology

Mentioned questionnaire was submitted in form of interview to 4 visually impaired Italians (2 students and 2 adults). As it stands in questionnaire, we have focused on the strategy and approach used by solving the problems. As well as on particular mathematical languages of interviewed persons. In order to have true image about interviewed persons, first they introduced themselves. We were interested in their "level" (knowledge) of mathematics and its particular parts, type of education and story of their blindness. Interview by itself has run as follows:

recorded interview in Italian language

⇓

translation into English

⇓

evaluation of obtained information

5.3 Second phase

After mobility in Sicily, the same interview was submitted to 5 Slovak visually impaired students in Bratislava during October 2004. The focus was the same, its continuance as follows:

recorded interview in Slovak language

⇓

translation into English

⇓

evaluation of obtained information

All interviews are presented in the Appendix 1. Next step of phase two was realized in November 2004, when 65 sighted students of Secondary Grammar School - Grösslingova, Bratislava, were asked to solve the same 4 problems. It dealt with students of common classes 4.C and 4.D, who had time of 20 minutes to solve given problems. We need this sample of sighted respondents in order to be able to find out if there were differences between non-sighted and sighted in approaches by solving of these mathematical problems. It is needed to say that time of 20 minutes was satisfactory for students to reckoning. Five solutions of sighted students we include in Appendix 2, they are marked as S1, S2, ..., S5 representing student1, student2, etc. The visually impaired respondents were not limited by time.

5.4 Analysis a priori

As it was already said, by interview we focused on strategy and approach of solving of given problems. That is why our solutions only allege assumed methods and important steps of solution. We do not consider the disability of interviewed persons.

1st problem:

- equation with variable
- arithmetical approach (try and error method)
- intersection of functions
- consideration of fact, that both of the father and son are getting older

2nd problem:

- reflection in a centre, rotation, reflection in a line
- equal triangles
- analytical approach
- symmetric point is placed on circle; on perpendicular line; on the opposite side of the bisector

3rd problem:

- equation with variable
- proportionality
- determination of basis of 30%

4th problem:

- recognition of similar triangles
- understanding of proportionality
- conscious of not changing the angles
- determination of the coefficient of the proportionality

5.5 Analysis a posteriori

1st problem

Classification of the solutions of visually impaired:

Solution	#	Comment
Equation with variable	2/(C1)	3.(16 + x) = 42 + x
Try and error method	1	16 + 14 = 30, 42 + 14 ≠ 90, etc...
Proportion	1	42 : 16 = 2,... {without sense}
Proportion and try	2	42 : 16 = 2,... =>in the past=> 41 ~ 15, ..., ***39 ~ 13***
Just father ages	3	16 . 3 = 48, 48 - 42 = 6

Classification of the solutions of sighted students:

Solution	#	Comment
Equation with variable	47/(C43)	42 + x = 3. (16 + x)
Try and error method	6	..., ***39 ~ 13***, 40 ~ 14, 41 ~ 15, 42 ~ 16, 43 ~ 17, ...
Proportion	2	42 : 16 = 2,625 => it will never happen
Just result	7	It was 3 years ago.
No answer	2	
Difference and ratio	1	42 - 16 = 26, 26 : 2 = 13

Analysis of obtained solutions and approaches:

The highest ratio of the same answers of visually impaired are from category: "Just father ages". 3 of them are not conscious of the same continuance of the time for both persons. By interpreting their (AL, MO, VL) solutions we can consider that it is only father who ages. VL2*:" By multiplying 16 and 3 I will find when father's age will* *be triple than son's."* On the other side, none of the sighted students solved it in this way, which might be the sign of very clear knowledge of passing time for everybody.

As we can read from the table, the sighted students are strongly used to solve this type of problems by equation with variable. 47 of them, which are 72,3 %, chose this approach (43 wrote correct equation). It is known that at Slovak schools is solving of the problems by equations trained quite enough, so it could be explanation of this great occurrence. Only 2 visually impaired (MT, PL) solved it by equation with variable, just one of it was correct.

It is interesting to see that 3 of visually impaired (DS, GR, DO) used proportion of the ages (42: 16) and consequently 2 of them realized very serious fact, and, indeed, by ageing the proportion of the ages decreases but difference of the ages is not changing, it is still the same. DS6: *" ...I have to go back into the past because now he is less than 3 times older."* GR2: *" If one year is passing for father, the same is passing for the son.... "* GR5: *" Because 42/16 is about 2,10 (approximately). And then by adding one year for each together this relation doesn't increase but gets less and less."* Only 2 sighted students considered this fact by proportion and as next they stated that father's age would be never triple than sons in the future. Here we must admit that text in Slovak language for sighted students didn't contain the last sentence about interpretation of obtained result. We didn't want to instigate them to any solution. Anyway, most of the students who get result -3, interpreted it as it happened in the past, 3 years ago.

Regarding to the try and error method, this approach should be very natural. Paradoxically, only 1 visually impaired (MH) would use this method to find the answer, even though 2 others who solved it by proportion continued in this way in order to get result. MH1: *" I would try until I get it. Now son's age is 16, father should be 48, but he is not. So I take random number and I will try, for example: 30 and 90. Son will be 30 in*

14 years, but father won't be 90. I shall just try." 6 of sighted students used it as well, but it is maybe more, because 7 students noticed only final result without any explanation. It is hard to say which strategy they have used; it could by try and error method that is very fast.

2nd problem

Classification of the solutions of visually impaired:

1st part:

Solution	**#**
In the mind, opposite signs	4
Hands, rectangle	1
Pens, don't know	1
Not solved	3

2nd part:

Solution	**#**
Vertical, equal distance	2
Calculation, analytical geom.	1
Drawing	2
Not solved	4

Classification of the solutions of sighted students:

1st part:

Solution	**#**
Drawing, without any answer	17
Drawing, correct coordinates	29
Drawing,	2
Just result	6

2nd part:

Solution	#
Drawing, vertical	14/(C9)
Drawing, incorrect coordinates	4
Same and inverted	6
Just result	2

Analysis of obtained solutions and approaches:

This problem is problem of analytical geometry, especially in the perspective of the Cartesian coordinate system. Four of sighted students didn't solve this second problem at all. 54 students solved first part; 26 students solved second part, 17 students solved both parts. We can interpret this more than half loss of not solving the 2nd part from experience, as students draws picture, finds the answer for the first question, he/she is satisfied and completely forgets about next question which left.

We have mentioned drawing of pictures, this is very important point, because 56 students used the picture to find the answer of whether 1st or 2nd part of the problem. It means 91,8% students of 61 who solved the problem. Hence, we can state very weak imagination of the students.

29 of them found the coordinates of symmetric points with the respect to the origin from picture by using congruent rectangles (see Appendix 2, S5). 9 students found the coordinates of symmetric points with the respect to the bisector correctly, all of them drew vertical line (see Appendix 2, S1) and 5 students mixed up the bisector with the x-axis, so they found the coordinates with the respect to the line y = 0. 17 students drew the picture but didn't write any answer.

The visually impaired affirmed our expectation of strong imagination, PL3: *" Now, I see in front of me the origin of the coordinate system, such that cross, I see the point in the right quadrant, and I know that I need to send it into the left down quadrant, I edit both signs so the point is on x-axis and as well on y-axis negative. And that it is. It is too easy. "* They have image in their minds and so 4 of them simply required just the edit of the numeral signs. Al3: *" I build in my mind Cartesian plane and then I drew approximately the point P. After that I imagined symmetric point of P, which I can get exactly, so its coordinates are [–3, -2]. The way I used is instinctive. The coordinates are opposite. "* Two visually impaired needed concrete model (personal tool) to be able to find the coordinates, hands (fingers) or two pens. In the case of MO, he found coordinates of first point correct: MO2: *" Now, I need the hands to construct Cartesian plane. I have to connect point P with the centre to get symmetric point. It means, the rectangular is divided into two triangles. I have to move to the other side from 0, to -infinity. Symmetric point will be placed in this part of the plane. That is how I get point [-3, -2].",* VL did not, she had problems: VL5: " *x = 5, I'm not sure. I'm not able to find it. Maybe with the gum table I could solve it …".*

As regard to the 2nd part of the problem which is a bit more difficult, 2 visually impaired had conception where the point is placed: MT3: *" y = x is line that contains 45 degrees with x-axis, with y-axis as well, naturally. I imagine point [3, 2], I make perpendicular line to that line y = x. The same distance like it is the point from the line, I spread on that vertical. "* Two others needed to draw and touch it, but finally haven't find the coordinates. MO9: *" I need your hands again. I have points P, Q. I cannot give you the answer; I need graph with units (measure). I connect the bisector and then I find the intersecting point."* , MO12: *" Yes, it is in this part. (he showed opposite half plane where P is) ".* Other approach was mentioned by PL who would just calculated it by analytical geometry, however he doesn't remember it now, PL8: *" No, because it is evident that line by-pass that point and now it is not so easy for me. I would calculate it. "*

This problem was not solved by 3 visually impaired at all.

3rd problem

Classification of the solutions of visually impaired:

Solution	#	Comment
Proportion	1	28 : 30 = 2,50 : 100
Proportion with variable	1	2,50 : 28 = 30% : x
Correct percent usage	4	(28/70).100 + 2,50
Incorrect percent usage	3	30% of 28 / 1,3 . 28 / 2,50 - 30%

Classification of the solutions of sighted students:

Solution	#	Comment
Equation with variable	38/(C31)	(x - 2,5) - (x - 2,5).0,3 = 28 or 0,7.(x- 2,5) = 28
Correct percent usage	18	28 = 70% => 100% = 40 + 2,50
Incorrect percent usage	5	28 = 100%
Just result	2	42,50
No answer	2	

Analysis of obtained solutions and approaches:

This problem is similar to the first one, but is more from daily life because it deals with money. That is why we expected it is acceptable for the respondents. MH2: *" One can solve it because it is from real life."*

Sighted students decided mostly for the solution by equation with variable again. In this way it was solved by 38 students, 31 of them wrote correct equation and get right result. On the other hand, just one of visually impaired (VL) would use variable with proportion of the prices. VL1: *" I have to start by relationship of proportionality.* $\frac{2,50}{28} = \frac{30\%}{x}$ *."* , in despite of, 2 others (GR, MO) have mentioned variable, but finally haven't use it. MO8: *" Initial price. So we can consider variable, let's call it whatever."* MO9: *" y, which is initial price. Maybe € 28 + € 2,50 = 30,50. And then € 30,50 –*

30%.." As regard to the solution of GR, we have to say that after translation from Italian to English we found here perhaps kind of influence:

GR5: *" x - 2,50 – 30%."*

R: *" 30% of what? "*

GR6: *" 30% of (x – 2,50) "*

R: *"Yes, might be, but where is 28. You haven't used it. "*

GR7: *" 28 + 30%.(28) "*

The first though was correct; it is pity my Italian colleague who managed the interview didn't let the respondent to continue. As a second, GR told us that he took course of accountancy. So we believe his solution is moot, nevertheless, we classify his final solution in the table as "Incorrect percent usage". Also DO didn't determine the second reduce in 30% right. DO1: *" Well, it is maybe 28 multiply by 1,3. So we have 30% upwards and then we add 2,50 and we should have result now. "* It is the same solution as sighted student S2 (see Appendix 2). Together we have 5 sighted students who didn't define the base of 30% correct.

We can read from the table that 18 sighted and 4 of visually impaired students would calculate it in right usage of percentage, e.g. DS1: *" In 30%, so it decreases to 70%. Thus, 28 = 70%. Next I would calculate 100% of it and next I would add 2,50. Then I have initial price."*

In the case of AL (which is a bit similar to VL) it is interesting to see: AL3: *" ... I usually don't care about problems like this one. They are not my favourite ones."* and his solution:

AL3 : *" What have we got? The final price is 28. Could I write the proportion? 28 : 30, I'm not sure, I don't want to say something wrong. 28 : 30 = 2,50 : 100. By solving this proportion I will reach the right solution. But I'm not sure. Maybe I said some nonsense... "* It seems like he is not conscious of having proportion only with all given data, but no meaning.

Again, we have few responses of sighted students only with correct result without calculus (2) and 2 cases without any answer.

4th problem

Classification of the solutions of visually impaired:

1st part:

Solution	#
Similar	2
Proportion	3
Don't know	3
Not solved	1

2nd part:

Solution	#
3 times bigger	6
Proportion 3	1
1 and 2	1
Not solved	1

Classification of the solutions of sighted students:

1st part

Solution	#
Similar	36
Congruent	1
Congruent, not equal	1
Reflection	1
Each other opposite	1

2nd part

Solution	#
Proportion 3:1	28
Proportion 1:1,6	1
3 times bigger/smaller	27
Smaller in 2/3	1
2 times smaller	1
Similar	1

Analysis of obtained solutions and approaches:

If we consider 1st part of the problem, we see from the table above that we have 3 different categories of answer of visually impaired respondents: similar, proportion, don't know. Two of visually impaired determined similar triangles, even though, also one more respondent clearly understood what is going on just didn't know the term for it. DS2: *" Clear, so the angles are not changing. Isn't it in some relation with direct proportion? But I don't know the name for it. "* We placed his answer to the category "proportion" as other 2: MO3: *" According to me, the triangles are maybe proportional. "* and VL4: *" No equal, but their sides are in relation of proportionality. "* Both of them presented the relation on the concrete model - hands and in the second case isosceles triangles.

The last category is represented by 3 visually impaired who didn't know to answer. Their first thoughts mostly regarded sides of the triangles, MH1: *" One is bigger and other is smaller... "* PL3: *" They should have equal sides, I think. "*, AL2: *" ... For example, we can consider two triangles which are isosceles. "*, but finally they were not able to find the relation between triangles.

As the second problem that is also from geometry, this one GR didn't solve.

As regard to the responses of sighted students, five of them didn't answer at all. It could be because in the text it stood as last problem and they perhaps didn't chase. 40 students gave answer for the first part of the problem, while for the second we have 59 answers. This difference might be caused by inattention and then they replied only on question they read last.

So, 36 students determined similar triangles. We have to admit that 47 students of 60 (78,3%) who solved the problem (1^{st} part or the 2^{nd}) drew the figure (see Appendix 2). It could help them by finding the relation between triangles and also between the sides of the triangles.

Here we come to the second part, which was solved by 59 students. 56 of them (94,9%) wrote correct relation, either proportion 3:1 or 3 times bigger/smaller or smaller in 2/3. On the other hand, 7 of 8 visually impaired (87,5%) said right relation, mostly 3 times bigger.

5.6 Conclusion of the pre-experiment and determination of the hypotheses

By our experiment we found that visually impaired people are able to solve mathematical problems, although their approach and way of solution is a bit different than approach of the sighted persons. With the regard to the algebra and arithmetic we discovered they mostly prefer arithmetic. They don't use variable very often comparing to the sighted students who do so many times. Nevertheless, we see analogies among the other used strategies.

In the next step we performed research in the field of geometry. Most of the sighted students drew a picture by solving given problems of analytical and Euclidean geometry. On the other hand, some of the unseeing used personal geometrical instruments and in general we can state that visually impaired have to use imagination, all object (solids and plane figures) are first touched and then stored. Geometry is for them kind of adaptation to the environment. Therefore, we think this adaptation is dynamic in the sense that they continually change the system of operation of environment they explore. Since every environment is a new environment s/he has to store all information (tactile, auditory, olphactive, etc.) and so make mental images.

It is interesting for us to investigate more in the field of space geometry in connection with visually impaired people, to see how they are adapted to various environments, what are their personal tools, since geometry can provide a more complete appreciation of the world. We have prepared new experiment (see Chapter 6) and we define our expectation in following hypotheses:

H1: *The sighted and non-sighted pupils perceive the space and its objects in different ways. The point of view on the space geometry of visually impaired people is point of perception and it is dynamic. The point of view on the space geometry of sighted people is static.*

H2: *Based on the senses the non-sighted pupils are able to differentiate and name basic geometric figures and solids.*

H3: *When exploring new room and objects in it, the non-sighted are using several senses; sense of touch, smell and ear; while sighted rely only on sight.*

H4: *The non-sighted pupils will describe objects in the space (shape and position) better and more exact as sighted pupils.*

H5: *The non-sighted pupils have better imagination about position of the objects in the space as sighted pupils and so they build more precise scale model of the room, even if they build it on the basis of given audio record.*

6 Experiment

6.1 Preparation and goals of the experiment

Since we decided to compare the approach to geometry from point of view of visually impaired and sighted people, we have prepared following experiment:

We have placed various subjects of different shapes in the room, such as following figures show.

Fig.6.1a

Fig.6.1b

Fig.6.1c

Except of typical office subjects (table, chairs, PC, cabinets) we placed in the room the fit ball, air freshener, the clock of pyramid shape and flowers as well. The lamp on the table was on, as well as the PC, water in the sink, which is in the closet, was on. The goal of the selection of above mentioned objects was to observe what sense the person in the room will use while exploring the room. Before realizing the experiment, we consulted about the location of subjects in the room with visually impaired university student M., who is experienced in exploring of new places. We wanted to know, what everything is he able to find, describe and feel in the room, which senses he uses. His role was to describe everything what he sees in the room; location, material of the subjects, which way he classifies the objects, possible problems and obstacles that seem to be all right for sighted person. It took him 14 minutes to explore and describe in detail the whole room; we have recorded it by Dictaphone (the whole transcript of record is in Appendix 3, p. 135). He acted as follows:

1) specification of the shape of the ground plan of the room
2) definition of the base point and its location with the regard the ground plan
3) description of the room along all walls
4) description of the centre of the room (regarding the base point)

After description of the room we have discussed with him the whole situation. Hereby we present some of his very interesting remarks:

- as he entered the room, he felt “aroma” of the electronics, concretely “aroma” of the computer that was on;
- otherwise he doesn’t follows, doesn’t rely on odors, so for the next experiment he is not sure about flowers and air freshener in the room;
- he felt the warmth on the table, so he predicted the lamp or reflex of the sun on the table (*it was nice warm day outside*);
- the wall that is between the computer and balcony is illusory with regard to the reflected sounds;
- the flow water in the sink seems to be illusory too, as it gives the feeling like the sink is in another room or behind the curtain (*the sink was inside the closed cabinet next to the door*);
- he measured dimensions of the room and walls by, steps, forearm, upper limb and snapping fingers;

- he determined the shape of the ground plan of the room as rectangle or square by snapping fingers;
- he described his actual position and position of objects in form: shorter side or longer side of the rectangle, right or left side of the rectangle, left upper corner of the room, the tables are connected like in letter T, vertically/horizontally to the door; which shows his good orientation and imagination of the explored room ;
- as he stand at window he suppose/he is looking for radiator; he also recognized the box on the table as box with CD's, which says about his rich experience.

The initial conception of the experiment was:

1) pupil A goes into the room and describes what s/he sees in the room (*recording by dictaphone*);
2) pupil A draws the schema of the room on the paper based on her/his memory;
3) according to audio record of pupil A, pupil B tries to draws the schema of the room on the paper;
4) pupil B can ask for more information, but only by asking questions to which pupil A can only answer 'Yes' or 'No'.

The question arose, whether the non-sighted pupil is able to draw 3-dimensional objects in to the plane, which is difficult, so we decided to substitute the projection in to the plane by building the model of the room using the various packets of different shapes. The sighted pupils (SP) who took part in experiment were selected randomly and all pupils were of 7th - 9th grade. Pupils of these grades know 2-D and 3-D shapes and their characteristics; they have their personal experience and they have learned it also in the school[15]. However, the problem was the number of pupils who took part in experiment. We wanted to form pairs of all possible combination of sighted and non-sighted pupils

[15] According to curriculum pupils of 1st grade of primary school should know to differentiate the geometric shapes as: triangle, circle, square, rectangle, cube, sphere and cylinder. Later on, in the 3rd grade they learn how to draw the circle, square, rectangle and triangle and learn name the edges and sides. In 6th grade pupils get in contact with cube, cuboid (they calculate volume and surface) parallelograms and trapezoids. In 7th grade they focus on prism, in 9th grade on cylinder, pyramid and cone.

(NSP), which means 4 pairs. It is needed to say we concentrated only on pupils who are non-sighted since birth and so do not have any visual imagination. That is why we were able to find only 3 non-sighted pupils (age 13-14) attending the special primary school for visually impaired children in Bratislava. Then we changed pairs for trinities and pairs as follows:

NSP_1-NSP_2-SP_1 NSP_3-NSP_2-SP_2 SP_3-SP_4

where always the first one of the trinity/pair went in to the room and verbally described what s/he sees and the others of the trinity/pair built the model of the room based on audio record. The first one of the trinity/pair built the model of the room as well, but based on her/his memory. In the first and second trinity is the same person (NSP_2), who was not told that she is building the model of the same room in both cases. This was caused due to the small number of non-sighted children.

Our aim during the experiment was to observe following items in order to be able to verify or negate the defined hypotheses:

- the orientation in the space;
- the way of description of the room and objects;
- the relationship between the image in the pupil's mind and the vocabulary s/he uses in the communication;
- what is the dominant attribute by description of the room;
- perception of the shapes, positions and dimensions;
- what senses s/he uses;
- how s/he perceive the space;
- what way s/he builds the model of the room;
- differentiation of the shapes and characteristics of the objects;

- what type of information pupils miss by building the model on the basis of audio records.

6.2 Description of the experiment

As written above we have divided children into the trinities and pair. We called the one who went into the room pupil A, pupil B is the one who didn’t go into the room. The tasks for the pupils were as follows:

Task 1

Instruction:

Pupil A: Enter the room. Within the twenty minutes explore it and tell me exactly what do you see. Tell me about everything, about all objects, their characteristics and their localization.

Task 2a

Instruction:

Pupil B: By using these packages and stuff try to build the model of the room based on audio record of Pupil A. The caps of plastic bottles represent the chairs. Later on you can ask for more information, but only by asking questions to which Pupil A can only answer ’Yes’ or ’No’.

Task 2b

Instruction:

Pupil A: By using these packages and stuff try to build the model of the room on the basis of your memory, on the basis of what you have seen. The caps of plastic bottles represent the chairs.

Applying the Activity theory we describe two activities, one that has been carried out in the room (Task 1) and the second activity that has been realized out of the room (Task 2).

The exploring and describing the prepared room is the activity that refers to the subject of Pupil A who goes into the room. The object of her/his activity is the room and all objects in it. The expected outcome is the as precise verbal description of the room as possible; consequently we are going to analyze this description in the sense of perceiving the space and its objects (see Chapter 5.4). There were no given rules concerning the progressing activity, just one restriction regarding the time was given. It has a implication that Pupil A can proceed as s/he wants, in the way s/he likes, so the are no horizontally segmented tasks of division of labor. Anyway, with the respect to action of university student M. and our experience we have expected the following possible actions which Pupil A could make in the room:

- to specify the shape of the ground plan and verify the dimensions of the room
- to seek points of the reference by means of the echo of the windows, of the doors, of the voice, etc.
- to individuate and memorize every possible obstacle
- to look for references in the noises and vibrations or in the odors
- to clap one's hands to grasp the dimensions and the volume of a room
- to move with the white stick and perceive the space, objects and obstacles
- to perceive the obstacles by air pressure on the face
- to touch all objects and describe them

The mentioned possible actions could be done by using the white stick, all senses, language, imagination, etc. and these are mediating tools or instruments by which the Pupil A can achieve the outcome of the activity. There is also no vertical division of status and power concerning the division of labor, since the community of this activity consists only of researcher who is present in the room in order to record the description and assist if necessary. It is needed to mention the whole environment in which the experiment was realized, as well as the researcher was new for pupils, so that is the reason why we are conscious of pupil's doubtful and sometimes reserved behaviour. All that might influence the objectivity of the experiment.

The second activity was carried out of the prepared room and its outcome is to interpret the room by building the model, which is also kind of description of the room and we can analyze it in the frame of perceiving and recognition of the space and its objects. The model of the room built by us is shown in the Figure 6.2.

Fig. 6.2

This activity has to be distinguished with respect to the pupil who is building the model (Task 2a, Task 2b). In both cases the object of the activity is the prepared room and the rest changes.

In the case of Pupil A who is the subject of the activity, the only rule given to him is to build the model of the room by using given packages and stuff, moreover the bottle caps have to be used as chairs. In the case of Pupil B we have two more rules about the building the model according to record and about the way of asking questions to Pupil A. All given packages and stuff of different shapes and sizes (playing cubes, packages of tea, matches, medicaments and cosmetics; tennis and squash balls, buttons, batteries, eraser, carton models) are for Pupil A and B instruments to build the model. The difference between Pupil A and Pupil B is that other instrument of Pupil A is her/his internal model of the room stored in her/his memory, while Pupil B has audio record of Pupil A at disposal. Pupil B can ask for more information that is becoming also his/her instruments. The community in both cases consists of researcher and her assistant and other pupils who took part in experiment. In the case of Pupil A all community except of researcher is just side, unimportant effect; they were just observers, no interfering into the process of building the model. On the other hand, important role of community in the case of Pupil B plays the researcher who moderates the conversation and Pupil A who answers to the questions. Since the instructions of Task 2a say to Pupil B first to build the model of the room based on audio record and later on to ask the supplementary questions, here we have horizontally segmented actions of division of labor (which is actually given by the rules). Also the succession: question, answer, and potential change of model represent partial horizontal division of the actions.

6.3 Qualitative analysis of the experiment

6.3.1 Analysis of the Task 1

In following we present some parts of protocols of pupils A. These sentences in certain way demonstrate the occurrence of explored items mentioned in Chapter 6.1. Consequently, we analyzed all three descriptions of the room assigned in Task 1.

Analysis of RB's description (protocol p. 139):

he pays attention to material of the objects:

R1: ... there is the wall inlaid by wood ...
R3: ... this one has showcase that is made of glass ...

he notices location/position:

*R1: So, on my **right** side ...*
R2: ... here are actually two cabinets situated one on the other one.
*R10: ... and the **room is curving** and I stay actually ...*
R14: Then you turn right ...
*R15: ... and here **down** is the chair.*

*R18: ... Then it is actually the **end of the wall**. I returned to the **beginning**, I walked around whole of it.*
*R22: ... In the **last** cabinet before the door, at the end of the room ...*
*R33: ... one is placed by **height** and one is placed by **width** beside it ...*
R37: From […] from the front […] actually, if I stay direct at the door, if I enter the room, I come to the table and at the right side ...
R44: ... On the right side from PC box there is ... on the right from the printer there is ...

he describes shapes of the objects:

*R2: ... cabinet, also shape of **rectangle**, classic cabinet with rectangular shelves. Below ... The door has holders of **circle** shape ...*
*R3: ... the holders are like **trapezoid** ...*
*R5: It is like […] it curves like in to **semicircle**. At this side …*
R14: ... Then here is a ball and ... (no mention of shape)
*R15: ... something like **cylinder** is placed on it, the cylinder is placed on it.*
*R26: ... It is something with **pyramid** shape ...*
*R28: On the sides it has **triangles** ...*
*R41: ... It is rectangle, no rather **square** ...*
*R47: ... They are […] it is neither **cube** nor […]*
*R49: ... It is shape of **cuboid**. Also the upper packet has had this shape. Yes […] it is cuboid.*

he describes properties:

*R6: At the front side it **curves** like in to semicircle, at the opposite side it is ...*
*R48: ... If I hold it like this […] it is longer than the other side. Actually, the **horizontal** side is longer then **vertical**. It depends how you hold it. I have it **along**, horizontally to me ...*

he describes sizes:

R7: That cabinet is high about […] something more than knees or like my thighs.

he uses senses:

R13: ... They are live flowers because they are ***not dry****, simply …*
R22: I have ***heard*** *water and then I went to see ...*
R31: Because […] you can […] when I bended down like this, I ***feel*** *the light.*
R32: Yes, and it is ***hot*** *...*

R. entered the room without the white stick, so he decided to touch everything and in this way to describe the room. First he walked through the room along the walls, then he determined the shape of the ground plan and afterwards he described the centre of the room. It took him 17 minutes. Concerning the whole description, one might state that it is quite exact. R. noticed almost all objects in the room, except of air freshener that was placed on the ground in the left bottom corner (regarding the rectangular ground plan). The truth is that it was not very intense aroma, but on the other side, the flowers placed on the table were lily of the valley and even their sharp distinctive aroma R. did not noticed them by sense of smell (*R12: I don't know […] flowers are in it. R13: ... They are live flowers because they are not dry, simply* …). It seems like the sense of smell does not play any role when exploring new environment, even he recognizes and distinguishes the odours. On the contrary, leading analyzer is naturally the sense of touch, which is supported in his case also by sense of ear (*R22: I have heard water and then* …).

R. noticed mathematical (*R6: …cabinet which has rectangular shape.*) and also nonmathematical (*R3: ... this one has showcase that is made of glass* ...) characteristics of the objects. He can differentiate and name the shapes of the objects, concretely: circle, square, rectangle, trapezoid, triangle, cube, cuboid, pyramid and cylinder. The interesting phenomenon in R.'s description we find when he talks about some objects of shape of cuboid:

R2: Then there is <u>cabinet</u>, also shape of rectangle, classic cabinet with rectangular <u>shelves</u>.
R3: …Then, it is shape of rectangle, here down, this <u>door</u> and the <u>glass</u>, whole it is rectangular shape.

R9: As next, window […] rather the door […] of balcony, I think, of rectangular shape…
R10: … And parapet is also of the rectangular shape …
R14: … The table has rectangular shape …
R16: … Also the doors of the cabinet have rectangular shape.
R41: … And the push-buttons are rectangular …

In spite of this incorrect categorization he knows what cuboid is and can identify its properties (2nd level of van Hiele' hierarchy):

R48: …this one side […] front […] If I hold it like this […] it is longer than the other side. Actually, the horizontal side is longer then vertical. It depends how you hold it. I have it along, horizontally to me …
R49: And under it is bigger packet which has shape of […] it is also not the shape of cube […] but it is shape of [...] what can I compare it to? It is shape of cuboid. Also the upper packet has had this shape. Yes […] it is cuboid.

The question is why he determined the shape of some objects (cabinet, door, table, etc.) incorrect? Why he attributed them 2-D shape?
We cannot state that he doesn't know 3-D solids, since he named correct the clock of pyramid shape (placed on the table) and the case of cylinder shape (placed on the chair). Noteworthy is that just these objects were "small", in the sense he could grasp whole of clock/case and touch it. On the contrary, cabinets, door, etc. were "big" in sizes, untouchable as the whole, and here he made a mistake. It seems like he perceives by touch only the length and height of the objects (dimensions that are immediately in front of him without any bend or stretching) and forgets about depth (in case of table about height). In the similar manner we can interpret his description of black case of interactive board (see Fig. 6.1b) that has also shape of cuboid. His description was as follows:

R5: It is like […] it curves like in to semicircle. At this side […].

E2: At which side?

R6: At the front side it curves like in to semicircle, at the opposite side it is […] it is normal […] It is something of semicircle shape and …

Again, this is the case when the object is too big to be grasped whole. Moreover, he really wanted to find out what it is, he stretched his right arm to the right and tried to touch the end of the case by his fingers, so he bended his wrist. Maybe this is the explanation why he was talking about semicircle. He could feel the semicircle in his arm or the case bend (it was flexible) and so he felt something rounded.

Concerning his ability to recognize and name the objects by touch, we see from the description that he has done it very well. Sometimes he even knew what type the object is (*R10: … on the left in the front is the chair, computer's chair …*), that happened when he get in contact with such a object before. In the case he has never seen before some object (e.g. untypical clock), he can only describe it but without giving it any name (*R26: …something with pyramid shape. I don't know what it is. It is something battery operated. R27: What it is? I really don't know. I can't remember. R28: On the sides it has triangles, but I don't know what it serves for. It can be anything, any definition for it.*).

Following his description one can see his orientation in space is well handled, he know in each moment where he is. After he first time walked through the room he determined its shape:

R18: And then I am actually […] then it is door. Then it is actually the end of the wall. I returned to the beginning, I walked around whole of it. The room is […] if I guess correctly […] it has rectangular shape or […].

He memorized step by step each object and its location in the room (he created cognitive map), which is shown at the point when he spilled the water from flowers' cup (*R33: I should know it that it is here …*). So it gives evidence about his dynamic approach of creation the images.

Analysis of MB's description (protocol p.144):

he pays attention to material of the objects:

M5: … The glass part of the cabinet …
M22: … the chair of […] wooden …

he notices location/position:

M22: I go at the moment by the wall. And down here is …
M33: …The equipment is situated on the table, not at the beginning but almost.

he describes shapes of the objects:

*M1: At the moment the **rectangular** door is here …*
*M4: … in **shape of heart** …*
*M6: … like […] maybe **cuboid** can it be like …*
*M14: … box here in shape of […] also like […] it can be maybe […] like **prism**.*
*M15: Then is here **round ball**.*
*M19: …actually this front part that is here forms like **square** …*
*M22: … is an item that is **cylinder** in shape.*
*M32: … such equipment in shape of […] **pyramid**, one can say it.*
*M38: … it is here the box in shape of […] such a **cube** …*

he describes sizes:

*M1: … it is **high** about 2 meters.*
M3: … it is high also about 2 meters, 1 meter.
*M8: Here is high box at a **height** about 1 meter…*
*M18: It is not so big […] like […] appropriate, **medium size** it can be.*
*M21: The table is from the floor about […] it is **small enough**, so…*
M24: …the door is like rectangle, height is about meter or two.

he uses senses:

M39: You can ***hear*** *here the whirr of computer and like […] as the water flows […] or like that.*

M. came into the room also without the white stick. He walked around the room and described its objects. He didn't determine the shape of the ground plan and we had to help him not to forget to describe the centre of the room.

Concerning the senses, except of sense of touch he didn't use during the exploration any of the other senses until we ask him whether he hears anything:

E14: And if you stop, what sounds do you hear?

M39: You can hear here the whirr of computer and like […] as the water flows […] or like that.

It shows that his primary analyzer is sense of touch, while he doesn't use sense of hear and smell in situations like that (he didn't notice the air freshener and didn't recognize the lily of valley as well).

Comparing to description of R. he tried to give also the information about the sizes, but rather in meters (R. used comparison to his body) or circumlocutionaly:

M3: …cabinet … it is high also about 2 meters, 1 meter.

M25: … These doors constitute of two rectangles, also the height of meter or two one can guess.

M18: It is not so big […] like […] appropriate, medium size it can be.

M21: The table is from the floor about […] it is small enough, so […].

These sentences say about his insufficient ability to estimate the sizes; as one meter is liken to two meters or things are appropriate and small enough in size.

Regarding the shapes of objects he recognized: square, rectangle, prism, cuboid, cylinder, pyramid and cube. Since his description is not so detailed as the one of R., he didn't mention shape of circle, trapezoid and triangle. On the other side, he noticed untypical shape of heart on the cabinet' shelf (*M4: …cabinet here, on […] in which is […] in shape of heart […] like […] maybe chocolate or something may be in it.*).

The same occurrence as in the case of R. was notices when talking about the shape of rectangle. M. determined following objects as objects with shape of rectangle:

M1: At the moment the rectangular <u>door</u> is here …

M3: Here is the <u>cabinet</u> that is also rectangular …

M5: <u>Shelves</u> […] also rectangle in shape. The <u>glass part</u> of the cabinet that is also rectangular.

M11: Then I see here <u>window blinds</u>, they are like […] it whole forms rectangle.

M12: It is <u>window</u> here that is also like rectangle and …

M13: … the <u>parapet</u> in shape of […] also rectangle.

M19: …On it […] it is here also case of <u>keyboard</u>, which forms shape of rectangle.
M20: … <u>mouse pad</u>, it is also in shape of rectangle.
M27: So, in the middle of the room is the <u>table</u> in shape of rectangle.
M30: The <u>book</u>s are here, for example this one is rectangle in shape…

On the contrary, in some cases he named the shape correctly:

M6: Other cabinet […] in shape of […] also, like […] maybe cuboid can it be like …
M8: Here is high box at a height about 1 meter and it is also like cuboid in shape.
M9: Under it is cabinet or small cabinet, which has shape of […] also of cuboid, one can say.

If we pay attention to the fact that sometimes he classified the shape correctly, we find possible explanation of it by looking at Fig. 6.1b, which actually displays the objects described in *M6, M8, M9.*

Fig.6.1b

In comparison with other cabinets, the bottom cabinet (red oval) described in *M6* is a bit shifted forwards so M. could perceive also its depth. The same with the small cabinet under black box (yellow oval) described in *M9*. It follows from the consequence he knows properties of solids (level 2 of van Hiele' classification), but they must be whole touchable.

Analysis of J's description (protocol p. 147):

she notices location/position:

*J2: **In the middle** of the room are two tables…*
*J4: At the wall, on the **right side** are cabinets …*
*J5: On the **left side** is cabinet, **up** is the box, **next** to the cabinet …*

she describes shapes of the objects:

*J7: …and some watch that has shape of […] **pyramid**?*

The description of J. was extremely short and could be caused by her shyness, but on the other side she has decided herself to go into the room instead of building the model on the basis of audio record of someone else (Task 2).

Her approach was expressly static in the sense she stood in one place and from this position described what she had seen (*J1: I came in to the room and from the door I see the window…*). She didn't have any necessity to walk through the room and so make sure she can see everything from the door. Her description shows strong visual nature of perceiving the space and objects since she didn't mention almost none shape of the objects (cabinets, packets, tube, etc.). We can interpret it in way that she rely on imagination and so doesn't consider the reference to shape of the objects as essential. For example: if someone says cabinet then we imagine cabinet that we have for instance at home and we know it has the shape of cuboid, certain colour, height, etc. She only speaks about untypical shape of objects (*J7: On the table we have… some watch that has shape of … pyramid?*), since usually the table watch has shape of cuboid or cylinder. In the same manner she specifies the form of location of the tables (*J2: In the middle of the room are two tables, which are situated in shape of T.*).
She also doesn't say about exact position or if so then insufficiently.

J3: <u>Behind one</u> is office chair and behind the other one are six chairs. (which one?)
J8: Then, <u>next</u> to the computer is the printer. (at which side?)
J12: <u>On</u> the chair is some tube. (which chair?)

At the end of the experiment, at Task 2, she comprehended that her description is deficient (see Chapter 6.3.2).
Except of all above mentioned facts we find here another difference in the comparison with descriptions of non-sighted R. and M. Since her leading analyzer is sight she notices also the space above her that is not possible to achieve by hands (*J9: Up there are four lamps.*).
We are conscious of the fact that from one description of the sighted pupil we cannot generalize. So it is hard to state the sighted pupils don't perceive the object of real world as geometric figures and solids and do not have any connections between them. We can only confirm the common claim the sighted people rely mostly on sight when exploring new space and the imagination and visualization play most important role.

6.3.2 Analysis of the Task 2

The first group of figures represents the models built by sighted pupils (Fig. 6.3a, 6.3b, 6.3c, 6.3d), the second group was built by non-sighted pupils (Fig. 6.4a, 6.4b, 6.4c, 6.4d).

Fig. 6.3a: Model of P.

Fig. 6.3b: Model of D.

Fig. 6.3c: Model of J.

Fig. 6.3d: Model of S.

All pupils B acted in the same manner: at first they listened to the audio records and parallel built the model, afterwards they asked for more information if needed.

At the first glance we see the difference; while models of sighted pupils are large the models of non-sighted are "small", tight, all objects are close to each other. The reason for it might be on one side the necessity of the control of the model by hands, on the other side the lack of experience with metrics.

Anyway, the non-sighted pupils showed that they are able to store the touched information and thereafter to visualize them in quite exact way (Fig. 6.4a, 6.4c).

Fig. 6.4a: Model of R.

Fig. 6.4b: Model of D_1.

Fig. 6.4c: Model of M.

Fig. 6.4d: Model of D_2.

In case of pupil D. who built the model based on audio records of R. and M. one can see she is able to interpret their description and so form an idea about the room. We are conscious that the learning experience derived from the first time she built the model in terms of the experiment is possible to have an effect on her way of building the model the second time she participates in the experiment (but she was not told that it is the same room). This puts the internal validity of the experimental design at risk and it may influence the results. It is obvious from the protocols that D. missed only few information and she have asked for them:

(the questions to R.'s description, protocol p. 148)

D1: … two cabinets that are on each other, are they at the wall?

D9: … cup was on the table where the computer is?

D15: Also three chairs were at the wall?

(the questions to M.'s description, protocol p. 154)

D1: … ball was at the cabinet?

Considerable is the fact, that despite of statements of R. and M. about rectangular shape of many objects (cabinets, tables, books, etc.) D. didn't have any questions and spontaneously used packets of cuboid shape. That indicates that she uses similar vocabulary as boys and image in her mind corresponds to the real look of the objects. She interpreted even her image of table as board with legs (red oval, Fig. 6.4d). She also expressed her image when trying to find out where the chairs were (*D28: At the short, shorter side of the table? D29: That shorter side of the table. It has two long and two shorter sides.*).

Regarding the models of sighted pupils and their creation we noticed interesting fact: before they started to build the model, they organized the packages by shapes in to cubes, cuboids, pyramids, and spheres which evidences to system of the work.

They were able to follow the description of non-sighted pupils and build the model without any question (case of P., Fig. 6.3a) or only few questions (case of S., Fig. 6.3d, protocol p. 152). Both of them made a mistake in setting the tables (shape T) in the centre of the model. Actually, the description was not exact:

R33: … Actually, at the other side of that table which is next to the second table … Basically, here are two tables, one is placed by height and one is paced by width beside it …

M27: So, in the middle of the room is the table in shape of rectangle.

M28: At it is the chair, another chair, third chair.
M29: It is such a table here, next to it.

In case of D. who built the model based on audio record of sighted pupil J. we see from the protocol (p. 154) that he missed more information:
D2: … Oh, those flowers were on that table of shape T?
D3: And that T was situated like that you looked at its bottom?
D4: On the upper part of that T, on that upper, was one chair, right?
D8: …actually that tube was on the chairs next to the cabinet and computer?
D9: And those chairs were between cabinet and computer?

As we have already mentioned in previous chapter, J. comprehended here that her description was inadequate. After D. compared his model with J.'s model, he saw the differences and said to J.: "But you didn't say about everything!" and consequently he asked for possibility to go into the room and have a look on it, together with pupil S. who was also curious.

6.4 The results of the experiment

Based on analysis that was carried out in previous two subchapters (Chapter 6.3.1, 6.3.2) we can state that:

H1: *The sighted and non-sighted pupils perceive the space and its objects in different ways. The point of view on the space geometry of visually impaired people is point of perception and it is dynamic. The point of view on the space geometry of sighted people is static.*

The sighted pupil really showed expected behaviour, right she entered the room she stated what is in there (sometimes very inexactly), while the non-sighted pupils detected the space gradually. So here we have development and dynamics of detection, which are actually facilitating the subsequent better description. If we would be able to bring the sighted pupils to such a dynamics, then the certain superficiality can be eliminated and hence also the superficial perception of the space.

In Task 1 pupils should describe the room and its objects, their characteristics and localization so pupil B in Task 2a can build the model of the room. We had seen that non-sighted pupils recognized and named many objects of different shapes (cube, cuboid, pyramid, cylinder, triangle, circle, trapezoid, square, rectangle), so hypothesis:

H2: *Based on the senses the non-sighted pupils are able to differentiate and name basic geometric figures and solids.*

seems to be true, although in some cases they used wrong terminology.

The hypothesis **H3**, which says:

When exploring new room and objects in it, the non-sighted are using several senses; sense of touch, smell and ear; while sighted rely only on sight.

has been confirmed only partially because the non-sighted pupils didn't use sense of smell, neither by finding the air freshener nor by flowers. The sense of touch is their leading analyzer and sense of ear is complementary analyzer. We can illustrate the usage of sense of ear by demonstrations from the protocols:

R22: I have heard water and then I went to see...
M39: You can hear here the whirr of computer and like […] as the water flows […] or like that.

Also in case of sighted pupil J. we cannot claim she only relied on sight. The true is she also didn't notice the air freshener, she saw the flowers, but she mentioned the sink in the cabinet even she couldn't see it since the door was closed, on the other side she didn't say anything about hearing.

J6: At the door are cabinets, where is for example the sink, in one there are books.

Since non-sighted pupils had to go over the whole room and touch everything, they described continuously and more exactly the objects in the room than sighted pupil, who stand in one point and described what she saw. Sighted pupil didn't mention lot of things, she didn't find it as necessary, even she was told to describe it precise. On the other hand, when building the scale model of the room, she did it very exact, which says about her strong visual memory. Based on these facts we can confirm:

H4: *The non-sighted pupils will describe objects in the space (shape and position) better and more exact as sighted pupils.*

The fifth hypothesis, which says:

H5: *The non-sighted pupils have better imagination about position of objects in the space as sighted pupils and so they build more precise scale model of the room, even if they build it on the basis of given audio record.*

wasn't neither acknowledged nor disproved since all Pupils A (sighted and non-sighted as well) built almost exact model of the room. In the case of pupils B we had noticed the ability to interpret the verbal description of the space and ability to create an image of solids and their location in the space. We cannot compare the results of sighted and non-sighted pupils who participated in Task 2 since there was the same non-sighted person participating two times in experiment. Anyway, regarding the mental representation of the space, the world of non-sighted is not different in comparison with that one of people who are sighted.

Except of determined hypotheses we came also to following conclusions that are applicable in pedagogic practice of the teacher. Right in the experiment, concretely at Task 2, the visiting maths' teachers from special school for visually impaired children pointed out that the same or similar tasks have considerable value as educational tools. They could be used for the diagnosis and assessment of pupils' levels of understanding of three-dimensional solids (van Hiele's levels) and metrics of the space and to develop their communicative skills about the solids. The Task 1 required the pupils to describe new space and its objects. This gave a very clear indication of level of vocabulary of the pupils and the communicative skills. According to some similar experiments (Littler, Jirotková 2004) when authors observed sighted children in process of tactile manipulation with solids and their verbal communication, this analysis help us also to construct the process of building structure of geometrical knowledge or even the process of creating new knowledge by extending the existing structure or its restructuring.

7 Conclusion

In the submitted thesis we dealt with questions concerning the study and teaching mathematics to visually impaired people. In order to fulfill the determined aims and better understand the given problematics we studied in second chapter how does the vision work, what are the possible diseases of the eye and how do they manifest. We have also become familiar with history of reading codes for the visually impaired and consequently we briefly presented the main problems of Braille notation of mathematics and limitations of Slovak Braille code, which are narrowly related to the actual situation at Slovak schools. Since there does not exist the notation that covers all symbols of mathematics at secondary and university level, visually impaired students are forced to create and use their own notation, which doesn't have to be understandable between each other. In the last years the solutions seems to have electronic form, there is software that are blind friendly and so the visually impaired students can do (calculate, read, write) mathematics the way that is also accessible for their sighted schoolmates and teachers. However, not all of the teachers are willing to accept it.

The trend of the last years is to integrate the impaired students into common schools and so in the third chapter we have described the change of classic didactic triangle in case of attendance of visually impaired student in the classroom. We have used for it the Theory of didactical situation and we have found out that integration in teaching process has rather social aspect. The other remarkable thing is the question of limit. Since in Slovakia there is no standard for teaching mathematics to integrated visually impaired students on the secondary level (the standards for common students are valid), the teacher has to determine requirements on these students by his own, on his subjective opinion. In this chapter we have also presented the Activity theory and van Hiele's levels of understanding the geometric solids in order to have a tool for describing and analyzing the realized experiment.

In pre-experiment that is part of fifth chapter we have compared the approaches and strategies of solutions of mathematical problems by sighted and non-sighted people, which was actually one of the determined aims. We have found out that visually impaired people are able to solve mathematical problems, although their approach and way of solution is a bit different than the approach of the sighted persons. Concerning the field of geometry we have noticed the most of sighted students drew a picture by solving given problems of analytical and Euclidean geometry. On the other hand, some of the unseeing used personal geometrical instruments or didn't solve these problems at all, because, according to their story, they were not taught the geometry since they are blind. That all led us to research more in field of space geometry in connection with visually impaired people, to see how they are adapted to various environments, what are their personal tools. So we have prepared new experiment that generates the sixth chapter. The task for the participating pupils was to describe the objects and their characteristics and location in the prepared room. Afterwards to built the scale model of the room based on the memory or in case of pupils who were not in the room based on audio record. We have analyzed obtained protocols and verified hypotheses *H1* till *H5*, which are presented in the introduction. The results of the research are summarized in following statements and they are also considered as verification of the determined hypotheses:

- Based on the senses the non-sighted students are able to differentiate and name basic geometric figures and solids. The only problem they had when describing the objects of cuboid shape. Sometimes they called it rectangular. Hereby we propose to use the similar tasks as Task 1, Task2 and tactile manipulation with solids for the diagnosis and assessment of pupils' levels of understanding of three-dimensional solids and to develop their communicative skills about the solids. These tasks should be realized during the process of education of non-sighted and sighted pupils as well.
- When exploring new room and objects in it, the non-sighted pupils are using sense of touch as leading analyzer, which is supported by sense of ear. They detect the space gradually and thus there is development and dynamics of detection. The sighted pupils rely mostly on sight and the imagination and visualization play most important role. If we would be able to bring the sighted pupils to the dynamics, then the certain superficiality can be eliminated and hence also the superficial perception of the space.

- At the first glance we saw the difference, while models of sighted pupils were large, the models of non-sighted were “small”, tight, all objects were close to each other. The reason for it might be on one side the necessity of the control of the model by hands, on the other side also the lack of experience with metrics. The other point is related to the estimation of distance and measure. It is shown in the protocols that non-sighted pupils compared the measures to their body. It could be meaningful to think about the usage and application of English system of measurements instead of metric system in their case.
- Both, sighted and non-sighted pupils built quite exact model of the explored room and thus, as regards the mental representation of the space, the world of non-sighted is not different in comparison with that one of people who are sighted. The difference is the way one gets information about the space. Through the sense of sight, one can obtain an overall knowledge of the environment, whereas one can achieve it through an analytic way, if s/he employs the haptic perception.

Since there does not exist any methodical guide for teachers of mathematics at special primary and secondary schools, interviews, remarks and observations of this research might be useful for them.

Except of some above mentioned proposals for future phase of the research we consider as interesting to observe the perception of the space and its object in connection with language as an individual tool. In what way the language and exactness of expression might influence the knowledge, but not only in the case of non-sighted pupils. The other improvement might be done in connection with realization of similar experiment with more pupils. However, we cannot influence the number of non-sighted pupils who will participate.

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9 Appendix

Appendix 1

Name: AL

Age: 21

Visually impaired since: 7 years old

Previous education: Liceo classico = oriented on humanistic subjects

Present: University Palermo – Airspace Engineer, 3rd year of study

1st problem:

AL1: 16 x 3 = 48

48 - 42 = 6

R: How did you get this result, what strategy did you use?

AL2: I got it spontaneously.

R: You haven't use any particular mathematical language.

2nd problem:

1st part

(…reading the first sentence….)

AL1: OK, we are in the 1st quadrant of the Cartesian plane.

R: Yes, we are in the 1st quadrant.

AL2: The point is (-3, -2) and we are in the 3rd quadrant.

R: Which strategy did you use to find symmetric point?

AL3: I build in my mind Cartesian plane and then I drew approximately the point P. After that I imagined symmetric point of P which I can get exactly, so its coordinates are –3, -2. The way I used is instinctive. The coordinates are opposite.

AL4: We have the point (–2, 3). We are in the 4th quadrant. The coordinates of the symmetric point are (3, -2).

R: Which strategy did you use? The same like before?

AL5: Yes.

2nd part

AL6: (he is thinking, he is repeating the point P (3,2)….). Wait a minute, I need time.

R: If you can talk loudly please, it is better for us, so we can tape your voice.

AL7: I'm not used to talk loudly. I have the point P (3,2). The bisector divides the quadrants into two equal angles. Each angle is 45 degrees. The equation of the bisector is y = x. I have to find the symmetric point. I need to draw. (he is preparing the drawing tools, he draws Cartesian plane). It looks bad, I drew the bisector in the bad way. Here, there is the bisector, here; there is the point P. The symmetric point should be (-3, -2). By connecting two points I have the symmetric points.

R: It is a good strategy.

AL8: I used to draw too many things as engineer, to be able to explain them. My professors wish me to draw all the time. And so I'm obliged to do so.

R: So you satisfied your professors.

3rd problem: (repeating the problem twice)

AL1: Usually, I don't care about problems like this, but I can try.

R: Would you like to write?

AL2: No. Because usually I don't need to write it. I'm used to write only when the steps are much more complex than this one.

AL3: What have we got? The final price is 28. Could I write the proportion?

28 : 30 (I'm not sure, I don't want to say something wrong)

28 : 30 = 2,50 : 100

By solving this proportion I will reach the right solution. But I'm not sure. Maybe I said some nonsense. I usually don't care about problems like this one. They are not my favourite ones.

4th problem:

1st part

AL1: How are the triangles placed?

R: …clarifying the problem… Would you like to draw them?

AL2: More less I understand. For example, we can consider two triangles, which are isosceles.

R: (repeating problem one more time)

AL3: I have no idea; I couldn't be able to solve this problem, to find the relations.

2nd part

AL4: The relation between these two triangles is 3. Yes, one triangle is triple than the other one.

R: I return to the 1st problem. How could you define the two triangles?

AL5: I could define them that one is triple than the other one.

Name: GR

Age: 45

Visually impaired since: birth

Previous education: Liceo classico, graduated in philosophy

Present: teacher of philosophy at secondary school

1st problem:

GR1: I'm going to try 60 (the age). By adding 18 to 42.

R: Why are you adding 18?

GR2: If one year is passing for father, the same is passing for the son. Now I'm trying to use the associative property. If I add 18 + 42, I have 60 and if I add 18 + 16, I have 34.

GR3: The father's age will never be triple than son's age in the future.

GR4: Because, at the beginning 42 and 16 aren't in a triple relation.

R: What is your procedure?

GR5: Because 42/16 is about 2,10 (approximately). And then by adding one year for each together this relation doesn't increase but gets less and less.

R: And if I would ask you about the past…

GR6: Of course. I have to adopt the inverse property. Let's try to subtract 4, we have 38 and 12. Their relation is more than 3. So let's try with 3. So we have 39 and 13. Now we have correct result, father's age is triple than son's.

R: How would you define your strategy?

GR7: Arithmetic strategy.

3rd problem:

GR1: At first we have to reason in the inverse way.

R: How in the inverse way?

GR2: From 28 + 30%. Now I'm introducing variable x.

GR3: You told me that x is initial price.

R: No, I didn't tell anything about x. So what is this x?

GR4: The initial price.

GR5: x - 2,50 – 30%

R: 30% of what?

GR6: 30% of (x – 2,50)

R: Yes, might be, but where is 28. you haven't use it.

GR7: 28 + 30%.(28)

GR8: (calculation 30% of 28) 28 + 7,4 + 2,50 = 35,4 + 2,50 = 37,9 (calculating in Braille)

R: What kind of approach did you use?

GR9: Empirical approach.

R: What do you mean by empiric?

GR10: Arithmetic approach. I believe my result is correct.

Annotation: No geometry at the school.

Name: MO

Age: 17

Visually impaired since: birth

Present: 5th year at the Social-Pedagogical School (lot of algebra, some analyse, analytical geometry)

1st problem:

MO1: By calculating 3 x 42. I need a calculator to find the result.

R: Don't worry about calculation, I'm interested only in steps = the way of solving.

MO2: Now I need to multiply 3 x 16. Then I think to have difference between 48 and 42. I'm not sure if it is correct…. Well, in 6 years the father's age will be triple than son's.

2nd problem:

MO1: all graphics are in my mind. The symmetric point is (-3,-2)

R: How did you get the solution?

MO2: Now, I need the hands to construct Cartesian plane. I have to connect point P with the centre to get symmetric point. It means, the rectangular is divided into two triangles. I have to move to the other side from 0, to - infinity. Symmetric point will be placed in this part of the plane. That is how I get point [-3, -2].

R: Do you use to draw so you understand?

MO3: Sometimes.

R: OK, what about symmetric point of Q [-2,3]?

MO4: I imagine where is Q placed. It is in 4th quadrant.

R: Are you sure?

MO5: Not in 4th....?

R: In the second.

MO6: [2, 3]

R: Not according to y-axis, but according to centre [0, 0].

MO7: Maybe in the 3rd, but I'm not sure. No, we have to move here. (he shows 4th quadrant)

R: If you have difficulties, do you need to touch concrete design?

MO8: Of course.

2nd part:

MO9: I need your hands again. I have points P, Q. I cannot give you the answer; I need graph with units (measure). I connect the bisector and then I find the intersecting point.

R: How do you draw this line?

MO10: What line? I will use the ruler.

R: But still, how? Line from point P is infinity long, what angle you will use?

MO11: It is not right angle.

R: Which way do you prefer, by touching or algebraic way? You should fine the symmetric point.

MO12: Yes, it is in this part. (he showed opposite half plane where P is)

3rd problem:

MO1: I have to do € 2,50 and then 30% remains. 28 + 2,50

R: Could you solve the problem in algebraic approach?

MO2: How? Do you mean by using variable?

MO3: I don’t like it, because the procedure to write it in Braille will become to long.

R: If you cannot write it, then try to say it by words.

MO4: € 2,50 – 30%

R: 30% of what?

MO5:…

R: What is the variable?

MO6: ....

R: The variable is what we are looking for.

MO7: Of course.

R: But what are we looking for?

MO8: Initial price. So we can consider variable, let's call it whatever.

MO9: y, which is initial price. Maybe € 28 + € 2,50 = 30,50. And then € 30,50 – 30%.

R: According to algebra how could you solve the problem, you didn't use the variable.

MO10: The final result is not € 28.

4th problem:

1st part

MO1: What does "proportional" mean?

R: For example, one side is 4-5 times bigger than the corresponding side of the other triangle, and so on for the other sides.

MO2: I need your hands again.

R: What is the relation?

MO3: According to me, the triangles are maybe proportional.

2nd part

R: So what is the relation of proportionality between the two triangles?

MO4: Three.

Name: VL

Age: 26

Visually impaired since: 14 years old (until 13 able to see only 1/10)

Previous education: Social-Pedagogical Secondary School

Present: student at University - to be a teacher at kinder garden

1st problem:

VL1: 16 x 3 = 48. 42 – 48, wait a minute … 48 - 42 = 6, so in 6 years father's age will be triple than his son's age.

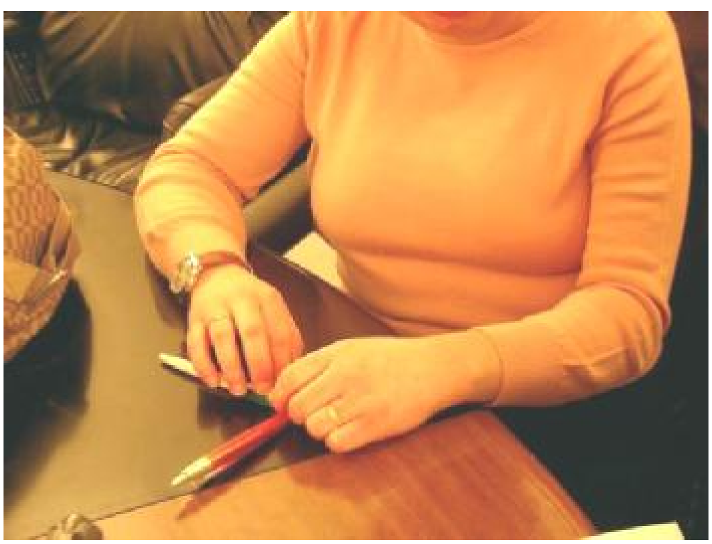
R: What was your strategy?

VL2: By multiplying 16 and 3 I will find when father's age will be triple than sons.

2nd problem: (…repeating the problem…)

VL1: I need 2 pens, to imagine Cartesian plane. (lot of confusion, mixing x- and y- axes)

VL2: Number 3 is approximately here, number 2 is approximately here and the point is placed approximately here.



R: Which is the symmetric point respect to the origin?

VL3: What does is symmetric mean? Parallel? I can't remember…

R: (explanation of term symmetric)

VL4: (she finds the symmetric point in 1st quadrant)

R: Could you find the coordinates of it?

VL5: x = 5, I'm not sure. I'm not able to find it. Maybe with the gum table I could solve it …

R: Do you want to solve the problem to find symmetric point of Q (-2,3)?

VL6: I need the pens again.

VL7: Where is –2? (she pointed right side of y-axis, finally she found the point Q)

R: Where is its symmetric?

VL8: The symmetric point is approximately here, (-4, y)

R: But y is general? Can you say it concretely, more less?

VL9: I'm not able.

2nd part

R: (explaining the last part of problem with bisector)

VL10: I don't understand how to solve it, because the bisector is not parallel neither to y-axis, neither to x-axis.

3rd problem:

VL1: I have to start by relationship of proportionality. $\frac{2{,}50}{28} = \frac{30\%}{x}$

R: What is the x?

VL2: Initial price.

VL3: I have to multiply 28 and 30% …

4th problem:

1st part

VL1: The triangles are isosceles.

R: Which strategy did you use?

VL2: Intuition.

VL3: The triangles are isosceles; one is bigger than the other one. They are equal.

R: What do you mean by equal?

VL4: No equal, but their sides are in relation of proportionality.

2nd part

VL5: Do I have to consider about numerical or geometric relation?

R: Numerical, because you have length of the sides.

VL6: 1 and 2 (consideration 9,6 and 3,2; 6 and 2)

R: How do you see (imagine) the two triangles?

VL7: I imagine they are isosceles.

R: So in what relation are 2 triangles?

VL8: 1 and 2.

Name: DO

Age: 21

Visually impaired since: birth (able to see 10%)

Previous education: Secondary Grammar school, Levoča, integrated student

Present: Faculty of ...UK, Bratislava

1st problem:

DO1: System of two equation, maybe? But if ..., no, system of two equations is a stupidity. Oh good, these logical problems are so easy!

DO2: Well, ... (laughing). I don't know. At first it would be good to find some relation between the years, let's say what is the difference. So, it is 26 years. Right?

R: Yes.

DO3: But, what is it good for? I feel now like damn moron, it is very easy for sure! Oh my goodness! Let's go to the next problem, maybe later I can solve it.

R: OK.

2nd problem:

DO1: Wait, what it is symmetric point? It will be maybe its image according to [0,0].

R: Yes.

DO2: If it is [3,2] it means, that it must be [-3, -2].

R: Why? What is your procedure?

DO3: You can say that you only add the number signs so you remove it the 4th quadrant.

R: OK, and what about point [-2, 3]?

DO4: It is [2, -3]. So I had moved it from the 2nd quadrant into the 4th quadrant. That before it wasn't from the 1st into the 4th, but to the 3rd then.

2nd part

DO5: ...

R: First, what is that line like? Can you imagine it?

DO6: It is line with points [1, 1], [2, 2], [3, 3], etc.

R: Ok, so it is the line, which divides quadrant into the halves. According to this line find the symmetric point of [3, 2].

DO7: It would be fine if the point pass the line, but now you have to ...

R: You don't have to tell me concrete coordinates, just procedure how you can get it, or just to tell me where is the searched point placed.

DO8: Maybe it is point [3, 4].

R: Why?

DO9: Because, I don't now. Now it could be symmetric according to that line.

R: What does it mean symmetric according to that line?

DO10: It should be also some turnover.

R: What do you mean by turnover?

DO11: I don't know. At the bottom, points should be equally distant from that line.

R: Is that enough?

DO12: Apparently you have to take into account direction of that line.

R: Ok, what will be that direction?

DO13: It divides the 1st quadrant as though into the half.

R: What if you connect point [3, 2] with the point you have found, so you have line. What about this line according to the line y = x?

DO14: It should be markedly perpendicular. Then I said it wrong, it is not correct point, it must be different.

3rd problem:

DO1: Well, it is maybe 28 multiply by 1,3. So we have 30% upwards and then we add 2,50 and we should have result now.

R: Good, so let's go to the last problem.

4th problem:

DO1: One more time please.

R: (...repeating the problem...)

DO2: Clear, I think they are similar.

R: Nicely. (reading the 2nd part of the problem)

DO3: The second one should be three times bigger.

R: Good. That is all. Do you want to return to the 1st problem or no?

DO4: Yes, let's try, it must be easy.

1st problem:

DO4: Clear, you cannot use difference, but portion. Hence 42 / 16, so we have some number...

R: What is representing this number?

DO5: It is some coefficient saying about difference between the ages at the moment.

R: Very good.

DO6: And I need to triple it. So that number I get I multiply by 3.

R: OK, I'm dividing 42 by 16; it is approximately 2 and something.

DO7: So I have the relation between the ages now and I want to triple it. So what should I do? Do we have some time limit?

R: No, no. As much as you want.

DO8: Then I can both of the numbers divide by that coefficient and the numbers I would get I would multiply by 3. Then I should have the result. Can it be like this?

R: Let's see: you said to divide 16 by 2 and something, it is almost 8. Then you said to multiply it by 3, so we have 24. And what should I do with 24?

DO9: The same procedure with 42. So we get 20 and then multiply by 3. I think it should be the wanted ages...

R: 20 multiply by 3 is 60.

DO10: Yes, 60 and before I get 24. But they are not exact numbers, only approximate.

Name: DS

Age: 25

Visually impaired since: 2 years old

Previous study: Special secondary school, subject: music

Present: Pedagogical Faculty UK, Bratislava, subject: music

1st problem:

DS1: Hmm, wait. Both of them are getting older. It will never happened because there is still the same difference between ages. 42 –16 = 26, it is all the time the same.

R: Really?

DS2: Yes, let's see, for example in 6 years: 42 + 6 = 48, 16 + 6 = 22, 48 – 22 = 26.

R: That is true, but I'm not asking about the difference of ages, but in how many years father's age will be triple than sons.

DS3: 3. 16 are not equal 48. How should I calculate it? I have to try all the time and divide it by that age.

R: Hmm.

DS4: Will it ever happen?

R: Why do you doubt of it?

DS5: Can the result be negative?

R: Of course.

DS6: There is just one moment like that; I have to go back into the past because now he is less than 3 times older. (thinking for a while)

DS7: 13, 39. That fits; it means it was 3 years ago.

2nd problem:

DS1: I shall move by 3 in one direction and by 2 in other direction. But I don't remember exactly how. I don't remember which one is x-axis and which is y-axis. I don't know.

3rd problem:

DS1: in 30%, so it decreases to 70%. Thus, 28 = 70%. Next I would calculate 100% of it and next I would add 2,50. Then I have initial price.

R: Excellent.

4th problem:

1st part:

DS1: What are proportional sides?

R: (explaining of the term)

DS2: Clear, so the angles are not changing. Isn't it in some relation with direct proportion? But I don't know the name for it.

R: Ok, let's continue with second part.

2nd part:

DS3: 3 times bigger.

Name: MH

Age: 21

Visually impaired since: since birth

Previous study: Secondary Grammar School - Levoča, special class

Present: Department of journalism Comenius University, Bratislava

1st problem:

MH1: I would try until I get it. Now son's age is 16, father should be 48, but he is not. So I take random number and I will try, for example: 30 and 90. Son will be 30 in 14 years, but father won't be 90. I shall just try.

R: OK.

MH2: Maybe I can construct system of two equations with to variables.

R: Well, how is it?

MH3: 3.y = x, where y is age of the father and x is son's age.

R: And what about the second equation?

MH4: I don't know. Suppose we shall use number 16 and 42, but I really don't know.

2nd problem:

3rd problem:

MH1: 28 Euro is 70%, so (28/70) * 100 + 2.50 and the result is initial price.

R: Very good.

MH2: One can solve it because it is from real life.

4th problem:

1st part

MH1: One is bigger and other is smaller. Should I say one word, one name for it?

R: Yes.

MH2: All angles are equal; I don't know what to say more, for sure.

2nd part:

MH3: The sides of second triangle are 3 times bigger then sides of the first one.

(Annotation: Almost no geometry at secondary school.)

Name: MT

Age: 18

Visually impaired since: 2 years old

Present: Secondary Grammar school, Bratislava, 4th year of study, integrated student

1st problem:

MT1: We will solve it by as follows:

42 x

16...... x

But I cannot construct the equation. Maybe x + 42 = 16 + x. I don't know. Maybe?

R: What about 3, you haven't use it?

MT2: I don't know.

2nd problem:

1st part

MT1: I would solve it by turnover through the point [0,0]. First x-coordinate, so I turnover 3 through 0 and I get -3. In the same way with y-coordinate, so I get -2.

R: What about the point [-2, 3]?

MT2: Similar procedure, from -2 I get 2. And from 3 I get -3.

2nd part

MT3: y = x is line that contain 45 degrees with x-axis, with y-axis as well, naturally. I imagine point [3, 2], I make perpendicular line to that line y = x. The same distance like it is the point from the line, I spread on that vertical.

R: Good. So it was geometrical solution. May I ask you how you will calculate it to get concrete coordinates?

MT4: I don't know. It is clear that it will be on the opposite side, it follows that y cannot be negative. I don't know.

R: Do you have idea in what quadrant it is placed?

MT5: I think, x will be negative and y positive. What quadrant it is? I don't remember.

R: Second.

MT6: Clear, I remember now. So in the second.

R: And what about point [-2, 3] according to line y =x?

MT7: So I will imagine it again.

R: In which quadrant is point [-2, 3]?

MT8: In the second. I by turnover I will move into the first quadrant. I think so.

3rd problem:

MT1: OK, so 28 Euro is 70 % of first reduced price. 28 is 70 %, so 1 % is ....

R: You don't have to calculate it in your mind, just tell me what is the next step.

MT2: I would calculate the basis, which is 100 %. Actually no. We have 3 different prices, 3 handbags. I have the third price, second price I will get: 28 Euro is 70%, 100 % will be x Euro, which is more than 28. Then plus 2,50 and the result is initial price.

R: Very good.

4th problem:

MT1: They are similar.

R: Excellent. (...reading the next part of the problem)

MT2: The second one is three times bigger.

R: What was your procedure?

MT3: 6 / 2 = 3, 9.6 / 3.2 = 3.

Name: PL

Age: 23

Visually impaired since: birth

Previous education: Special secondary school, subject: music

Present: Faculty of Mathematics, Physics and Informatics UK, Bratislava, 4th year of study, subject: informatics

1st problem:

PL1: So, it will be fraction, maybe.

R: Hmm.

PL2: So, we will construct equation with one variable that we will calculate.

R: Could you tell me your mentioned equation?

PL3: OK, so one more time, father is 42, son 16. Well, in how many years? Thus 42 +. No, not that way. Hmm, hmm. Three, 3 times 16 + x = 42 + x.

R: Good, and hence you will calculate x somehow. Right?

PL4: Yes.

R: OK and what is x?

PL5: I don't know now.

R: I mean, that x is your answer.

PL6: It is variable which says in how many years it would happen.

R: I'm satisfied.

2nd problem:

1st part

PL1: Symmetric point according to [0,0] ? And now you want some procedure? I 'd imagine how is that point looking like, actually it is enough just to edit the number signs.

R: All right, this is what I'm interested in. What does it mean to edit the signs? What is your result?

PL2: [-3, -2]

R: Why? How did you get it? What is your procedure?

PL3: Now, I see in front of me the origin of the coordinate system, such that cross, I see the point in the right quadrant, and I know that I need to send it into the left down quadrant, I edit both signs so the point is on x-axis and as well on y-axis negative. And that it is. It is too easy.

R: OK, the same with the point [-2,3].

PL4: Again, I see it and I know that I need to send it into the .... Well, again, I just see it, nothing else. Just to reverse the signs.

2^{nd} part

PL5: I would calculate it by analytical geometry.

R: OK, but could you tell me, according to the line y = x where is the search point placed? What is your image?

PL6: y = x, well, again it will be the same as I already said. Because that line pass point [0,0], so there will be not too much to change.

R: But still, according to the mentioned line, where is the point placed? If you consider some half plane?

PL7: No, it will be not as I said. Oh, I would only calculate it.

R: So you don't know to image it?

PL8: No, because it is evident that line by-pass that point and now it is not so easy for me. I would calculate it.

R: How would you calculate it? What would you use?

PL9: .... At the moment I really don't know, I don't remember the things of analytical geometry. I don't know.

3^{rd} problem:

PL1: Well, I can ... It is a bit inexact, 30% afterwards as the price get lower in 2,50...

R: It was reduced first about 2,50 and then about 30 %, so the final price is 28.

PL2: I would divide actual price by 70, because 70 % left of the primary price, and then I would multiply it by 100, so I would have the price after reduction about 2,50. And last, I would add 2,50 to it. Now I would have initial price.

R: How would you call your approach?

PL3: Algebraic.

4th problem:

1st part

PL1: Once again please.

R: (repeating the problem)

PL2: They should be equal. Right?

R: In what relation are these two triangles?

PL3: They should have equal sides, I think.

R: But if I talk about proportion, for example, one triangle has side 2 cm long and the second triangle has corresponding side 6 cm long. Are these two also equal?

PL4: No, no, no. Well, what does it mean proportional sides?

R: The proportion between all pair of sides is preserved.

PL5: (silent for long time)

R: What are you thinking of?

PL6: I don't know what we are solving now.

R: (explaining the problem on named triangles ABC and A'B'C') So in what relation are these triangles?

PL7: I don't know.

2nd part

PL8: The sides are triple. The angles are same, of course, one triangle is three times bigger than the other one.

R: Can you say something more?

PL9*:* No.

R: As you said now, they are not equal now. Do you know a term for it that we used to use?

PL10: Oh my goodness, how we call it? I don't remember. It is some ratio diminution...

Appendix 2

Appendix 3

M – University student, 20 years old

E – Experimenter

M1: Well, first I do this (*he snaps his fingers and paces out the room*) to find out what shape is the room.

M2: I would say, rectangular perhaps square, but, maybe rather rectangular shape is this room. At least, it seems to me like that.

M3: And we will describe it, let's say, the base point is the entrance door.

E1: All right.

M4: So, we have entered the room […] I don't know […] I don't know whether it is rectangle or square […] let's say it is rectangle.

E2: Ok.

M5: So we have entered the room on the one side of the rectangle, the shorter side.

M6: The door is in the recess. Ups, I have found the coding system, shall I encode it?

E3: Rather no, because it will start to peep.

M7: So in the recess is the coding system situated. Shall I describe it like this? In details, everything?

E4: Yes.

M8: On the right sight of the recess I have found light switch, coding system and lath with cables.

M9: The recess is much lower than the room; its high is perhaps 2 meters, maybe not, something less than 2 meters.

M10: On the door I have found some poster; the door is made of banded glass, it has no standard door handle, only some pulling.

M11: The recess is on the exterior bounded by metal laths.

M12: So we go right. Here I find some cabinet. On the right side is the cabinet, which is … that is part of the recess.

M13: We come to the longer side […] exactly in the corner of the room and we continue along the longer side of the rectangle, the right side.

M14: Here is shelf cabinet with the door, other drawers. Do I have to go under the cabinet?

E5: No.

M15: Above the shelves is other cabinet. Next, there is small display cabinet that is part of cabinet, other cabinet.

M16: I'm still on the right side and I find […]What it is? (*He is "looking" at it by hands.*) It looks like trunk, which is situated on the […] something like bed table.

M17: We continue along the longer side and we come to the corner of the room that is untouchable. I have found printer.

M18: Now we are opposite […] opposite wall, opposite the door, it is again shorter side of the rectangle.

M19: Here is the window with the jalousie. Oh, it is balcony. I don't see the radiator. Yes, I see it […] under the parapet.

M20: I continue along this side. I have found telephone.

M21: It is […] I would say that balcony door is opposite the entrance door.

E6: How do you know it?

M22: I estimate. We can verify it. (*He is straight going to the entrance door.*) Well, it is almost. This door is narrower than the balcony door. (*He comes back to the balcony.*)

M23: On both side of the balcony there are windows with jalousie. The balcony's jalousie is either pulled up or balcony doesn't have jalousie.

E7: it is pulled up.

M24: Ok, on the left parapet is situated […] I don't know what, under is radiator, next the waste basket.

M25: Here I find interesting thing. It is pillar, which function is for me stranger. Perhaps, once it has been two rooms.

E8: I would say, it is rather supporting function [...]. Even I don't know, why it is here.

M26: It is quite big, cuboid's shape. (*He is measuring it by forearm and arm.*) Maybe 30 times 50, maybe 60 centimeters.

M27: We are coming in to the left upper corner of the room, considering the base point.

M28: Next to the parapet is the chair and on the chair is printer. Next to the chair, along the wall is table, standard PC table, it is a bit small.

M29: The PC is on the table, I see the mouse and the keyboard, which are on the classic telescopic pad, under the table is nothing interesting. After all, speakers. Then monitor, it has quite long diagonal.

M30: Here I see same cables on the ground.

M31: I continue by long wall. I have found chair, it is hard […] the second chair.

M32: The double-wing cabinet follows; I estimate it is hanger cabinet.

M33: Now we are in the left lower corner of the room, some boxes and air freshener is here. Shall I count them?

E9: Yes, you can.

M34: One, two, three, four, five, six […] six boxes.

M35: We come to the shorter wall, here is again some cabinet. We can not open it because of boxes.

M36: Behind this cabinet is the sink.

E10: How do you know it?

M37: Because the water flows.

M38: And it is all. No, still the centre of the room.

M39: So it was be edges, now we go in to the centre.

M40: This in the centre looks like a table. It is not office desk; it is like table at conference because there are chairs around it […] chairs with soft upholstery for long conferences. They are six, three at each side.

M41: In the front of the table there is no chair because you can not shift it because of leg of the table. At the other side of the table there is also no chair because of the leg. (*He goes to the other side of the table.*)

M42: No, here is another table. It looks like it is the same like the first one. They are connected like in letter T.

M43: At his second table that is […] or like this: the first table is settled vertical to door and the second horizontally. Simply, from the view from the door it is classic letter T.

M44: On the first table I have forgotten some boxes; they are the same as the ones in the corner and some CD box.

E11: How do you know it is box with CDs?

M45: I know it, because I use to buy CDs very often.

M46: At the second table, on one side there are no chairs, on the second side there is wheelchair and some office things on the table.

M47: Some books, one, two, three, four […] four books, the lamp that is on, au. And what else? Nothing more. No, something more.

M48: I wouldn’t give it to here, at least not so close. Flowers. So they don’t blow it down.

M49: This table has drawers as well. Two, four. Are they also on the other side? No, they are not. So these tables are not the same, I was wrong.

M50: I think it is all.

R – 14 years old boy, 9th class of special primary school, non-sighted since birth

E – Experimenter

R1: So, on my right side, as I entered the room, there is the wall inlaid by wood, it has rectangular shape.

R2: Then there is cabinet, also shape of rectangle, classic cabinet with rectangular shelves. Below, there is door, on the cabinet. The door has holders of circle shape. What is here onwards? Other cabinet, which is a bit different, it has door up, the holders also circular, the same down, it has also rectangular shape and here are actually two cabinets situated one on the other one.

R3: Then here is other cabinet, but this one has showcase that is made of glass and the holders are like trapezoid, it seems to me like that. Then, it is shape of rectangle, here down, this door and the glass, whole it is rectangular shape.

R4: What is it?

E1: Maybe you don't know what it is, try to only describe it.

R5: It is like […] it curves like in to semicircle. At this side […]

E2: At which side?

R6: At the front side it curves like in to semicircle, at the opposite side it is […] it is normal […]. It is something of semicircle shape and it is put on the cabinet which has rectangular shape.

E3: And how big is the cabinet?

R7: That cabinet is high about […] something more than knees or like my thighs.

R8: The other things that are here […] what is it? It is […] what it is? It is something of shape […] rectangle? I rather make sure. Yes, it is something of rectangular shape and the typewriter is on it.

R9: As next, window […] rather the door […] of balcony, I think, of rectangular shape. Whoops, it is maybe parapet and here are […] I don't know how it is called but it is for sun […].

E4: Jalousie.

R10: Jalousie, maybe. And parapet is also of the rectangular shape, the same as the first one, previous. Then it is here wall and the room is curving and I stay actually […] at my right side is the wall and here, on the left in the front is the chair, computer's chair […] most probably.

E5: How do you know it?

R11: Because computer's chairs are like that. And in front of it is the table, rectangular. Some cups […] something is in it.

E6: What?

R12: I don't know […] flowers are in it.

E7: How do you know they are flowers?

R13: Thus, because I dropped it. They are live flowers because they are not dry, simply […] they are flowers. I don't know how to define it.

R14: Then you turn right, here is a bit of wall. Then here is a ball and what else is here? This one is computer; it is actually computer, the hard part of computer, the box that is placed on the table. The table has rectangular shape; then here is also the monitor. And the table is made like […] it is here telescopic rectangular shelf; on it is the keyboard with the mouse and with pad for the mouse.

R15: Then further, here […] so I don't forget something […] maybe no. Then here is again the wall, and here down is the chair. What shape? Simply, typical chair, next to it is another one, but on it is something […] something like cylinder is placed on it, the cylinder is placed on it.

R16: Then, is the cabinet, the big cabinet, not like the previous […] the big cabinet. It is not parted, it is double-wing and it has rectangular shape. Also the doors of the cabinet have rectangular shape.

R17: Then it is here another cabinet, but this one is built in, also it is rectangular and double-wing. Then it is here another cabinet, the same as the first one. Between the cabinets are gaps. Between the doors of two cabinets are gaps. They have rectangular shape as well.

R18: And then I am actually […] then it is door. Then it is actually the end of the wall. I returned to the beginning, I walked around whole of it. The room is […] if I guess correctly […] it has rectangular shape or […].

E7: You can check it, you still have 12 minutes. You managed it quickly.

R19: I go to see whether I overlooked something. (*He is walking through the room one more time and repeating quietly what he "sees".*)

R20: This cabinet is a bit ajar.

E8: And what is in it? Can you say it?

R21: Something […] (*he opened the door*). It is the sink.

E9: How did you know it?

R22: I have heard water and then I went to see and it is the sink. In the last cabinet before the door, at the end of the room, it is actually the sink. And the water flows in it.

E10: Did you know it according to the sound?

R23: Yes, I have heard that something flows here, but first I thought that something flows in to that cabinet or […] I didn't suppose it is the sink. The water flows by poor current. It is typical sink.

E11: I close the door a bit again. Did we walk through all the room?

R24: I was thinking about […] I don't know how to say it, but at the beginning of the walls is something […] I don't know what it is, but it has shape of rectangle.

E12: It is not important. This whole building is in that style.

R25: I was thinking what it is good for […] Anyway; I go to see here in the centre whether I haven't forgotten something. Yes, I have.

R26: As there is the centre that table, so along the rectangular table, there are chairs and beside it is another table. On it is something […] what is it? It is something with pyramid shape. I don't know what it is. It is something battery operated.

E13: So what it could be? Rather do not remove the batteries out of it.

R27: What it is? I really don't know. I can't remember.

E14: It doesn't matter.

R28: On the sides it has triangles, but I don't know what it serves for. It can be anything, any definition for it.

R29: I go this way. Moreover, on the table are books, several of them are placed on each other.

E14: How many?

R30: Two, three […] maybe four. And next to them is the table lamp, which is on.

E15: How do you know?

R31: Because […] you can […] when I bended down like this, I feel the light.

E16: Are you light sensitive?

R32: Yes, and it is hot. (*He spilled the water from the cup in which the flowers were.*) I didn't mean to do it.

E17: It doesn't matter, it doesn't matter; it is my fault, I should pour out the water.

R33: I should know it that it is here. Actually, at the other side of that table which is next to the second table […] (*he is laughing*). Basically, here are two tables, one is placed by height and one is paced by width beside it. I think it is like that.

E18: So check it!

R34: So this one is by height, in the shape of rectangle and this one is placed by width. Yes, it is like I said it.

R35: At the table that is placed by height, there are chairs at both sides.

E19: How many?

R36: At this side, here are three.

E20: What does it mean at this side?

R37: From […] from the front […] actually, if I stay direct at the door, if I enter the room, I come to the table and at the right side are three chairs and at the left side also three chairs. Thus, six chairs are at the table, six people could sit here, at this table.

R38: Have I for gotten something more?

E21: I don't know; it is up to you. You can check it, you still have 7 minutes.

R39: Some details […] whether I […] by chance.

E22: You can walk through it one more time.

R40: Here is something glued. As I enter the room, on the right side is something […] I don't know […].

E23: Rather do not press it because it is alarm.

R41: And this one? These are two switches actually. It is rectangle, no rather square. And the push-buttons are rectangular. It leads the cable out of it.

R42: (*He walks through the room.*) Perhaps I have forgotten that […] as here is the first cabinet, so on one side there is showcase and on the other one there isn't. So this I overlook in the first case.

E24: You don't have to notice it.

R43: Some cables […].

E25: Where we are now?

R44: At the computer. On the right side from PC box there is printer. If it is printer […] then there is […] actually, on the right from the printer there is pushed the chair under the table. And here is a cable.

E26: You don't have to put attention to it.

R45: At the ground, there is also cable so nobody stumble.

E27: You don't have to look for things on the cabinet. We are interested only in big objects of this room.

R46: I try to go into the particulars so I don't forget anything. That is why I check it so exhaustively. So now I came again to the door, which is also rectangular. It is made of glass. It is something glued on it.

E28: A poster.

R47: And didn't I forget what is on table? Maybe yes. Moreover, on the table there are packets. They are […] it is neither cube nor […]

E29: How do you know it is not cube?

R48: Because this one side […] this one […] this one side […] front […] If I hold it like this […] it is longer than the other side. Actually, the horizontal side is longer then vertical. It depends how you hold it. I have it along, horizontally to me. I don't know what is inside it.

R49: And under it is bigger packet which has shape of […] it is also not the shape of cube […] but it is shape of [...] what can I compare it to? It is shape of cuboid. Also the upper packet has had this shape. Yes […] it is cuboid.

E30: This is water of flower. Is here something more?

R50: I consider […] I try to find whether I overlooked anything […] this is lamp […] books […] it is all, perhaps.

E31: So is it all?

R51: According to me yes.

E32: So now you are able to imagine the room and able to build the model of the room by using the small packets.

R52: Maybe, I don't know, I'm not sure about that, if I could build it.

M – 13 years old boy, 7th class of special primary school, non-sighted since birth
E – Experimenter

M1: At the moment the rectangular door is here, it is high about 2 meters.

M2: The wall is here, at which are three like square equipments, the lath is here that leads to the roof.

M3: Here is the cabinet that is also rectangular; it is high also about 2 meters, 1 meter.

M4: It is also the cabinet here, on […] in which is […] in shape of heart […] like […] maybe chocolate or something may be in it.

E1: You don't have to say about details, rather observe big things.

M5: Shelves […] also rectangle in shape. The glass part of the cabinet that is also rectangular.

M6: Other cabinet […] in shape of […] also, like […] maybe cuboid can it be like or small cabinet. Above it is other cabinet situated.

M7: Other cabinet with glass part of rectangular shape. Also down here is small cabinet.

M8: Here is high box at a height about 1 meter and it is also like cuboid in shape.

M9: Under it is cabinet or small cabinet, which has shape of […] also of cuboid, one can say.

M10: The parapet is at the window here. And on it is equipment, it is typewriter.

M11: Then I see here window blinds, they are like […] it whole forms rectangle.

M12: It is window here that is also like rectangle and high about 2 meters, it can be like that.

M13: Other window blinds of rectangular shape, the parapet in shape of […] also rectangle.

M14: Then I see here cable that leads towards to computer. And computer, right now is computer box here in shape of […] also like […] it can be maybe […] like prism.

M15: Then is here round ball.

E2: And where is the ball?

M16: The ball is situated on the floor and it is […] it can be under the table, one can say it.

E3: How big it is?

M17: It is like […] like […] about […] .

E4: Try to guess only. What do you think?

M18: It is not so big […] like […] appropriate, medium size it can be.

M19: It is monitor here in shape of […] also like […] actually this front part that is here forms like square and under the monitor is such a table. On it […] it is here also case of keyboard, which forms shape of rectangle.

M20: On the table we can find more mouse pad, it is also in shape of rectangle.

E4: You don't have to say these details. Rather try to say how high this table is.

M21: The table is from the floor about […] it is small enough, so […].

E5: Ok, further .

M22: I go at the moment by the wall. And down here is the chair of […] wooden and on the second chair is an item that is cylinder in shape.

M23: It is here […] yes; item that has shape of cylinder.

E6: It is me .

M24: The cabinet is here, also such a […] the door is like rectangle, height is about meter or two.

M25: Here is the wall, other cabinet. These doors constitute of two rectangles, also the height of meter or two one can guess.

M26: Another, the same cabinet. And here the door is. And on the wall is paper…

E7: You don't have to [...].

E8: And what is in the middle of the room? You didn't tell me.

M27: So, in the middle of the room is the table in shape of rectangle.

M28: At it is the chair, another chair, third chair.

M29: It is such a table here, next to it.

M30: The books are here, for example this one is rectangle in shape. The lamp, we can find here.

M31: And here is presently turning chair.

M32: And on it is […] such equipment in shape of […] pyramid, one can say it.

E9: And where is that equipment?

M33: Such a small pyramid. The equipment is situated on the table, not at the beginning but almost.

M34: Then it is here also one item.

E10: What is it?

M35: Here is […] it is […] it is flowers. But, pad in which they are is shape of cylinder.

E11: What flowers it is? Can you say what flowers?

M36: I think no.

E12: All right.

M37: Flowers […] and it is perhaps all.

E13: And is here something on this second table?

M38: It […] it is here the box in shape of […] such a cube […] it can be. Another box under it also like […] such a cube also, one can say.

E14: And if you stop, what sounds do you hear?

M39: You can hear here the whirr of computer and like […] as the water flows […] or like that.

E15: And where the water flows?

M40: It is like […] like in the cabinet […] you can hear it like that.

E16: Yes, it is in the cabinet.

M41: (*He went to check.*) Yes, in the cabinet is the sink.

E17: All right.

J – 14 years old girl, 8th class of primary school, sighted

E – Experimenter

J1: I came in to the room and from the door I see the window and balcony with jalousies. There are radiators at the window.

J2: In the middle of the room are two tables, which are situated in shape of T.

J3: Behind one is office chair and behind the other one are six chairs.

J4: At the wall, on the right side are cabinets, where the books are.

J5: On the left side is cabinet, up is the box, next to the cabinet are two chairs and further table with computer and under it is ball.

J6: At the door are cabinets, where is for example the sink, in one there are books.

J7: On the table we have packets, lamp, flowers, books and some watch that has shape of […] pyramid?

J8: Then, next to the computer is the printer.

J9: Up there are four lamps.

E1: We can skip it.

J10: I know.

E2: Do you see here some more objects?

J11: The telephone at the window, above the radiator where is the parapet or how you call it. So there is the telephone situated.

J12: On the chair is some tube.

J13: And perhaps nothing more.

E3: Ok.

D – 13 years old girl, 8th class of special primary school, visually impaired since birth

R – 14 years old boy, 9th class of special primary school, non-sighted since birth

E – Experimenter

D1: Rišo, those two cabinets that are on each other, are they at the wall?

R1: Yes.

E1: Dominika, do you want to ask more?

D2: […] (*Nobody can understand her.*)

E2: So can you describe what do you have here? Or tell me, what is what, regarding to your packets.

D3: Here are two cabinets on each other; here is the wall that leads into the room. Then there are two cabinets on […] on each other […] Then here is the ball, then chair […].

E3: How is the chair?

D4: Those are the two at the computer.

E4: And where is the computer?

D5: On this table.

E5: And which one of these is the computer?

D6: Wait, this one, no, this one.

E6: What about this next to it?

D7: It is the cup.

E7: The cup?

D8: […] Rišo, the cup was on the first table or on the second one?

E8: You can not ask like that. You must ask in the way I told you. You know.

D9: Rišo, the cup was on the table where the computer is?

R2: No.

D10: (*she is moving things*)

E9: Ok, so what have you done now?

D11: I put the cup on the other table.

D12: Then here are down cabinets […] cabinets on which is that cube.

E10: What cube?

D13: (*she is thinking for long time*)

E11: Don’t you know? It doesn’t matter. And further?

D14: (*few packets fall down*) Three chairs and […].

D15: Also three chairs were at the wall?

R3: No.

E12: That means you have to ask him, if you need something.

D16: Were they in the centre of the room?

R4: Yes.

D16: Were they at the table?

R5: Yes.

D17: The other three as well?

R6: Yes.

D18: (*she bears down few packets*)

E13: It doesn’t matter. I will help you to make it like it was. Well, so what have you done now?

D19: I put these three chairs to this table.

E14: Tell me what are you looking for?

D20: Some long […] packet […] some thinner, but wider […].

E15: Here you have one.

D21: Not like this one. (…*she found another one*)

D22: Those two cabinets were at the table? The ones that are on each other?

R6: No.

D23: At the wall?

R7: Yes.

D24: At the wall that you enter the room by?

R8: No at that one.

D25: No? So yes or no?

E16: You can not ask him like this.

D26: No? I didn’t understand what he said.

R9: I said no.

D27: Were they somewhere in the middle?

R10: No.

D28: What did you say, I didn’t understand.

E17: He said no.

D29: […] (*she mumbles something*)

D30: Were they by the other cabinet? Those two that are on each other?

R11: I don’t know what she means. Which cabinet? You must ask concrete.

D31: What did you say?

R12: You have to ask concrete.

E18: He doesn’t know what cabinets you mean.

D32: There, where the cup was? Wait, no. There, where was something circle on the cabinet?

R13: Yes. In this case yes.

D33: Was it in front of that cabinet?

R14: yes.

D34: Was it very close?

R15: Yes.

D35: Was it sidelong? That cabinet? Those two on each other…

R16: I don’t know how you mean it.

D36: I say about that cabinet, on which was something placed. Something round.

E19: Don’t you know Rišo? You can say, you don’t know.

R17: I don’t know.

E20: Dominika is rebuilding it.

D37: Shall I say what I have done?

E21: Yes, say it.

D38: I put these two cabinets that are on each other to the other cabinets where something round is on them. I put it sidelong. Closer to the door; there, at that cabinet […] As you enter the door, there where is the cabi […] the wall, more on […].

E22: Well, so there you built the wall […] right?

D39: I extended it.

E23: Ok and next? Do you want to ask something? Ups, it is falling down, I will help you […].

D40: And the other three chairs that were at the wall, were they closer to the cabinet?

R18: No.

D41: So close to the wall?

R19: Neither to the wall.

D42: (*silence for the long time*)

E24: Well, Dominika, so you need to know where the other three chairs were?

D43: Yes.

E25: Where else they could be?

D44: At that […] at that cabinet on which is that cube?

R20: No.

D45: Close to the ball?

R21: No.

E26: Ok Dominika, I permit you to ask him what you need.

D46: Can I ask him where those chairs were?

E27: Yes.

D47: Where were those three chairs?

R22: They were also at the table, the first one were from one side, the right side and the other one from left side.

E27: Dominika, here I'm giving you chairs. (S*he tries to put the chairs in to the model.*)

D48: And those first three chairs were there, where is the computer, right?

R23: No.

D49: (*She is rebuilding the model.*)

E28: Dominika, what have you done now?

D50: I put opposite those chairs and one more […].

E29: Here you have one more.

D51: […]

E30: Do you want to ask something?

D52: No more.

E31: No? So it is good like it is now? Do you think so?

D53: Maybe.

E32: Well, I turn it off.

S – 13 years old girl, 8th class of primary school, sighted

R – 14 years old boy, 9th class of special primary school, non-sighted since birth

E – Experimenter

S1: So those two boxes were somewhere close to the door?

R1: No.

S2: Was there in the middle of the room some table or something like that?

R2: Yes.

S3: Were there in the room four tables?

R3: Wait, four? No.

S4: I don’t have more questions.

E1: Don’t you?

S5: No.

J – 14 years old girl, 8th class of primary school, sighted

D – 14 years old boy, 9th class of primary school, sighted

E – Experimenter

D1: Were there nine chairs?

J1: Yes.

D2: It is probably all. Oh, those flowers were on that table of shape T?

J2: Yes.

D3: And that T was situated like that you looked at its bottom?

J3: Yes.

D4: On the upper part of that T, on that upper, was one chair, right?

J4: Yes.

D5: Ok.

E1: Is it all?

D6: Yes.

E2: But you wanted to ask more […] I remind you […] about the tube.

D7: I have done it.

E3: Did you ask about it already?

D8: Yes, actually that tube was on the chairs next to the cabinet and computer?

J5: Yes.

D9: And those chairs were between cabinet and computer?

J6: Yes.

D10: Ok. Good.

E4: Is it all?

D11: Yes.

D – 13 years old girl, 8th class of special primary school, visually impaired since birth

M – 13 years old boy, 7th class of special primary school, non-sighted since birth

E – Experimenter

D1: Mišo, that ball was at the cabinet?

M1: It was not at the cabinet, it was at the […].

E1: One moment, you can only answer yes or no. Ok? As she asked you: was the ball at the cabinet?, you say no. Nothing more. She will ask you. Do you understand?

M2: Yes.

E2: All right.

D2: Was it at the table?

M3: Also yes.

D3: Was it between the table and the cabinet?

M4: No.

D4: Was it at the table? Too much? So, […] Was it under the table?

M5: Partially yes.

D5: So then the chairs were also under the table?

M6: The chairs under the table?

D6: Under the table!

M7: The chairs were not under the table.

E3: Dominika, what do you mean by this?

D7: Where should I put the ball?

E4: Don't you know where to put ball?

E5: Is it the only thing you want to ask about, that ball?

D8: Yes.

E6: It is the only one.

M8: Or you can ask other way.

D9: I don't want to ask about something else, only about the ball.

E7: All right, you want to ask him only about the ball. So let's think about where else it can be.

E8: All right Dominika, we do it one more time from beginning. Tell me where the door is and where you enter the room. Describe what you have built.

D10: (*She mumbles something*.) […] this is that […] I don't know how to say it. That iron.

D11: Here I put this.

E9: What is this? That packet for me. What it is?

D12: It is wall.

E10: All right.

D13: Here are those two cabinets on each other.

D14: Other cabinet. Then, then, this is table on which is the computer. This, around it are the chairs.

D15: (*she bears down few packets*)

E11: It doesn't matter. I will put it back.

D16: Wait, wait, wait…

D17: So this is another cabinet, it was at the computer another such a cabinet. Then here is other cabinet.

D18: Then, in the middle is also the table, these are the chairs.

D19: And that ball I don't know where to put it.

E12: Only the ball is missing, right?

D20: Yes.

E13: Ok, so let's go from the beginning of the room and you will ask Mišo [...].

D21: Was it at the door?

M9: No.

D22: At the wall?

M10: No.

D23: At those first cabinets that are on each other?

M11: No.

D24: At the single cabinet?

M12: No.

D25: At the computer? Actually, between computer and cabinet?

M13: No.

D26: At the computer?

M14: Yes, at that table.

D27: At that short side?

M15: Where?

D28: At the short, shorter side of the table?

E14: If you don’t understand, say I don’t understand. Or […].

M16: Now I don’t know what shorter side you mean, whether […].

D29: That shorter side of the table. It has two long and two shorter sides.

M17: But it was not at shorter side.

D30: In front of the table?

M18: Yes, it was in front of the table. One can say yes.

D31: In front of the table?

M19: Yes.

D32: (*She is placing the ball.*)

D33: Was it closer to that table which is in the middle?

M20: No.

D34: Closer to the one with computer?

M21: Yes.

D35: Was it closer to the other cabinet?

M22: No.

D36: Was it exactly in front of the table?

M23: Yes.

D37: Maybe it is good now.

E15: All right. So is it good now?

D38: Yes, maybe.

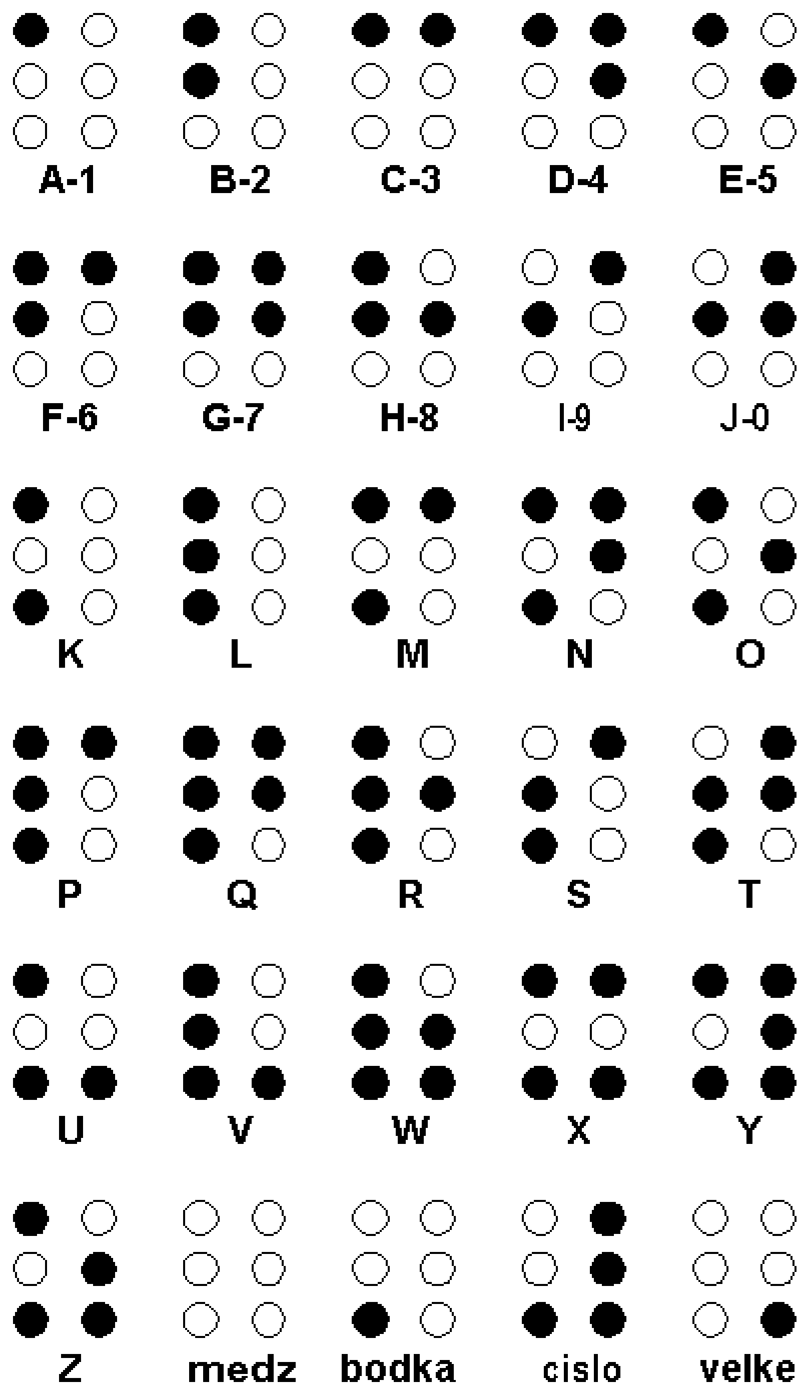
E16: Ok, so you don’t want to ask more?

D39: No.

E17: All right.

Appendix 4

Braillova abeceda



Appendix 5

Codice Braille italiano 1998

Letters:

a ⠁	b ⠃	c ⠉	d ⠙	e ⠑	f ⠋	g ⠛	h ⠓	i ⠊	j ⠚
k ⠅	l ⠇	m ⠍	n ⠝	o ⠕	p ⠏	q ⠟	r ⠗	s ⠎	t ⠞
u ⠥	v ⠧	w ⠺	x ⠭	y ⠽	z ⠵	à ⠷	â ⠡	ä ⠜	è ⠮
é ⠿	ê ⠣	ë ⠫	ì ⠌	î ⠩	ï ⠻	ò ⠬	ô ⠹	ù ⠾	û ⠱
ü ⠳									

Numbers:

0 ⠼⠚	1 ⠼⠁	2 ⠼⠃	3 ⠼⠉	4 ⠼⠙	5 ⠼⠑	6 ⠼⠋	7 ⠼⠛	8 ⠼⠓	9 ⠼⠊

Other:

! ⠖	" ⠦	& ⠯	' ⠄	(⠶	) ⠶	* ⠔	, ⠂	- ⠤	. ⠄
/ ⠲	: ⠒	; ⠆	? ⠢	“ ⠈					