Proceedings CIEAEM 61 – Montréal, Quebéc, Canada, July 26-31, 2009
Quaderni di Ricerca in Didattica (Matematica)", Supplemento n. 2, 2009.
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Introduction to the Working Group on „Problem Solving and Institutionalization of Knowledge"

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A main characteristic of 'mathematical problem solving' is the concept's ambiguity, vagueness and impreciseness of delimitation against other types of mathematical tasks. The subsequent research reports and developmental activities represent the broad spectrum of mathematical tasks that have been classified as problem solving, whether open or close problems, contextualised or abstract-symbolic problems. Apparently, problem solving in mathematics is an only loosely defined area.

However, it would be simplistic to only classify the papers of this section according to the type of problems that they discuss. A classification that draws on the functions of mathematical problems in mathematics teaching and in research on mathematics education is more pertinent as it penetrates those surface descriptions. In the working group on problem solving and institutionalization of knowledge, problems have been presented and discussed in various settings and with different aims. For instance, mathematical problems can serve diverse research purposes:

- As test items and activators in order to survey students' mathematical errors or metacognitive skills. These are classical normative research settings in which the students' responses are measured against a fixed mathematical or psychological agenda.
- As a task that allows the study of students' construction of mathematical knowledge. This is again a research setting but the grounding research paradigm is interpretive.
- As a strategic device to analyse social interaction and social pathologies in mathematics classrooms. The grounding paradigm of this kind of research is again interpretive, however it rather aims at a critical deconstruction of educational practice.
- As key incidents of crystallizations of an implicit structuring of mathematics teaching and learning. The research paradigm behind is critical-emancipatory as mathematical problems used in the classroom are always products of the development of ideologies.

Mathematical problems play a crucial role in the development of mathematical practice in the classroom. They can serve:

- As examples within the construction of a didactic-technological learning environment, where the particularities of the technology influence the ways in which the students can act mathematically on the given problems.
- As a means for activating creativity and mathematical autonomy in the mathematics class. Apparently there is a dialectic relationship between the quality and introduction of the problems and the guidance provided by teachers.
- As a tool to make the students construct new mathematical knowledge.
- As a trigger for the transition from iconic to symbolic forms of representation. In these classroom activities the necessity of communication is deliberately used as a didactic model for students' representational development.

- As a catalyst for cognitive conflicts, for instance in transitions from arithmetical to algebraic thinking.

Finally, the relevance and power of mathematical problems in the process of teaching and learning mathematics and, particularly, for the learning to teach mathematics is systematically explored and thus offers a multilayered grid for the conception and analysis of the training of teachers of mathematics.

Interestingly, many papers, explicitly or implicitly, give rise to important questions about students' access to the knowledge and experience that is necessary to fruitfully and successfully engage with the many different mathematical problems. These questions are of importance for the improvement of mathematics education as well as for its study. Access is not only meant in terms of the mathematical knowledge which students need in order to understand the mathematical problem but also in terms of their understanding of the intention behind using the particular problem. On one hand, teachers choose or adapt mathematical problems with specific didactic intentions that often are not conveyed to the students. Researchers, on the other hand, use mathematical problems for their specific research purposes mostly without explaining broadly to the test persons or research subjects what the research questions are about. In both situations, although in very different fields of activity, mathematical problems come attached with hidden assumptions and implicit expectations. Often the mathematical problems cannot be approached adequately without sufficient tacit knowledge about these assumptions and expectations.
As a consequence researchers need to be very careful about the validity of their research results. The framing of a mathematical problem as a means to investigate learning processes or students' competences can distort the students' mathematical behaviour, in principle. For mathematics teachers and their students the difference of insight into the didactico-functional aspects of problem posing and problem solving has been described as a matter of framing. However, in both settings, in teaching as well as in research, the difference of framing is a conditio-sine-quanon for the very existence of a teaching or research interest. It is still an open question how to provide access to mathematical problems for all students.