# The potential of mathematical competitive activities as a social equalizer and creativity stimulus

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## Introduction:

Man has engaged in various forms of competition since the dawn of history. It is hard to find any area today - whether cooking, music, or space exploration - without competition in it. The competitive drive that leads man to strive for ever higher achievements is also linked to the acquisition of math skills, and this competitive spirit is a foremost feature in the atmosphere of the math club.

The Kidumatica Mathematics Club, located at the Ben-Gurion University in the south of Israel, offers young people an array of math activities. The club has 400 members, all talented pupils between the ages of 10 and 16 who participate in an after-school enrichment program twice a week. The youngsters come from diverse socio-economic, ethnic, cultural, and religious backgrounds. There are Jews, Moslems, new immigrants, and native-born Israelis; some youngsters are indigenous, others very wealthy.

The research-based program developed at the Kidumatica Math Club is designed to enhance logical reasoning, combinatorial thinking, quantitative sense, and visual discernment. The club teaches problem solving strategies, optimization methods, number theory, and cultural aspects of mathematics. Pupils work in small groups, "micro-scientific teams," with top-notch teachers. However, social boundaries often persist within the teams. One of Kidumatica's goals is to break down these boundaries and reassemble the groups' identity on a new, common basis. Competitive math activities have been successful in achieving this goal.

We claim that the competitions:

- can serve a social, in addition to intellectual, function.
- are an effective way of creating a multi-cultural environment.

## Working assumptions:

A number of factors need to be considered in order to motivate pupil participation in competitive activities:

1. The activity should include an element of "fun."

2. The rules should be clear and simple so that the emphasis will be on the math rather than on the rules of the games.
3. The math should contain elements of surprise and challenge but should not be overly difficult.
4. The competition should be skillfully managed.
5. Peers and tutors should maintain a supportive and democratic atmosphere.

**Group, pair, and individual competitions**

The following examples illustrate three types of competitive math activities:

a) Group competition (between two or more groups).

b) Competition in pairs.

c) Individual competition.

All three cases exhibited effective cognitive social interaction. In pair and group competitions an "esprit de corps" united team members to a single goal.

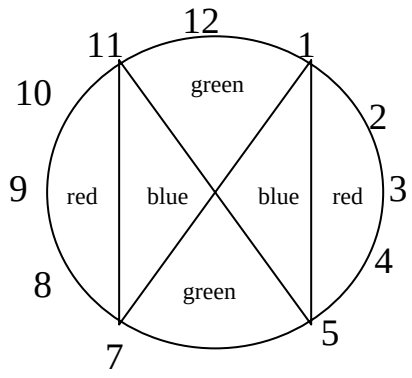
**Group competitions:**

One of the best examples of group competition is **Math Hockey**. As the name suggests, it comes from the world of sports:

Rules of the game: Each group (or team) is made up of 6 players: 3 "hitters," 2 "defenders," and 1 "goalie." The hockey "puck" is an unconventional math problem. The problem has to be solved by the first string – the "hitters". If they fail, the problem goes to the second string – the "defenders" – and if they "flub it" the problem "lands in the lap" of the third string - the "goalie." If the team fails to score a goal after three tries, the other team "scores" one point. If it solves the problem then the "puck" goes to the challenger, who is given another problem to solve. The object of the game is to be the team with the least number of "goals" scored against it.

The following problems appeared in a "math hockey" game:

1. *Mrs. Rambo loves pets. All of her animals are dogs, except for two. All of them are cats, except for two; and all of them are parrots, except for two. All the rest of the animals are cockroaches!*
   *How many animals of each kind does Mrs. Rambo have?*
2. *A clock has been painted in 3 different colors according to the picture below. Which color takes up the most space? Explain your answer!*




**Intermediary remarks:**

Math hockey is based on “constant dynamics.” Many pupils are in a constant state of alertness. Furthermore, math hockey not only has a cognitive aspect but also a strong social one: group structure, player selection, and task allocation are determined efficiently and democratically by all the members of the team. As one pupil said, “We don’t care who you are or what your parents named you [meaning your origins] all that matters is that you’re good.” We have observed that socio-ethnic barriers in groups disintegrate for the sake of a common purpose. The game – as the Kidumatica Mathematics Club plays it - has proven itself very successful for this reason.

In addition to the social aspects just mentioned, math games of the “hockey” type also contain important mathematical-cognitive benefits since they demand of the players:

1. Concentrated mental energy on the spur of the moment.
2. Use of intuition in the specific area of knowledge.
3. Quick reference to a body of relevant knowledge.
4. Immediate formation and corroboration of assumptions.
5. Synchronization of peers’ ideas.
6. Flexibility in switching from one thought track to another.
7. Associative thinking – moving quickly from one discipline to the next.

Unlike sports, math games ask the pupils to justify their victory both during and after the game. Several skills are required for this:

1. The ability to persuade peers by verbal, graphic, or illustrative argument.
2. The ability to construct a logical chain of reasoning and present it to peers.
3. The ability to conduct a focused Socratic-style discussion in a micro-group.

4. The courage to defend a minority view, withstand majority pressure and the lure of charismatic dominating personalities.

Solution paths of the painted clock problem (above) illustrate the diverse abilities that pupils need to develop. First, visual intuition tended to deflect pupils from the right solution track. In order to arrive at the solution they had to draw on their knowledge of equivalence of areas and recognition of geometric rules, courage to add auxiliary lines, and flexibility to break down a complex problem into smaller and simpler elements.

**Competition in pairs:**

This type of competition is generally based on games of strategy, as the following two examples illustrate:

**"The car race"**

This game is designed for 2 players. Each player must develop a strategy in order to advance along the grid squares while blocking the opponent.

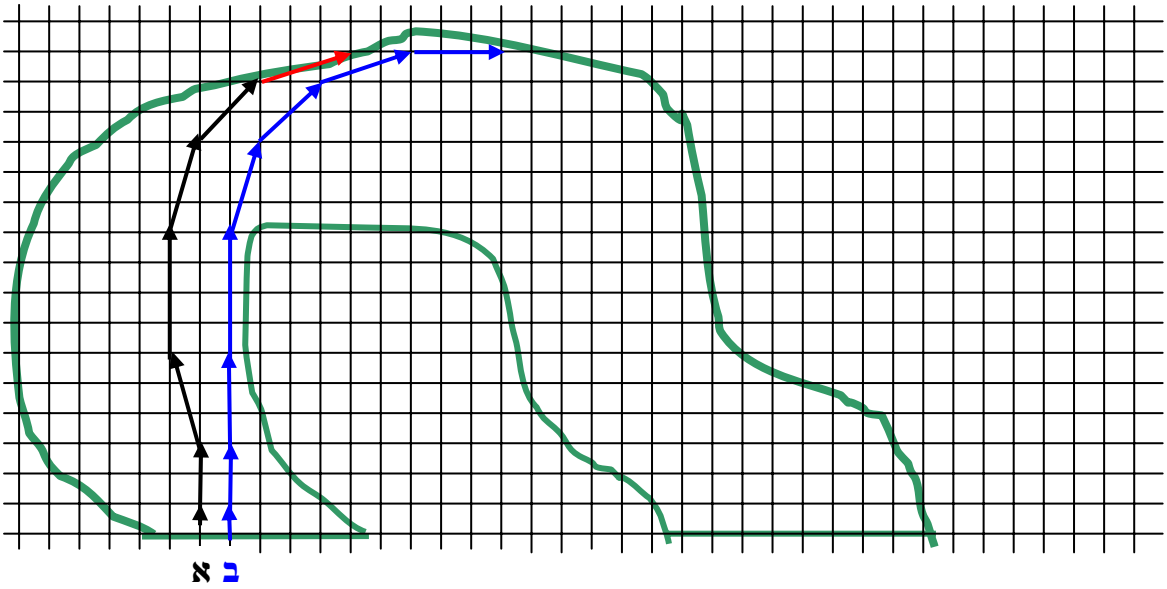
Rules of the game:

**1.** The player advances along the grid by moving from one square to the next. At the start the players are allowed to move to a contiguous square only in a vertical or horizontal direction. Later in the game, they may add or reduce the number of steps.

**2.** It is forbidden to go beyond grid boundaries.

**3.** A player cannot move to a square occupied by an opponent.

**4.** The player who crosses the finishing line in the smallest number of steps is the winner.



**“No Choice”**

Rules of the Game:

There are six cards numbered 1, 2, 3, 4, 5 and 6.

At the start of the game someone randomly decides which player picks the first card.

The first player chooses a card (the numbers on the cards are visible).

The second player decides who takes this card, himself or the first player.

In each round, the player who gets to pick a card is the one who has accumulated the largest number of points on the cards. In the case of an equal number, the player who picks a card is the one who began the preceding round.

The winner is the player who has accumulated the highest number of points on his cards.

Note: The sum of the numbers 1, 2, 3, 4, 5, and 6 equals 21. Therefore the player who reaches a sum of 11 or higher is the winner.

A general rule of this game: To increase the game by a factor of 'n' of cards, the cards are numbered from 1 to 'n'.

Games played by pairs are generally held in a “play-off” format.

Three stages have to be accomplished in order to reach success:

1. **The preparatory stage**. Approximately one week before the competition, tutors explain the rules of the game and make sure that pupils understand the algorithms and are given an opportunity to practice them. For didactic reasons algorithmic thinking, as well as the games mathematical aspects, have to be strengthened.

We observed that math skills were reinforced through consistent practice. The pupils’ desire to succeed in the competition motivates them to develop a deeper understanding of math concepts and increases their natural curiosity. Unfortunately, these constructive learning habits are often dormant in the classroom.

2. **The actual competition stage.** In this stage, pupils who invested time in preparing for the competition honing their problem solving strategies are at a distinct advantage. Sometimes during the game they realize that the strategy they developed is in effect a “micro-strategy” applicable only in certain cases. When this happens they have to go through a number of micro-strategy cycles until they find the local optimum (the global one may not even exist in this case). During the competition, cognitive and meta-cognitive interaction results in the empowerment of thinking resonance and the optimization of potential.

3. **The "conclusions and analysis" stage.** In this stage maximum knowledge of the game is attained (excluding the emotional factor) and the principles of the victory strategy (if there was one) are proven. Pupils become aware of the underlying difference between the victory strategy and the particular algorithm that made victory possible. They analyze the micro-strategies, and pitfalls they encountered on the way to victory (or failure).

**Individual competitions:**

**"Weight Lifting"**

A reservoir of approximately 100 unconventional problems at ten levels of difficulty is developed. The problems are presented as barbells. The weights range from "very light" - 30kg - the lowest level, to 120 kg – the heaviest weight and highest level of difficulty.

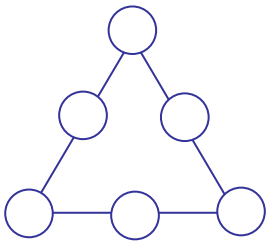
Rules of the game:

1. Game begins when the pupils choose a "very light- or light weight" problem.
2. The correct solution allows them to move up a notch to a heavier weight.
3. They have two opportunities to solve each problem.
4. A double failure "bounces" them back to the starting point.
5. There is no time limit.
6. The winner is the pupil who manages to "lift" the most weight.

An extra value inherent in this kind of game is that it leads pupils to develop realistic self-appraisal. In order to win, the pupils not only have to understand math and employ creativity, but must also know how to realistically evaluate their own abilities and act accordingly.

**The Sum-100 Triangle.**

Place each of the following numbers 31, 32, 33, 34, 35, and 36 (Each number only once) in the 6 circles of the triangle, so that each side adds up to 100.



**The mysterious number.**

Find the smallest natural number that fills the following conditions:

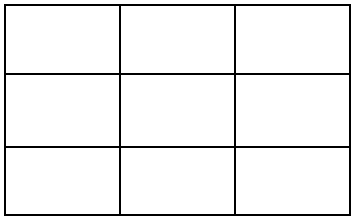
If it is divided by 3, the remainder is 3.

If it is divided by 5, the remainder is 3.

If it is divided by 8, the remainder is 1.

**Anti-magic square.**

Build an “anti-magic square” in which each line, column, or diagonal adds up to a different sum.



**Conclusion:**

Experience has taught us that competitive math activities in formats such as the Kidumatica Mathematics Club are capable of altering the pupils’ mathematical conceptualization, deepening their informal understanding of the math laws, and contributing to personal and social reformulations (such as creating Jewish-Bedouin friendships).

Competitive activities require a strong sense of team cooperation. All players have to follow the game rules in solving mathematical exercises. Competitive activities also **develop positive personality qualities** such as honesty, risk-taking, patience, flexibility, acceptance of failure, and the ability to learn from failure.

Competitive activities serve as a **catalyst for learning** and can contribute to the improvement of cognitive and meta-cognitive abilities.

**Cognitive abilities** develop strategies for problem solving and discovering links between mathematical fields. **Meta-cognitive abilities** enhance the pupils’ awareness and control of thinking, e.g. when and where to apply knowledge, confirmation strategies for convincing others and developing proofs, and evaluation if a strategy is valid for a particular case.

As a **learning mediator**, competitive activities are powerful devices for encouraging students to experiment with ideas that have an immediate effect on their mathematical understanding. In sum, competitive activities are conducive to the learning process.

Competitive mathematical activities require students to work simultaneously in two rules systems:

The rules of the competition

The rules of the game’s mathematics

The competition cannot be won if either of the rules systems is ignored.
In the pupils' eyes, since the rules of the game are taken from everyday situations, they "*legitimize and humanize mathematics, and the rules of the games serve as a 'trigger' for learning math rules*."(Pupil's quotation)
Competitive activities are common to all cultures. The "shared fun" aspect of the games induces players of dissimilar socio-economic-ethnic backgrounds to work and learn together, and share their ideas, failures, and successes. All Kidumatica pupils learn that one cannot expect to win or stand out in every game, that everyone is bound to fail on occasion, but that everyone can also learn from failure.
Competition call for a variety of skills: flexibility, tolerance, leadership, adaptation to changing situations, and so forth. It enables different groups to excel, but none to dominate the game. In short, it promotes a **sense of equity**.
Mathematics is often perceived as a tool or means to succeed in engineering, natural science, and other exact sciences. Competitive math activities may change that **perception about math** since they require pupils to use intuition, confront paradoxes, and deal with intellectual challenges such as associative and integrative thinking. In this light, the study of mathematics is a fascinating endeavor and not just a tool for other disciplines.
The enormous potential of competitive pedagogical activities as **a social equalizer and creativity stimulus** has not been fully appreciated yet.
We observed that a major mathematical **and** social breakthrough was achieved when Kidumatica decided to focus its activities on game playing and competition. This orientation has developed a spirit of cooperation and motivation for excellence among the pupils. The advancement and support of this systematic pedagogical infrastructure must become a primary educational goal. So far we have taken only one small step in this direction.