THE PENELOPE STORY: LEARNING THROUGH ACTION, SPEECH AND GESTURES

R. Barbero[*], L. Bazzini[**], F. Ferrara[**], B.Villa[***]

[*] IRRE Piemonte, Torino,

[**] Dipartimento di Matematica, Università di Torino,

[***] Scuola Elementare di San Mauro Torinese, Torino e N.R.D., Università di Torino

Introduction and theoretical background

Recent studies have pointed out the significance of perceptuo-motor activities in mathematics learning. In particular, summarising some findings of the last past years in neuroscience, Nemirovsky (2003) states that "while modulated by shifts of attention, awareness, and emotional states, understanding and thinking are perceptuo-motor activities; furthermore, these activities are bodily distributed across different areas of perception and motor action based on how we have learned and used the subject itself. [Moreover,] that of which we think emerges from and in these activities themselves". As a consequence, "the understanding of a mathematical concept rather than having a definitional essence, spans diverse perceptuo-motor activities, which become more or less active depending of the context" (ibid.). Some years ago in cognitive science the theory of embodied cognition (Lakoff & Núñez, 2000) was already beginning to stress the fact that abstract and formal mathematics is rooted in concrete sensory-motor experiences via metaphorical thinking (that link language to visual and motor experiences). According to this perspective, mathematics is a product of the human and bodily activity in interaction with the world we live in. Behind metaphorical thinking, the sensory-motor experiences "might variously be structured by those common neurophysiological predispositions human beings genetically possess and might variously be mediated by environmental factors including those developing cultural and symbolic systems into which specific human beings and groups of human beings are variously and progressively initiated" (Schiralli & Sinclair, 2003). So, the role of the context in which mathematical concepts are learnt or taught comes to be essential. This claim agrees with the belief that it is difficult to know something by studying only its components separately (Vygotsky, 1992). Vygotsky (ibid.) used a metaphor to explain what he meant: it makes no sense to study separately hydrogen and oxygen in order to study water since those two do not have the properties of water. Just assuming such a viewpoint, Arzarello (in press) has introduced a new construct that can be seen as bringing the Vygotskian metaphor into being for mathematics education, the so-called APC space (Action, Production and Communication Space). The APC space is "an environment for cognition, which may be built up, developed and shared in the classroom", whose main components are: the body, the physical world and the cultural environment (Arzarello, ibid.). One cannot get a realistic picture of what is going on in the classroom without considering all these active and interactive elements. It is a space "built up in the classroom through the interactions among pupils, the mediation of the teacher and possibly through interactions with artefacts" (ibid.). In this space we can situate the so-called semiotic means of objectification (Radford, 2003). As Radford (ibid.) argues, it is important "a broader context large enough to conceive of tools, things, gestures, speech, writing, signs, and so forth, in relation to the individuals' activities and their intentional goals. In this broader context, we called semiotic means of objectification the whole arsenal of intentional resources that individuals mobilize in the pursuit of their activities and emphasized their social nature. The semiotic means of objectification appear embedded in socio-psycho-semiotic meaning-making processes framed by cultural modes of knowing that encourage and legitimise particular forms of sign and tool use whereas discarding others" (ibid). According to the sub-theme of the present Conference relative to cultural values, a relevant issue is whether an embodied cognition approach and the new trends in neuroscience, together with recent studies in Math Education can help to reconcile the "mind" and the "body" in the teaching and learning of mathematics. Particularly, taking into account the results aforementioned, can a learning context where perceptuo-motor activities, the use of the body and of semiotic means of objectification, and the contact with culture and experience are favoured by the task, be fruitful and effective in terms of mathematical understanding? Within such a theoretical perspective and with these issues in mind, our research aims at studying the students' cognitive processes when making mathematical sense of a situation. Action, gestures and words become the lenses through which trying to reach the aim.

The research study

In this paper, we consider the case of a classroom activity that is based on a popular legend, the Penelope story from Homer's Odissey, and requires its mathematical modelling. The choice of a legend as starting point of the activity allows linking knowledge outside of mathematics with mathematics itself. The teacher can approach important concepts, as the ones of variable, rational number, division, beginning from a simple story. The task also entails an active and creative involvement of the students. In fact, they are required to imagine things, and to this purpose they are given materials of different kinds: paper, colours, cloth, scissors, glue, etc.

From the research point of view, our interests are on the students' cognitive activity in the process of constructing meaning on the given situation. The analysis focuses on the fundamental elements of the students' dialogue and interaction: words and gestures. Following the semiotic approach of Radford (2001), we distinguish between different functions of language: the *deictic function* and the *generative action function*. The first one is used to indicate something or somebody (examples are given by words as "here", "there" "this", etc.); the second one refers to actions to be performed or previously performed (examples are verbs as "to cut", "to add", "to stitch", "to take away", "to increase", etc.). Furthermore, there is a third function that has a relevant role in mathematics learning: the *logic function* (Ferrara & Robutti, 2002) expressing functional relations ("if…then", "so", etc.).

For what concerns gestures, we take into account the theoretical notion of *semiotic node* developed by Radford et al. (2003), who define it to explain those "pieces of the students' semiotic activity where action, gesture and word work together to achieve knowledge objectification". The authors reflect on the objectification of knowledge as the process that leads students to successfully construct mathematical concepts, starting from their perceptions and interacting with cultural artefacts through gestures and language. The lens is developed taking into account the integration of different semiotic systems (Radford et al., 2004): body actions, artefacts, graphs and speech. The role of gestures has also been widely studied in the field of psychology, where some researchers (as for example Kita, 2000; Goldin-Meadow, 2003) highlight their significance not only in the processes of communication but also in those of thinking.

From a methodological point of view, we assume that the mathematical discussion plays a crucial role in the construction of knowledge. The mathematical discussion is meant here as a "poliphony of articulated voices on a mathematical object, that is one of the motives of the learning activities" (Bartolini Bussi, 1996, p.16). In a Vygotskian perspective, the mathematical discussion is a promising context for the social construction of knowledge.

The teaching experiment (The Penelope story)

A teaching experiment has been carried out in a grade 5 classroom; it approximately took a period of four weeks (November-December 2004). The involved pupils were familiar with problem solving activities, interacting in small group work. The didactical contract, as established in previous grades, was consistent with the methodological choice of such a kind of learning, grounded on the co-operation, participation, and guided enrichment of tools and skills of the students in the activities. Furthermore, a rather common routine of the classroom work consisted in individual production of written solutions for a given task (if necessary, supported by the teacher in one to one interventions with individual students), followed by collective discussions of the students' results, led by the teacher. The didactical contract also included exhaustive written production of doubts, discoveries, heuristics, etc.
The story submitted to the pupils is the following:

> *… On the island of Ithaca, Penelope had been waiting ten years for the return of her husband Ulysses from the war. On Ithaca, however, a lot of men wanted to take the place of Ulysses and marry Penelope. One day the goddess Minerva told Penelope that Ulysses was returning and his ship would have employed 50 days to arrive to Ithaca. Penelope immediately summoned the suitors and told them:*
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> *"I have decided, I will choose among you my bridegroom and the wedding will be celebrated when I have finished weaving a new piece of cloth for the nuptial bed. I will begin today and I promise to weave every two days; when I have finished, the cloth will be my dowry". The suitors accepted. The cloth had to be 15 spans in length. Penelope immediately began to work, but one day she wove a span of cloth, while the following day, in secret, she undid half of it…*
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> *Will Penelope choose another bridegroom? Why?*

The teaching experiment has been developed according to the following steps. The teacher reads the story, checking students' understanding of the text. Then the story is delivered to the groups and the work group begins. The students work in their groups and produce a solution, through written materials. The teacher collects the solutions, which are successively compared in a collective discussion. Finally the students work on new requirements and produce a number table and a graph representing the story.

An example from a dialogue

We present here a brief extract from a dialogue of some students working in a group. The dialogue shows richness in gestures, as well as in deictic and generative action words; their use supports students in developing understanding of the situation and constructing mathematical meaning for it.

1. Edoardo: *It is the same as you do so much cloth* [his hands, open and parallel one in front of the other, are indicating a space interval]. *She makes so much amount, then she cuts so much amount…*
2. Ornella: *No, look…because if she was making a span and then the day after she was undoing its half* [her hands are repeating the same gesture as Edoardo, first representing making and then undoing], *a half remained, then the day after…*
3. Davide: *A half always remained*
4. Simona: *No, she was always taking away a half*
5. Edoardo: *It increased* [with index finger and thumb placed to shape a distance, his hand is moving to mime an increasing in the length of the cloth]
6. Simona: *Yeah*
7. Edoardo: *But imagine that she arrives at the end and we take the calculation… but once she has finished the cloth, if it didn't take her all the days, the fifty days…*
8. Ornella: *It didn't take her fifty days*
9. Simona: *Because we have to count the nights* (…)
10. Edoardo: *So it took her twenty five nights to undo*
11. Davide: *We have to count whether she was able to make the cloth in twenty five nights because she makes a span and then she takes away a half* (…)

12. Simona: *So twenty five half span*

A first analysis of this extract points out that the students perform some gestures with their hands in order to represent lengths in spans (#1) and changes in length (#2, #5). In the first case, the gesture is static, because its shape resembles what is expressed in speech: an amount of cloth ('so much cloth'). In the second case, the gestures are dynamic: the movements of the hands give the idea of the change that is uttered: 'making a span' and 'undoing its half'; 'it increased'. In #2, the gesture refers to an imagined action of Penelope; in #5 the gesture refers to an imagined result of an action on the length of the cloth. Verbs with generative action functions are then used both for speaking of Penelope acting ('to make', 'to undo', 'to take away', 'to arrive', 'to finish'), of results on the cloth ('to increase', 'to remain'), of students' action ('to count'). They can be distinguished through the pronouns, subjects of the sentences: 'she' (Penelope), 'it' or 'a half' (the cloth), 'we' (the students). A first generalisation arises from the use of 'always' in #3 and #4, but it still concerns the process of making the cloth. A step further is made when students begin to reason on what they are looking for (#7-#12). At this point, the students also find out consequences (expressed by the word 'so' that has a logic function) and the result of the process: the number of spans after fifty days ('twenty five half span'). A passage from deictic and generative action to logic functions in speech, and from static to dynamic gestures occurs, marking an evolution in the mathematical sense-making of the situation.
An extended description of the teaching experiment, as well as extracts from the dialogues and the collective discussions that support the research analysis, will be provided during the presentation. To finish, very striking issues to be considered come from the results of an evaluative questionnaire in which students were engaged. These issues are relative to the way of learning, which occurs through working in groups on the one side, and working on a legend in an interactive way on the other. To the questions: (1) "Was it helpful to talk in your group?"; (2) "Give one example where you used your imagination in this project", there were respectively answers of the following kind: 'Yes, because if you think alone, you don't understand a lot; but if you work in a group, you think more clearly', 'Yes, because groups understood that there are different ways to solve the problem' (1); 'I imagined to be Penelope…the whole story went on in my mind', 'When we had to do and undo the spans using our hands' (2). These answers of the students highlight the significance of the learning context and of the use of imagination and gestures for understanding the situation.
As a consequence and preliminary result, we would like to stress the importance of an embodied perspective in order to deepen the analysis of students' cognitive dynamics when they are involved in mathematical problem solving. It is a relevant subject under debate, together with new challenges that are entailed by such a perspective, as far as the didactical contract in the classroom, the role of the teacher and the use of new technological devices are concerned.

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