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Importance of the use of didactic, new technological materials in mathematics

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Many examples are given in order to illustrate the importance of the use of didactic, new technological materials in mathematics classrooms and to show how this “participating environment” can improve mathematical learning, teacher’s practice and mathematical production.

In brief the principles upon which the research is based are:

1) an important objective for didactic research consists in providing some solutions, which must be both effective and convincing, to the requirements of having the contents and methods of teaching satisfy the emerging needs of society and of the individual
2) in order to achieve some objectives of innovation in didactics it has been seen to be important to consider:
   a) the presence of the computer, automata and mass media and the consequences derived from these from the aesthetic, expressive, cognitive and social standpoints. Amongst these is the significant reduction, for the individual, in the requirements stemming from the routine aspects which occur in scientific investigation and in production of goods;
   b) the greater complexity of the problems deriving from the increase in the population and from the effects of globalisation.
3) to resolve the problems in a useful manner while keeping the aforementioned aspects in mind, it becomes increasingly important to have the active, aware and co-ordinated participation of all the components of society, of the culture and of the production, these deriving from a dynamic relationship which renders it possible to combine their positive peculiarities: the deductive and inductive components of learning and behaviour;
4) in order to bring about the diffusion and experimentation of proposals for didactics which are innovative from the standpoints indicated, it can be important to take into account the positive aspects resulting from personal discovery, both for students and teachers, and from the dynamic relationship between the deductive and inductive components of learning and behaviour.

To link what has been stated with research into the didactics of mathematics, it is considered, in brief, that:

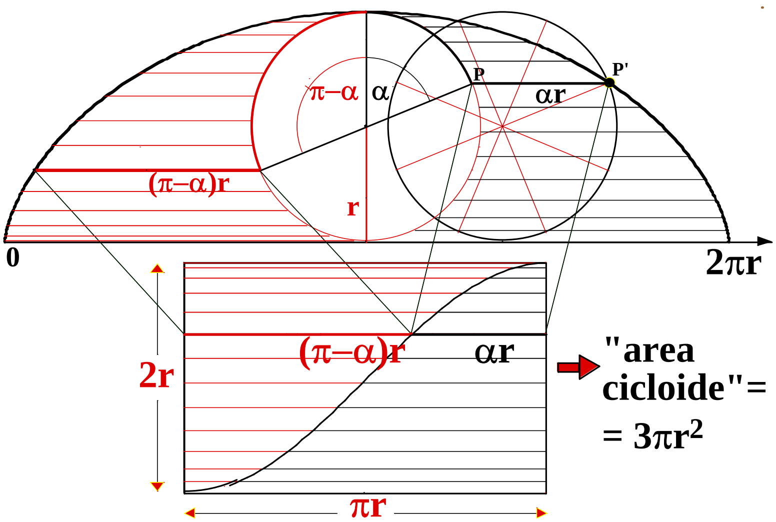
- mathematics must increasingly be thought of as a tool with first-rate attributes for cognitive growth and for an understanding of all the instruments which can be used for solving problems;
- applied mathematics is based above all on the polynomials which, to be integrated and derivated, can require a lesser consideration of the “upsilons and deltas”;
- it can be important to focus on, highlight or “create” those topics, properties and expressions which are useful for establishing a bridge between natural and scientific reasoning, thus demonstrating in a convincing manner the reasons for the appropriateness of and the enjoyment to be had from this linking.

In particular, emphasis is placed on visualisation, learning based on dynamic instruments and concepts, geometry presented in combination with other sectors of mathematics and with other disciplines in line with the "Fusionism" philosophy of Klein and de Finetti. Greater space is given to analogy and induction as put forward by Laplace and Polya, …, to manipulation techniques, to "Discrete Mathematics" and to "Concrete Mathematics".

More space is given to the relationship between synthetic and analytic geometry, between the dimensions of space and between discrete and continuous geometry, which are seen as language for expressing various not necessarily geometric aspects properly, following the didactic suggestions starting from the Pythagorean school and up to the proposals of the mathematicians cited above and of other authors. Even more particularly, emphasis will be placed, for example, on the following subjects, strongly interrelated: probability, simple way to integrate, equidecomposability, tessellations of space, transformations, sections, projections, … , colours, the treatment of aesthetic aspects and the effectiveness of the presentation and communication of a subjects.

Now you find the pictures of some examples that will be presented dynamically with Cabri:

The Cycloid[1] is the path of a point of a circle rolling upon a fixed line. Two proofs:



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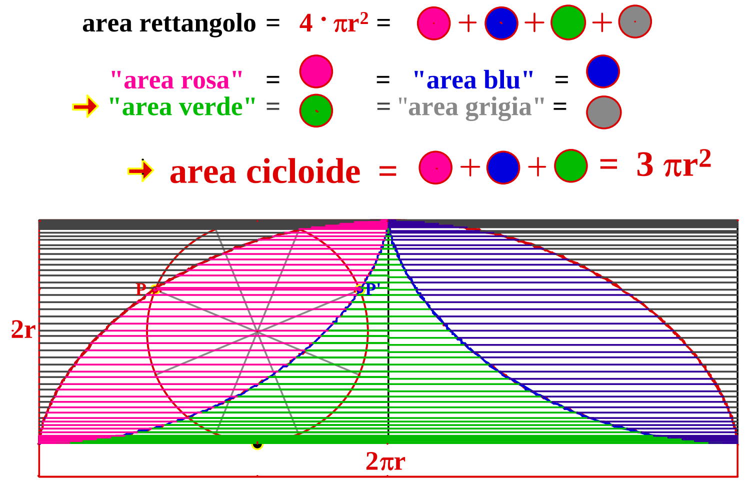
[1] In this way, dynamically, you have the possibility to understand the proof, only seeing the pictures, in a "proof without words". For the first of this two proofs, expressing in a different way, see: Barra M.: 1975, The cycloid - A didactic Experience - A new proof, *Educational Studies in Mathematics*, 6, 93-98. For the second and a third new proofs, see: Barra M.: 2004, La regina della matematica: la Cicloide. Tre nuove dimostrazioni "senza parole" che la riguardano, "difficili" da capire senza Cabri. Baricentro della Cicloide. Come possono nascere le idee con Cabri, *Progetto Alice*, N. 14, Vol. 5, Ed. Pagine, 331-344.



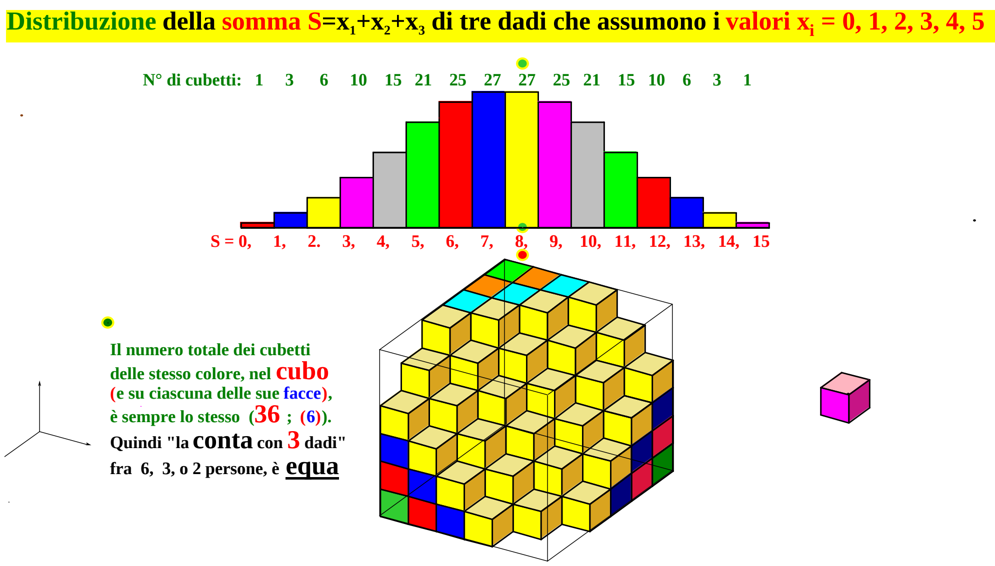
In many examples and in a confrontation with the classical or with the historical proofs, it will be possible to convince you that the software Cabri permits:

- to test new possibilities to communicate, utilizing:

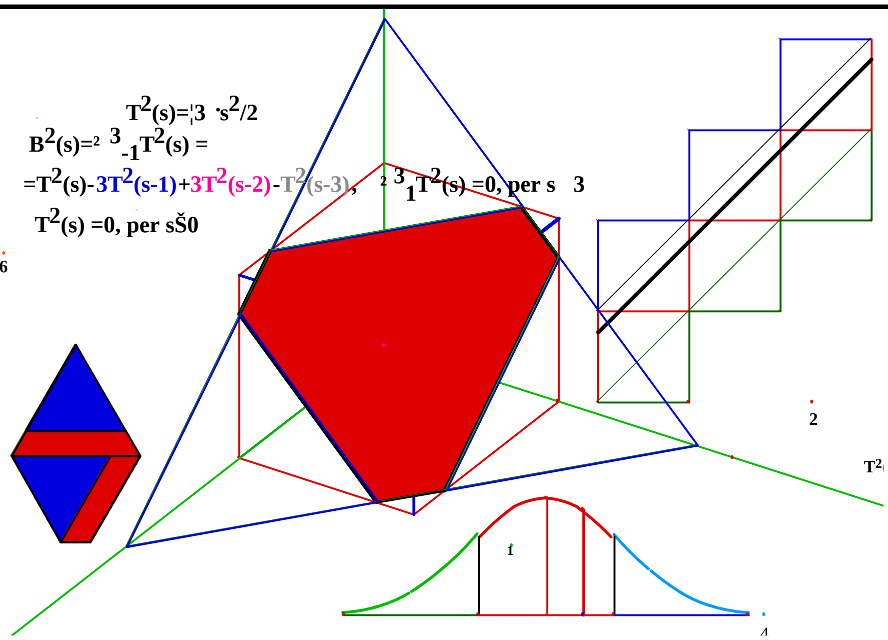
colours, dynamic aspects, graphical techniques, …, up to the time of understanding how move a picture, dimensioning a text, …, can be very important: - to understand and memorize mathematical aspects in a better way, - to give more motivations to study, - to make a less difference between natural and scientific languages and reasoning, …

- to verify some suppositions
- to discover new properties
- to found new proofs of known properties
- to proof new theorems
- …

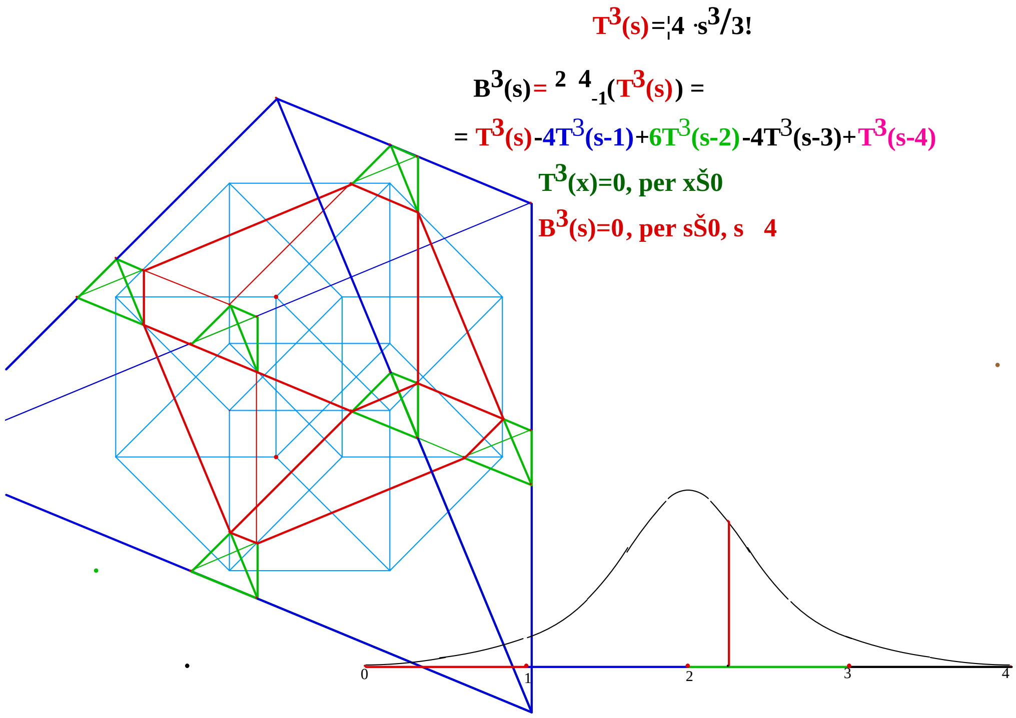
For instance:



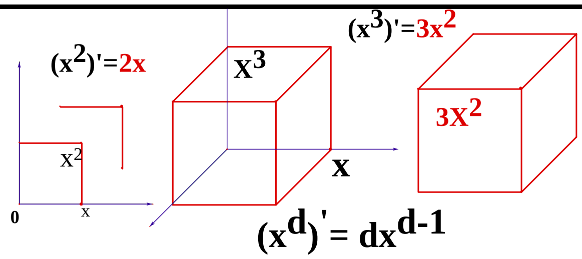
The same in the continuous space:



Starting with a square and a cube, we can see and prove better: - the analogous in the hypercube:



and new way to understand: - the Low of the Large Numbers, the Central Limit Theorem (the tendency to the Gaussian distribution), - new ways to obtain the spline functions and to prove some of their new properties, and, at the same time, - the derivatives of a polynomial, the tessellations of the space (also with a new polyhedron), the equidecomposability (The third Hilbert’s Problem), …


(x2)'=2x
x2
0
x
x3
x
(x3)'=3x2
3X2
(xd)'= dxd-1