



NOT LOGICALLY CONNECTED STEPS OF REASONING: INTERPRETATION OF HISTORICAL MATHEMATICAL PICTURES

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This paper refers to the sub-theme *Cultural and political changes in primary schooling: information, knowledge, technical tools, and education.*

In our society communication is becoming faster and faster. Visual images can give a large amount of information, in a very short time. Our students live this situation, which is a drastic shift with respect to the past. There is a negative consequence of this fact: they have less skill to establish logically connected steps of reasoning, in a traditional manner. This is a challenge for mathematics education and the teacher is stimulated to create situations in which the students are able to put into practice their skills by enjoying visual messages.

Mathematics has a peculiarity, not only nowadays: in a mathematical document we can find many forms of communication (words, formulas, illustrations). Mathematics is intrinsically a field for multimedia.

The history of mathematics may be a resource to teach mathematics in a better way today: old and new Italian national proposals for mathematics education contain historical aspects. History is a way to a new view of mathematics as sociological and cultural process.

Mathematics should be viewed in a broad context, instead of in a narrow disciplinary field. Such a contextual approach can be used in teaching mathematics. It gives the opportunity to emphasise how mathematics is firmly fixed in culture and nature.

The new proposals for education in Italian school highlight the hermeneutic approach to knowledge. This fact is a new challenge for mathematics education. Hermeneutics contrasts with some aspects of mathematical epistemology: hermeneutics is “the art of interpretation” and interpretation is an individual act that not immediately fits with the characteristics of the mathematical objects.

A team of teachers is working on ‘Didactical activities of history of mathematics’ for the IPRASE (Istituto Provinciale per la Ricerca, l’Aggiornamento e la Sperimentazione Educativi, Trent - Italy) a Provincial Institute for educational research. Within the team, the debate has produced the conclusion that the original sources analysis is the most suitable way to introduce history (see also, e.g., Fauvel & Van Maanen, 2000, 291-328). Original’s interpretation doesn’t require supplementary notions and is paradigmatic of the hermeneutic approach to mathematics. At the web address http://www.iprase.tn.it/prodotti/materiali_di_lavoro/Matematica/laboratori_abaco.asp you can find some proposals of didactical activities inherent to *Larte de labbacho (Treviso Arithmetic)*. These proposals are for pupils of primary school and are about the interpretation of documents.

In a previous article we described a didactical experience about the interpretation of a picture (Demattè, in press). Students analysed *Circle Limit IV* from M. C. Escher. Their writings contained affective and cognitive statements. The cognitive statements were based on ‘elementary’ mathematics and not on concepts of Calculus, in spite the fact they had studied it during the last year.

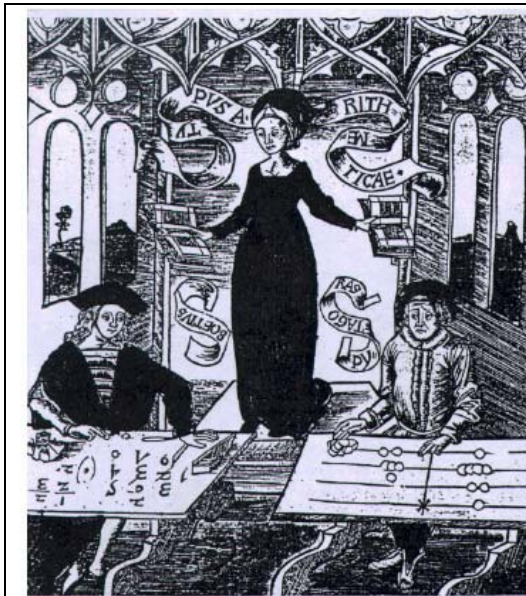
In the present article we will engage with an almost paradoxical situation: to elucidate the motivations to use old historical pictures, with the aim to tackle an actual didactical challenge.



The activity

The activity was implemented in a secondary school (Istituto ‘Rosmini’ Trent – Italy), students aged 14-18, and was considered a facultative task during non curricular lesson times (substitutions of absent teachers, activities with small groups of students) or was assigned as homework.

One copy of a picture was given to every student. Every one of the pictures showed a scene in which some people are working, using instruments or numbers. Here are some of the pictures used during the activity:



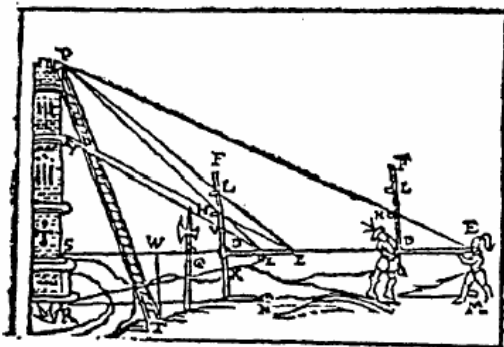
Picture 1. Gregor Reisch's *Margarita Philosophica*.

Picture 2. A mural painted at Abd-el-Qurna, Egypt, around 1400 B.C.



Picture 4. Depicted in *Das Stadebuch* (*The Book of Trades*) by Jost Amman, Frankfurt, Germany 1568.

Picture 3. In a field manual (1590)



Source of illustrations: Katz & Michalowicz (Project Directors), 2004.

The students were asked to read instructions in the following form:

RESEARCH PROJECT
WHAT KIND OF INFORMATION CAN YOU OBTAIN FROM A PICTURE WHICH IS INHERENT TO
THE HISTORY OF MATHEMATICS?

The students' ideas

- In our society, we communicate by using visual images: television, computer, posters and so on.
- Mathematics was born and developed in a social context. Many people had used it.

Interpret this picture:

look at the people on the picture and describe what they are doing according to you (examine the context and the details; try to use your mathematical knowledge).

Thank you.

Students' responses

Picture 1. Let's examine an example of student's protocol, referred to *Margarita Philosophica*. One female student writes: "This picture struck me because I see a woman in mathematics, at last! The picture represents two men during a mathematical competition. The man who is using figures is young, like his mathematics. He looks richer than the other one. The other man is using the abacus. He is worried and old like his mathematics. Their names are *Boetius* and *Pythagoras*. The woman is looking at them. On her skirt I can see figures and, in my opinion, this means that she prefers *Boetius*. I can see a Latin sentence: *Typus Arithmeticae*. The mathematics with ciphers is faster and more precise. The other one was used in the Middle Ages and seems much more laborious."



It's interesting to note that “the woodcut probably does not symbolise the algorist winning out over the abacist, as is frequently mentioned, but simply represents aspects of the theoretical arithmetic tradition stemming originally from the classical Greek tradition.” (Katz & Michalowicz, 2004; p. 6). We can reflect on the student's writing. She interprets the allegorical aspect of the picture, using common sense. She reports the Latin sentence and the Latin names: this fact is significant in the context of her classical-pedagogical studies. She and her class had studied only a little of the history of mathematics (evolution of arithmetic and algebra): this fact probably prompted her remarks about old and new mathematics.

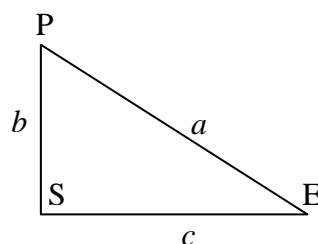
Picture 2. “In the first picture, the slaves are measuring the area of a wheat field. There are also some children who are helping to measure with a rope. In the second picture, the horse is carrying some wheat to the granary in which other slaves are recording.”

Only a few students refer to the rope, besides this one. One of them (a female student) points out the rope has equidistant knots and this fact is considered proof that Egyptians are measuring the area. She also writes: “Egyptians calculate the ratio of the wheat's amount and plots”. She also reconstructs the story which is showed in the second part of the picture: “On the left, the cart which is carrying two king's servants is represented; they have the task of rewriting the data with the aim of giving an account of them to the king.”

Picture 3. “In this picture, people are checking if the tower is perpendicular with respect to the ground. They are using Pythagorean theorem inherent to rectangular triangle ($a = \sqrt{b^2 + c^2}$).

Another hypothesis: people are trying to measure the height (side B) and they are doing $B = \sqrt{a^2 - c^2}$.”

The student marked the sides as in the following triangle:



The student assumes the problem to interpret the picture but his immersion in the situation (“What would I do if I were one of those people represented in the picture?”) is not completely satisfactory. In what circumstances must people calculate the height of a tower in that way? The student's interpretation proposes an application of the Pythagorean theorem and this fact is enough to show that the student has used his mathematical knowledge (as requested in the instructions).

Picture 4. “A mathematician is in his study and he is examining a globe. I see other terrestrial globes on the floor. He is studying Earth: he is drawing the meridians.”

This student's writing is less significant from a cognitive point of view, but we would like to point out a sentence which is a suggestion about the mathematical beliefs (Leder, Pehkonen & Törner (editors), 2003). The sentence is: “A mathematician is in his study”. The student really thinks the man in the picture is a mathematician or does he write this because his mathematics teacher asked him to interpret the picture? What is his image of mathematicians? Picker & Berry in their article (Picker & Berry, 2000) illustrate an experience in which the pupils were asked to draw a mathematician at work and to answer some questions about the mathematician's profession. Do the students' image of mathematicians vary as time goes on?

Not every question can be answered. In the present paper I am reporting only an exploratory study; other studies are necessary in order to invest further around suggestions and questions.



Conclusions

The students were free to expose many aspects of their picture interpretations: to say personal sensations, to observe details or to consider the global situation described in the illustration, to remember their past experience in mathematics. This multifarious student analysis is made up of not logically connected steps of reasoning,... may be... The logic derives from mental activity, establishing connections among different disciplinary aspects. The hermeneutic approach to knowledge breaks the epistemological structure of disciplines. From the students' point of view, the study of a logically structured topic requires a preliminary deconstruction and a successive reconstruction using the personal knowledge bricks: the epistemological structure is broken...

The main aim of the activity was to create a possibility to use visual media to increase the student's historical view of mathematics. Another aim was, conversely, to inquire about the students' readiness to apply their previous mathematical competencies in the interpretation of a historical picture. The analysis of students' protocols helps us to discover if the aims have been reached. Previous didactical experiences, similar to that reported in (Demattè, in press), show that students remember many visual details, even a lot of months later.

Mathematics is a cultural and evolutionary heritage. The cultural value of mathematics can be preserved in primary and secondary education. The activity illustrated in the present article regards history and images: are these topics so far from "making mathematics"? The students' writings refer to so many mathematical subjects which are traditionally part of the primary school curriculum. In my opinion the use of historical images could be used as a tool in order to teach mathematics in a better way. We could reorient the task and choose others pictures too. The student could use historical pictures in problem solving or he could explain how mathematics is embedded in culture...

I do not accept the perspective of a purely technical preparation of the students in primary and secondary school!

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