



CIEAEM 57 – *Italie – Italy*  
Piazza Armerina,  
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**Présentations orales**  
dans les groupes de travail  
**Oral presentations**  
in Working Groups

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## **“Dignam mathematicam docere” - Axiomatics meets axiology**

Paola Ghiringhelli e Gemma Testin. Scuola Elementare “A.Scarpa” via Clericetti. Milano  
Margherita Locatelli. Scuola Secondaria di primo grado “Aldo Moro”. Presezzo (BG)  
Paolo Longoni, “Didactics Laboratory of Mathematics and Philosophy” – Bergamo  
Gianstefano Riva, Istituto “Maironi da Ponte” – Presezzo (Bergamo) [giansri@tin.it](mailto:giansri@tin.it)  
Ernesto Rottoli, Istituto “Secco Suardo” – Bergamo [ernerott@tin.it](mailto:ernerott@tin.it)

*The link between mathematics and Western society resides in the idea of universality.*

*Today crisis of universality produces bewilderment in teaching mathematics too.*

Dignity of particular opens a new ethic horizon to thought mathematics, that becomes so the text of a hoped society.

Paper folding shows didactic dignity of thinking by hands

Teaching is going across a phase of bewilderment that seems new and nearly unforeseen if compared with enthusiasm that accompanied it along most of last decades, at least in Italy. We believe that this fact is due to the crisis of universality that interests many aspects of actual Western society and invests its criteria of truth, values and, in general, references and ideal tensions.

But it is just in the idea of universality that the link between mathematics and Western society resides: since the beginning, mathematical truth becomes necessary for the “universe” of considered objects, a prototype of rationality and a chance of discovery.

In some historical times this interconnection is so deep that we can say, by freely inspiring to Ricoeur’s philosophy, that mathematics becomes a text to read some characteristics of society and to individuate the ways of its tension to the universal.

But a double movement is implicit in the concept of universal, as we can assert by referring to the philosopher Lévinas, (*Totalité et Infini*): the opening one and the closing one. The opening happens when the becoming of world’s things rises paths to open spaces: mathematical truth unveils itself as “in-finite” event, as “necessary” course to a “focus”, of which sometimes historical time takes marks. But there is also a structuring movement towards totality, in which the dimension of closure prevails: mathematics set itself as an enclosed system.



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Euclid's *Elements* are an example both for the function of text for mathematics and the double movement that interests the aspiration to the universal. On the one side they contain and tell the stages of forming western rationality. On the other side, however, in its final draft, they constitute a form of closing in regard to debate and potentialities of Greek thinking in two preceding centuries.

Non Euclidean geometries are another example: with them the idea of universality discloses itself as plurality of truth and centrality of freedom. On the other side, plurality of truth advocated by non Euclidean geometries rises strong oppositions, and finds resistances, not only among mathematician but on the inside of society too, mirroring difficulties to accept that idea. Soon an opposed view of universality as a closure matches together mathematics, science and society: the project of a total axiomatisation agrees with the aspiration of unification of science and some of its aspects reverberate in the totalitarianisms that have grown in the society.

The current time sees the defeat of totalitarianisms, the difficulties of science in front of complex systems and the disillusion of mathematics as regards the axiomatisation. Mathematics, science and society live together a sense of bewilderment in face of the idea of universality. At the same time, technology, the main product of interaction among Western society, modern science and mathematics, is pushing to globalisation. So the idea of universality of Western society has to front the active presence of other cultures that makes reason, criteria of truth, values and rights relative.

The crisis of universality produces a bewilderment that involves, in particular, teaching mathematics. Teachers are left without certain references, and their teaching risks to result in training to technicism and in mere memorisation. In particular, teachers whose mathematical formation has been grounded on axiomatisation feel this bewilderment more deeply.

In order to rediscover the "flavour" of universality in teaching mathematics, we start from a reflection about the original meaning of term "axiom". The Greek term "*axios*" means *dignum, id*



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*est worthy*; teaching mathematics needs to invent dignity of what is being taught. Styles and subjects of teaching must be reconsidered in function of moment, of background, of context: dignity is sometimes criterion of choice, but, above all, a challenge to make taught subjects re-flourish.

“*The whole stays in the fragment*”. Just because of the fact that, in Italy, primary teaching seems less involved in the loss of centrality of axiomatisation, our path of discovering starts from it. This teaching becomes an opportunity to explore and to rediscover the dignity of particular as a starting point to a new attention to universal: it could bring out those single moments and processes which are fundamentals of mathematics and unveil a new path of opening. Starting from dignity of the particular, our taught mathematics and ethnomathematics meet.

Two examples show what we mean for dignity of particular, referring to teaching. In one example, dignity lies in recovering deep, basic meaning of quantity; in the other, dignity lies in recuperating corporeity in teaching and learning geometry, through the use of paper folding. The first example is developed in another presentation of these proceedings: *Algebra for primary grades: construction of sense of quantity (second part)*. Here we develop the second example.

We have explored many activities of paper folding and we have experimented them in our classes, from primary grades to high school. Aims differ according ages: a generalized aim is to widen and to deepen students knowledge about geometric forms, by creation, modification, investigation; for younger children the aim is to facilitate memorization, logical capabilities and communication; for higher students the aims are to project the sheet of paper and to structure the succession of folds, to experiment and explore spatial reasoning by practical activity, to verify and check results, to communicate through practical demonstrations.

The following observations of the teacher that worked in sixth degree show as, by means of simple activities, she recovered dignity to her teaching.

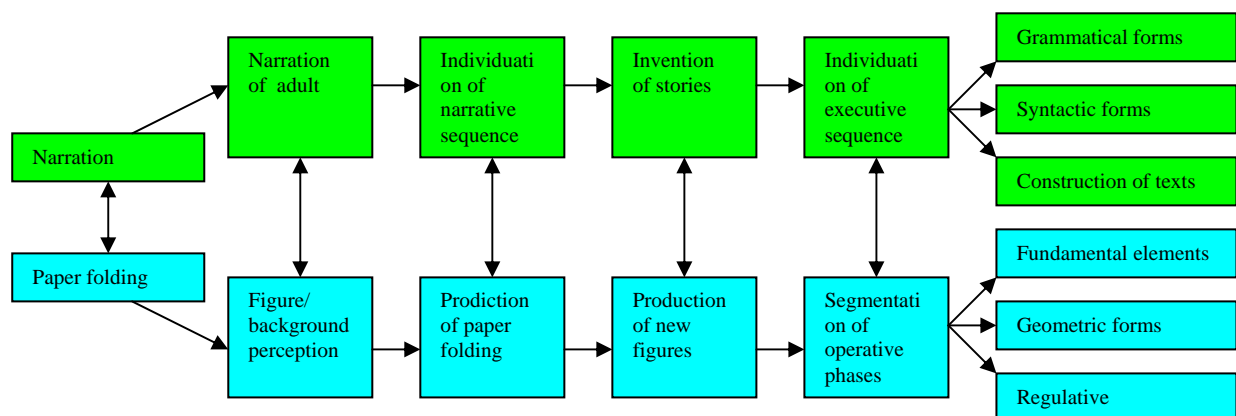
“It is a normal class of middle level and without particularly brilliant pupils. Pupils immediately show unexpected difficulties in manual skill of paper folding. Acquiring manual skills could be itself an aim of proposed activities. These difficulties forced us to renounce to the more complicated among prepared activities and to limit ourselves to propose the ones about the construction of geometric figures (rectangle, isosceles triangle, square, equilateral triangle) and the determination of some of its properties.



Study of properties of figures was the most interesting part of activities. The labour of constructing facilitated their determination and memorisation: while some properties learned in other classes remained vague and poorly memorized, manually constructing, for example, median, bisectors etc., shaped in pupils a deeper knowledge that they can employ in analysing properties of figures. Pupils have been completely involved and motivated. Notwithstanding difficulties in manual skills, they don't have perceived this part of mathematics as an obstacle. In the choice for portfolio, all pupils have proposed to insert some figures constructed with paper folding; no other part of the program of mathematics has been proposed by pupils to be inserted in it."

This activity finds its dignity in involvement, corporeity, effectiveness and easy transferability.

In activities developed in forth and fifth degree, teachers analysed analogies between functions used in the production of a literary text and a geometric one.



Then they, running through the found scheme, guided children toward the production of a geometric text, by which they can learn the properties of geometric figures.

So, for example, a story about Egyptian and Mayan pyramids is expanded in the production of pyramids by means of paper folding, following a succession of suitable rules, and in the representation of the story in the exercise book. The terms used by children in indicating different folds are compared with right language. Then the scene is inverted: children, using the same basic folds, create other forms and, referring to them, invent a new story. This story gives the opportunity to classify folds used by children, to translate them in a geometric language and to produce a text about relations among straight lines.

In this example dignity is in involvement, corporeity and communication.

In conclusion, the need of dignity for what is taught demands a different way to be a teacher. It is an opportunity of dignity for the teacher itself. The "axios" discloses as ethical dignity.



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So mathematics is not only a site of formalisation but open itself to axiology, that is to a horizon of ethical values, chosen in function of human dignity. It could become a text of a hoped society.

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