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THE USE OF TECHNOLOGY AND MATHEMATICAL RESULTS IN PARALLEL. A CASE WITH IMPROPER INTEGRATION

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Abstract

This paper addresses a teaching sequence developed to improve the students' understanding of improper integration. We briefly describe our theoretical framework (semiotic registers of representation as the cognitive dimension of a Didactic Engineering) and some activities with the aim of operationalising some theoretical results while working with computer software (using the theory of instrumentation).

1. INTRODUCTION

Nowadays it is undeniable that the impact of new technologies affects our everyday life. Even if we do not use them directly, the world that surrounds us does. As a consequence, many countries explicitly mention the use of Information and Communication Technologies (ICT) in their curricula (see, for instance, CFEM, 2004) and encourage teachers to train their students with the use of them. However, we think that a misuse of these technologies (particularly, graphic calculators or computer software) may produce some problems which may raise questions like:

- What kind of skills will our students develop?
- If the students learn how to carry out complex calculations with the help of these tools, what can we teach them?
- Will our students be interested in understanding theory if they are aware of the power of these tools?

Nevertheless, a correct use of these technologies, based on an adequate theoretical framework, may transform the previous questions into the following ones:

- What new skills will our students develop and how can they be combined with the *traditional* ones?
- Can students be aware of the importance of controlling the tool's outputs and that sometimes it fails?
- What kind of activities may we propose to help them to be aware of the importance of knowing theoretical results?

In the following, we will summarise the most important points of a Didactic Engineering (see Artigue, 1992) we have developed in order to teach the concept of Improper Integral[1] at the undergraduate level, which was complemented with some sessions in the computer lab.

[1] The Riemann integral is defined under two important conditions: bounded interval of integration and bounded function within this interval. When one of them fails, or both, we define the improper integral as follows:

2. THEORETICAL FRAMEWORK AND METHODOLOGY

Our previous investigation showed us that our undergraduate students do not achieve a good understanding of the concept of Improper Integral (González-Martín & Camacho, 2002a, 2002b, 2004a) and face a number of obstacles and difficulties. Moreover, traditional teaching does not allow a coordination between semiotic systems of representation, so many students are not able to interpret the results they obtain. As a consequence, the students adapt themselves to this situation and just develop an algorithmic understanding of the concepts related to improper integration.

In accordance with Duval's statement that *"there's no knowledge without representation"* (Duval, 2000), we consider that the use of different semiotic representations of a mathematical object is necessary to achieve a good understanding (Duval, 1993). In particular, we developed an epistemological analysis of the origins of improper integration, which showed us that it was born linked to geometric considerations. As a result, we decided to design a teaching sequence which took into account the use of both the algebraic and graphic registers to minimise some lacks we had observed in our students.

The methodology we decided to use consisted on the design of a Didactic Engineering (Artigue, 1992) which allowed us both to modify the traditional teaching sequence and to observe the students' reconstruction of their knowledge. Our fundamental aims were (González-Martín & Camacho, 2003): 1) To analyse the processes involved in the development of this concept; 2) To investigate the most common obstacles, difficulties and errors that arise in this context; 3) To develop a teaching sequence that included the use of new technologies.

On the other hand, our most important contributions in the design of our sequence are: 1) Systematic use of examples and counter-examples (in particular, those in the graphic register); 2) Combination of the graphic and algebraic registers; 3) Explicit turn to knowledge about series and definite integrals; 4) The student is given more responsibility in his learning process; 5) Use of the software *Maple V;* 6) Use of the obstacles and errors previously detected as elements to improve learning and teaching.

Some of the results of our Engineering (in particular, those addressing the use of the graphic register) have been showed in González-Martín & Camacho (2004b).

3. THE DESIGN OF THE COMPUTER SESSIONS

We agree with Guin & Trouche (1999), who state that *"transforming any tool into a mathematical instrument for students involves a complex 'instrumentation ' process and does not necessarily lead to better mathematical understanding"*. Bearing this idea in mind, we developed an analysis of some restrictions *Maple V Release 5* imposes when working with improper integrals (that allowed us to design our activities) and the possibilities of *orchestrating* our sessions with the use of computers (which we do not show here for lack of space). The main idea was to design some tasks which showed the students both the potentials and constraints of the software, making it necessary to operationalise the theoretical results institutionalised during the Engineering sessions. This way, it is possible to develop a feeling that computers will not solve all the questions, what can encourage the students to use their knowledge to overcome these limitations.

$\int_a^b f(x)dx = \lim_{t \to b} \int_a^t f(x)dx$ (supposing f is locally integrable in *[a, b))*.

Our main objectives were to continue reinforcing the mathematical status of the graphic register (as we did in the Engineering sessions) and to show the students the importance of handling mathematical tools in order to control the outputs of the computer. We also aimed at observing some instrumental genesis and whether the obstacles detected by Drijvers (2002) may be generalised to the use of *Maple V*.

4. SOME RESULTS

We do not discuss in this preliminary version of our paper the questions concerning the viability of orchestration using a computer lab nor the collective introduction to the software and the appearance of some phenomena which seem to generalise the obstacles observed by Drijvers.

We focus on one of the activities designed to give importance to the use of mathematical criteria even when working with a computer. We asked our students to analyse

the convergence

of the integral: $\int_0^{\infty} \frac{\sin^2 x}{1+x^{3/2}}dx$

However, if they tackled this question directly, they met one of the constraints of the software, which gives an unintelligible output (Fig. 1).

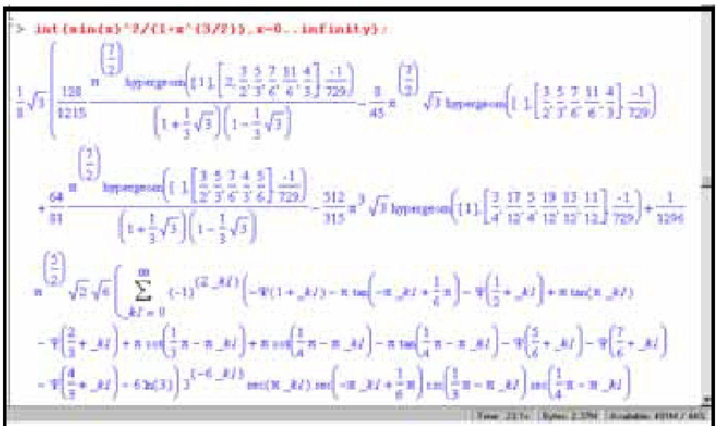

Figure 1

We intended that the students operationalised the convergence criteria institutionalised in the classroom. In particular, we expected them to use the Quotient Criterion (calculating the limit of *f(x)/g(x)* to get a finite constant, being *g(x)* an easier function with convergent integral).

For instance, the function

$g(x) = \frac{x^{1/3}}{1+x^{3/2}}$ has a convergent integral (what can be showed using both *Maple V* and other criteria) *and f(x)/g(x)* $\rightarrow$ 0 as *x* increases (what proves the convergence of the initial integral). But many of our students had serious difficulties with the use of algebra.

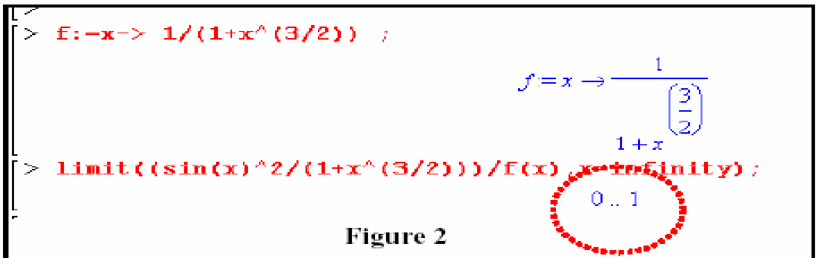

Figure 2

Some of them tried to compare using

$g(x) = \frac{1}{1+x^{3/2}}$ but when

calculating the limit they found such a disconcerting answer as the one showed in

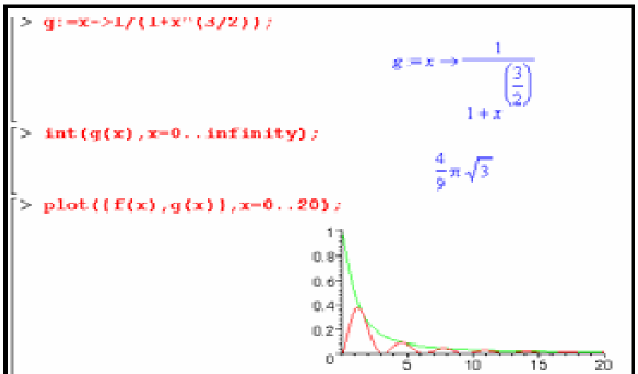

Figure 3

Fig. 2., which they could not interpret. Faced to this, they tried to solve this difficulty (and their own algebraic limitations) and they chose to use the Comparison Criterion (what we did not expect) and checked that $g(x)$ had a convergent integral and that $g(x) > f(x)$, so they could accomplish the task (Fig. 3), giving great importance to the use of the graphic register.

5. DISCUSSION AND CONCLUSIONS

We agree with the importance of observing the students' interactions with the technological tool; moreover, observations lead to state that many technical obstacles may be related with conceptual difficulties (Drijvers, 2002). Paying attention to technical obstacles will often involve conceptual aspects, what may provoke a conceptual development. The observation of our students' activities allows us to identify some steps in their instrumental genesis and the operationalisation of institutionalised results. We will give further examples of these activities and will analyse them in the light of our theoretical foundations.

Referring to our initial questions, it can be said that the learning of Mathematics in a computer environment may be combined with theoretical results, so that the students feel the necessity to control the software's outputs. This can be made by proposing appropriate activities which allow the operationalisation of both the technical and theoretical skills (showing the limitations lf the software, for instance).

In our case, the use of the software is not an impediment to the use of theoretical criteria, since it is possible to give the students activities where the software fails (if it is not used adequately). On the other hand, we have seen that our students take advantage of the software to develop activities which imply a coordination between registers (one skill that can be promoted with the use of a software), reinforcing the status of the graphic register (and producing graphics which are not easy to draw by hand). We think that the dialectic between the work in the Engineering sessions and the work in the computer lab has reinforced both the use of the institutionalised results and the graphic register to overcome the software's limitations.

In this brief space, we have showed one activity which allowed the students to be aware of the limitations of the software, which created a positive effect: the students got engaged to use the institutionalised results to overcome this situation, not only by recurring to the algebraic register, but also coordinating it with the graphic register.

The recourse to technology and its learning cannot be let completely in the hands of the student; it must be done under adequate theoretical foundations which allow a correct use of it and the development of an instrumental genesis. Its misuse may lead to new problems to be added to the ones caused by incorrect ways of teaching.

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