MATHEMATICAL DISCUSSION IN MULTIMEDIA LABORATORY: A NEW OPPORTUNITY FOR THE STUDENTS, A NEW CHALLENGE FOR THE TEACHER

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The growing importance of technology in everyday life and in school life is at the very core of international debate in Mathematics Education and the focus of our research. One of the main questions is if and how the impact of new technology in the classroom can be considered positive. Examples of a clearly productive integration of new technologies in the teaching and learning of mathematics are advocated. We present here a telling example of integration of new technology, namely the dynamic geometry software Cabri Géomètre. Our aim is to deal with some specific didactic choices that make the introduction of a new technology really useful in the classroom. In particular, we focus on the integration of Cabri within the mathematical discussion.

INTRODUCTION AND THEORETICAL BACKGROUND

It is widely reported in literature that the introduction of new technologies may change the way of teaching and learning mathematics. Lagrange & Grugeon (2003) present a review of several international studies concerning the potentialities of new technologies and find out the main "streams" of research, including the matter of integrating new technology into classroom life. They observe that, in order to find out the factors that may contribute to a fruitful introduction of new technologies, it is useful to take into account situations in which technology is already in the everyday classroom practice, exactly as paper & pencil. Our research can be inserted into this stream, since we present some reflections arising from the observation of a teaching experiment carried on in a classroom where activity with technology is widespread. In particular, we refer to the dynamic geometry software Cabri Géomètre.

A recent study by Assude&Gelis (2002), concerning Cabri, captured our attention: they give a sharp analysis of the "old-new dialectic relationship": they find out some constraints linked to the introduction of Cabri in ordinary classroom life and underline that it is necessary to find a good balance between old and new *ways of doing mathematics*, old and new *tasks* and *techniques*, traditional and innovative *learning environments* (namely, paper & pencil and Cabri). The matter of integrating Cabri within the *usual way of working* of the classroom seems crucial to us. In this paper, we take into account the situation of a classroom where the mathematical discussion is an *usual part of doing mathematics* and we focus on the viability of the integration of Cabri *within the mathematical discussion*.

The mathematical discussion is defined as a "polyphony of articulated voices on a mathematical object, that is one of the motives of the learning activities" (Bartolini Bussi, 1996, p.16). The interaction of *voices* is crucial: there is the *voice of the teacher*, who represents the *mathematical culture* and has the role of transmitting the official knowledge; there are the voices of the students and, as in a polyphony, not only each isolated voice, but the imitating voices are important. Adopting a Vygotskian perspective, the mathematical discussion is a promising context for the social construction of knowledge.

Activities of social interaction and classroom debate are also valuable because "students would rehearse the sort of intellectual courage, intellectual honesty, and wise restraint that Polya considered essential for doing mathematics" (Lampert, 1990, p.41). This sentence suggests us that a mathematical discussion is important in fostering students' construction of meaning as well as in improving students' view of mathematics. Furthermore, we believe that a mathematical discussion is an *engaging* experience for the students.

THE TEACHING EXPERIMENT: AN OVERVIEW

In this contribution we refer to a classroom episode taken from a research project aimed at introducing equations and inequalities from a functional standpoint, i.e. the notion of function as preliminary concept to solve equations and inequalities (Bazzini & Tsamir, 2001). There are different ways of conceiving functions in terms of variables and their changes (Slavit, 1997): to our aims, the difference between independent and dependent variables is relevant and leads to conceive a function in terms of *covariance*. According to these perspectives, we planned a teaching experiment (7 session of two hours each) with 8th grade secondary school students. Starting from a geometric situation of covariance, the students were guided throughout a series of activities, which led to discover the relationship between dependent and independent variables. The activities were set in different environments (paper & pencil, Cabri), and carried out with different modalities (individual work, group work). The transition from one activity to the other was enriched by fruitful *mathematical discussions*.

All the activities were video taped and the written production were collected. Furthermore, we have at disposal the field notes taken by an external observer.

FOCUSING ON THE MATHEMATICAL DISCUSSION

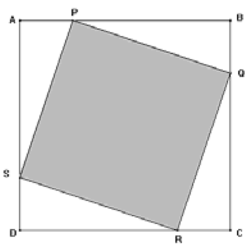
In a previous paper (Bazzini, Bertazzoli, Morselli, 2003), we analyzed the students' production in individual and group work and we argued that the sinergy of the two environments (Cabri and paper&pencil) may support students' mental dynamics. We also claimed that this integration may *respect* each student's preferred way of thinking and working, fostering a *personal* way of doing mathematics.

In the following, we'll detail on one mathematical discussion that took place in the multimedia laboratory, with only one computer, held by the teacher, and a projector. The episode we refer to took place in the third session: the students had already worked individually (in paper & pencil environment) and in groups (in paper & pencil environment and in Cabri environment).

We want to focus on the peculiarities of the mathematical discussion when it takes place in the **multimedia laboratory** and it is held by a teacher who is already accustomed to hold it in a traditional environment. Referring to the students, we ask ourselves if the mathematical discussion is still a rich and powerful context in which to carry out a social construction of knowledge, and if such an activity is still an *engaging* experience for the students. Referring to the teacher, we wonder whether her role changes, whether there are new constraints or necessities.

A priori, some difficulties may arise: the students have not at their direct disposal the computer, and the fact of being obliged to ask for the mediation of the teacher may hinder the spontaneousness (and fruitfulness) of the discussion. Furthermore, specific features of Cabri (e.g., the dragging modality as a powerful means to support exploring, producing a conjecture, validating**,** and proving, Arzarello et al., 2002) are known to be productive when it is the student to work with them, but what about the use of the software through the mediation of the teacher?

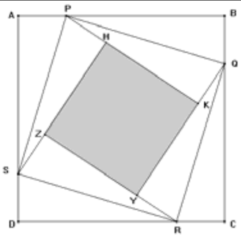
Our methodology consists in a **double analysis** of the transcript of the mathematical discussion: the analysis of the students' interventions and the analysis of the teacher (T)'s interventions. Here we present a short excerpt (other interesting excerpts will be discussed during the oral presentation). The students' interventions are analyzed according to two dimensions: the way of communicating own ideas and strategies (with special attention to the way of referring to the use of Cabri, of giving instructions to the teacher etc.) and the way of



dealing with the other students' projects and strategies. The teacher's interventions are analyzed linking each intervention with a goal and judging the effect of the intervention according to the goal.

The episode: A square is inscribed in another square. Starting from the previous group work, the teacher is leading a balance discussion, aimed at working on limit cases (minimum and maximum area of the inscribed square), symmetric squares having the same area. The aim of the discussion is to ground the relationship between the position of the inscribed square and its area. After half an hour of discussion, Thomas proposes his conjecture.

1. Thomas: I would like to say something. If we took the triangles that are under and put them into the square (*his gesture reproduces the act of moving the triangles*), maybe something would happen… maybe there would be another square… in the square.	Thomas proposes his conjecture, which comes from a tranformational thinking on the figure which is on the screen.
2. T: Wait, I'll do this: we understood that the two symmetric squares have the same area and that area is bigger than the area of the limit case. Then, I can close this figure and go back to the previous one, otherwise I can not follow you, Thomas. (*she opens the initial file, with the square in the square*). These triangles? (*she points QCR, SRD, APS, PBQ*).	The teacher opens another figure, in order to have a new figure to work on. She wants to clearly separate the previous discussion, which led to an institutionalisation ot the concept of limit cases, from the new explorative activity proposed by Thomas. Her concern is to make Thomas's conjecture accessible to the other students.
3. Thomas: And if we put them into the square, maybe we have another square. Because they take some space away and we have another square.	Thomas's instructions to the teacher are led by his dynamic thinking.
4. T: (*she "puts" the triangle RDS in the square PQRS, that is she gets the triangle symmetric to PQR according to the line RS*) In this position or the other? (*she asks whether she has to consider the generic inscribed saure or the square with the minimum area*)	The teacher is acting as a prothesis of Thomas's hand. She makes Thomas to be more efficient in communicating the instructions.
5. Thomas: in this position. I don't know… maybe not in this position but in the other one. For me, putting the triangles in the square we have a square.	Thomas is obliged to revise his conjecture, in order to give more efficient instructions to the teacher. He states that the conjecture refers to the limit case (minimum area), but he seems puzzled. Verbalizing the cnjecture requires a deeper reflection.
6. T: (*she erases the triangle just drawn*) So, you think that if I reverse these triangles into the square, I have another square?	The teacher mirrors the conjecture to Thomas, with the double aim of encouraging him and clarifying the conjecture to the other students.
7. Thomas: yes, but I'm not sure of it.	
8. T: Ok, I must draw these triangles! Shall I draw them? I hear some "yes", some "impossible"… Shall I work with the symmetry again? (*she draws the symmetric of PBQ*). In this way, Thomas? Did I understand? Shall I do it? Andrea, are you convinced?	The questions have the aim of involving the other students. The students are reflecting on the conjecture.
9. Andrea: Yes, because if we go on, we have a quadrilateral which is smaller than the yellow one, but it is a square.	Andrea agrees with Thomas: from this point on they represent the same *voice*. He adds that the new square will be smaller.

10. T: (*she draws the other triangles. All the students observe in silence the screen*). Yes or no? 	
11. Many voices: Yes.	
12. Stefano: It is smallest.	Stefano enters the discussion; he wants to add his own observation.
13. Thomas: I didn't say it was the same area!	
14. T: Yes, Stefano, what did you say?	The teacher takes care of giving space to all the voices.
15. Stefano: no, I had understood that Thomas said it was the same area.	
16. T: The same area of what?	
17. Stefano: Of the yellow one. (*the yellow square is the square PQRS*)	
18. T: But there will be a case when the area will be the same of the area of the yellow one, or not?	Stefano's observation is an occasion to promote a further reflection.
19. Pietro: In the limit case.	Pietro produces a conjecture.
20. T: In the limit case? (*she drags the point P to A; the square PQRS is coincident with the square ABCD*).	The teacher mirrors the answer and drags the point, to test the conjecture (dragging test).
21. Many voices: There is not.	
22. T: Be careful, what is not there? (*she drags the point back to an intermediate position between A and the middle point of AB*).	The teacher drags the point back in order to foster the reflection of the dynamic situation; now, she acts without waiting for the students' instructions.
23. Andrea: There is not the square.	
24. T: Why? (*she drags back to the limit case*)	
25. Marta: It is the centre.	Also Marta and Andrea are involved in exploring the situation proposed by Thomas.
26. Andrea: There is the square, but we can not see it.	
27. Thomas: It is the point.	
28. T: What did you say? Is it a point? (*she drags the point P along the side AB*)	
29. Thomas: I mean… finally that is like (gesture)… how can I say… the focus of a camera.	Thomas's metaphore is efficient in transmitting the dynamyc image of the square that disappears.

PRELIMINARY CONCLUSIONS AND FURTHER DEVELOPMENTS

As a preliminary conclusion, we claim that the mathematical discussion that takes place in the multimedia laboratory is still a promising context in which the students socially build their knowledge and experience an engaging mathematical adventure. The analysis of the students' interventions shows that Cabri plays the role of a *shared workplace* in which students carry out their reflections (Olivero, 2003). Cabri's being at disposal via the mediation of the teacher does not seem to constitute an obstacle: on the contrary, the necessity of explicitating instructions to the teacher fosters the student's verbalization and the involvement of the peers. Furthermore, the students' interventions introduce into the discussion many ideas and issues, making the discussion a rich and *engaging* mathematical adventure. The role of the teacher is crucial in assuring the success of the activity: she has to take into account the social, mathematical and technical (related to the software) dimensions of the discussion. Referring to the particular situation (one computer at disposal), we could say that she acts both as a route-keeper and as a prothesis of the students' hand.

New issues seem worth to be developed. The analysis of the discussion shows that the presence of Cabri supports the discussion and makes the social construction of knowledge even more dynamic. This growing dynamism may become problematic from a double point of view: for the teacher's orchestration and for the real involvement of all the students into the discussion. For this reason, it would be interesting to re-analyse the transcript focusing on the effective possibility for the teacher of giving space to all the ideas emerged and on the real participation of each student. Another issue, deeply related to the previous one, concerns the matter of the personal *time* of each student. In our previous work we claimed that the alternation of activities in paper & pencil and Cabri might support a *personal* development of meaning, because activities in one environment helps understanding in the other. We wonder now if the discussion in multimedia laboratory can help the students to settle their personal times of reflection and understanding in the collective time.

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