



## Analytic geometry and vector calculus at secondary school

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The aim of our work was to describe situation of teaching the analytic geometry and vector calculus at secondary schools in the Slovak Republic, to specify possible problems, those are due to the application of methods of analytic geometry and vector calculus in various geometric tasks and to propose their possible solution. This aim we realized in following phases:

- 1 *Historical and epistemological development of analytic geometry and vector calculus.*
- 2 *Survey of the actual situation and historical background of teaching the analytic geometry and vector calculus following textbooks used in Slovakia.*
- 3 *Analysis of actual mathematics French textbooks of secondary schools and their comparison with Slovak textbooks.*
- 4 *Actual situation and specification of problems in the teaching of analytic geometry and vector calculus at secondary schools.*
- 5 *Proposals of possible problems' solutions and compilation the collection of tasks that are leading to enhancement of the teaching the analytic geometry and vector calculus at Slovak secondary schools with specific recommended strategies of their solution.*
- 6 *Verification of efficiency the selected tasks from our tasks' collection by comparison the abilities and capacities of students that took part in experimental teaching (with utilization our propositions and strategies) and students that participated in the teaching with utilization the methods and processes, which are typical for Slovak secondary schools.*

### ***1 Historical and epistemological development of analytic geometry and vector calculus***

Importance of working-out this part have been, first of all, in research the background of students' problems with application of methods and processes in analytic geometry and with utilization the vector in geometric problems at secondary school, by reason of analogy between ontogenetic formation of everyone individual, which is a consequence of evolutionary development the world and development ours mathematical thinking with historical evolution of mathematics as a discipline.

### ***2 Survey of the actual situation and historical background of teaching the analytic geometry and vector calculus following textbooks used in Slovakia.***

Progressive introduction the methods of analytic geometry in solving the mathematical problems, the definition of vector and the constitution the analytic geometry on vector's ground based in the history, result in ambition to integrate this part of mathematics in the teaching at secondary schools and colleges. Teaching the analytic geometry and vector calculus at secondary schools progressed, what indicate using of different methods and processes in mathematics' textbooks which were edited during the development of Slovak educational system. The analysis of this textbooks

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included with analysis of actual textbooks used at Slovak secondary schools permit us to understand the reliable methods and practices in teaching the analytic geometry and vector calculus at secondary schools.

### ***3 Analysis of the actual mathematics French textbooks of secondary schools and their comparison with Slovak textbooks.***

Concerning our positive pedagogical experiences with teaching the analytic geometry and vector calculus following the French textbooks (at French-Slovak bilingual secondary school), we proposed to solve problems in analytic geometry and vector calculus at Slovak secondary schools by application some strategies from French mathematics which prefer geometrical approach to problem solving.

### ***4 Actual situation and specification of problems in the teaching of analytic geometry and vector calculus at secondary schools.***

As the method of work we chose the content analysis of students' works (109 participants), which consisted of two tasks (one was situated in the plane and the second in the space). Our research was realized following the analysis of possible students' solutions in accordance with *theory of didactic situations* (Brousseau 1986, 1998, Chevallard 1989, Margolinas 1994, Sarazy 1995, Sierpiska 2001) and by analysis of historical and epistemological development of analytic geometry and vector calculus (the first part of our work) and analysis of secondary schools' textbooks (the second part of our work). We described the research results by the help of statistical software C.H.I.C. (Gras et al. 2003).

The principle aim of our research was to verify five hypothesis:

- H1** Students' capability in solving the open problems and in „problem solving“ is developed deficiently in the schools' practices, the students are using mainly acquired abilities and algorithms in solving the problems.
- H2** The effective combination and using the knowledge from different domain of mathematics in solving a particular geometrical problem make difficulties for the students.
- H3** The students prefer arithmetic and algebraic approach (methods of calculus in geometry) in comparison with methods of analytic geometry and vector calculus in solving the geometrical problems.
- H4** Students' access to analytic geometry is isolated from the other parts of schools' mathematics, the vector calculus represents for students only a mathematical tool for the analytic geometry.
- H5** The teaching of geometry in the space has a decreasing trend in regard to deficient students' motivation and their attitude towards tasks situated in the space.

We resumed the results of our research in following statements which are at the same time verification of our hypothesis and which explain in more detail the problems in teaching of analytic geometry and vector calculus at secondary schools (Regecová 2003):

- ❖ Teaching the vector calculus and analytic geometry at secondary schools is separated from this parts of geometry that used the methods of calculus and constructions and also they are not indicated the possible interdisciplinary relations, what cause for the students many difficulties at effective combination and using of knowledge from various area of mathematics in solving a particular geometrical problem.
- ❖ The students prefer methods of calculus in geometry (arithmetic and algebraic approach) in comparison with methods of analytic geometry and vector calculus in solving the geometrical problems.



- ❖ Content of vector calculus represents for the students almost a mathematical tool for the analytic geometry, forgotten is possibility of its individual usage in solving some geometrical problems.
- ❖ Comprehension of algebraic and geometric parts of analytic geometry is isolated in students' minds. Even though the students prefer analytic expression of the geometric figures in the solving of geometrical problems, connection between algebraic expression and geometrical representation the figure it is not clear for them.

Concerning the analysis of historical and epistemological development of analytic geometry and vector calculus, the analysis of secondary schools' textbooks, as well as frequent exploitation of different mathematical software and graphical calculators at secondary schools in France we formulated following statements in addition, which are according to us one of possible background of students' problems:

- ❖ Students of secondary schools are not sufficiently motivated by historical background of development the vector calculus and analytic geometry, as well as by possibility to applied obtained knowledge in other areas of mathematics, physics, informatics and in practice.
- ❖ Minimal using of information and communication technologies at the teaching of geometry decrees students' experiences in work with geometric figures and their complete view on geometry.

***5 Proposals of possible problems' solutions and compilation the collection of tasks that are leading to enhancement of the teaching the analytic geometry and vector calculus at Slovak secondary schools with specific recommended strategies of their solution.***

Following the analysis of actual Slovak and French secondary schools' textbooks and also for a consideration of results from the content analysis of students' works (part 4 of this paper) we arranged set of tasks that has an objective to improve the teaching of analytic geometry and vector calculus at Slovak secondary schools. This collection includes 113 tasks. With majority of those tasks we present their solution or instruction for solution. In our tasks we utilize at the same time Slovak and French strategies as their useful combination is a supposition to improve and to increase the efficiency of teaching the analytic geometry and vector calculus at secondary school.

***6 Verification of efficiency the selected tasks from our tasks' collection by comparison the abilities and capacities of students that took part in experimental teaching (with utilization our propositions and strategies) and students that participated in the teaching with utilization the methods and processes, which are typical for Slovak secondary schools.***

The efficiency of tasks and strategies that we proposed in our collection of tasks, we tested by an experiment which should have verify the validity of following hypothesis:

**H:** Application of methods and strategies of analytic geometry and vector calculus typical for French secondary teaching will enable to obtain better results for students at the end of our experimental teaching (in didactical test – students' papers) in comparison with students who know only methods and strategies typical for Slovak secondary teaching.

In this experiment we compared products of students that was participated in experimental teaching with French methods and strategies (31 participants) and students that took part in teaching



with typical Slovak methods and strategies (28 participants). As method of work we used an experimental method (**Gavora, 1997**) in which we have two equivalent students' groups and in each of them we apply an experimental change (experimental group) in comparison with the second group (control group). Process and experimental conditions excepting experimental change were identical in both groups. Results of experiment were worked by statistical induction, which permits to generalize results obtained from research of selected set to all population. As a method of statistical work we chose verification of hypothesis by  $t$  test (Kerlinger, 1972). Results obtained in our experiment are in favour of teaching by tasks and strategies from our tasks' collection, so by tasks which come from French secondary schools.

At the same time, results of experiment propose us an another proposals for next research. In the first place, we prefer to evaluate our experiment by the help of statistical software C.H.I.C., which permit to analyze students' works in more detail and which underline relations between particular variables in experiment. In next phase of research we plane:

- to realize an experiment with more tasks from our tasks' collection,
- to realize an experiment with more students,
- to give more time for experimental teaching,
- to repeat our experiment later in term of verification stability obtained knowledge.

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*CIEAEM 57 – Italie – Italy*  
*Piazza Armerina,*  
*July 23-29, 2005*

**Présentations orales**  
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**Oral presentations**  
**in Working Groups**

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