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From motion to graphic and symbolic representation: a teaching experiment in primary school

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Summary
Our research aims at studying the cognitive strategies used by primary school pupils during the construction of models in problem solving situations. The focus of the teaching experiment is the passage from perceptuo-motor experiences to conceptualisation of uniform motion and its mathematical model (direct proportionality) through symbolic representations: the graph and the formula. In order to do this, the teaching experiment has been planned with a gradual approach to the mathematical concepts, in which the children analysed the variation of the quantities space and time, using two technological artefacts: a pedometer and a calculator.

Resumé
Notre recherche a comme but d'analyser les stratégies cognitives que les enfants de l'école primaire activent pendant la modélisation de situations problématiques. Le noyau de l'activité projetée se fonde sur le passage des expériences perceptuo-motrices à la formalisation du mouvement uniforme (relation de proportionnalité directe) par des représentations symboliques: le graphique cartésien et la formule. Dans ce but on a bâti un parcours graduel au cours duquel les enfants ont analysé la variation des relations entre les variables place et temps, par l'apport de la technologie: un pedometer et une calculatrice graphique.

Theoretical framework

This study has been planned with both didactical and research objectives. On the didactical side, we completely agree with the recent curricular project presented in most countries all over the world, not excluded Italy, with the publications by UMI (Unione Matematica Italiana, Italian Mathematical Association): UMI, 2001 and UMI, 2003. Moreover, we are aware of the influence of technology in every aspect of students' life, involving their action, perception, imagination and control, most facilitating also the solution to school standard problems. The use of technological devices allows students also to share school tasks with school mates and teachers everywhere, favouring learning in social contexts. According to our research interests, we used results coming from psychology, mathematics education and cognitive science to investigate how technological devices influence students' learning processes. Being aware of the fact that simply giving students the possibility of using artefacts does not guarantee by itself they learn mathematics (Mariotti, 2002), we recognise the role of the teacher in planning activities and coordinate students during them.
Here we show an example of integration of two technological artefacts in a teaching experiment aimed at the construction of the meaning of direct proportionality (sub-theme 3, CIEAEM 57). A pedometer[1] collected space/time data and a graphic calculator elaborated these data in numerical, graphical and symbolic environments: children of 5th grade used them in a learning context of

[1] The pedometer is an artefact for athletic purpose, in order to collect measurements of space covered during a running performance in a certain time. It detects the number of steps made by a person: receiving as input the average length of a step, it gives as output the covered distance.

measurement, working in groups and discussing their results all together, with the coordination of the teacher.

Introducing the distinction between two learning modalities: the perceptuo-motor and the symbolic-reconstructive, Antinucci (2001) recognised the important role of the former in his studies of psychology. Recently this importance has been also recognised in mathematics education (Arzarello & Robutti, 2004), together with the importance of artefacts introduced in classroom activities for approaching mathematical concepts (Nemirovsky & Borba, 2003). The perceptuo-motor learning does not exclude the symbolic-reconstructive one: they are two complementary way of learning, which can be integrated with continuous interactions and mutual enrichments. On one side, the theory of *embodied cognition* (Lakoff & Nuñez, 2000) claims that the cognitive structure of mathematical concepts is based on conceptual systems (metaphors) with the role of linking language to visual and motor experiences. On the other side, the interaction with artefacts (more or less technological), mediates the construction of meaning of mathematical objects. Integrating these two sides of the theoretical framework, an interesting research point could be the analysis of the effects the use of an artefact can have on the use of conceptual mechanisms as metaphors. In some cases the interaction with an artefact deeply and constructively affects at cognitive level the students' mathematical production (e.g. Maschietto, 2004; Robutti, 2004). Introducing an artefact in classroom activities involves many problems: one can be the so-called *transparency* of the artefact with respect to the knowledge involved in using it. Referring to a graph obtained with a technological artefact, Ainley (2000) speaks of *transparency of the graph*, intended as the possibility to see the graph and the information it conveys, together with the phenomenon it represents. In any case, transparency cannot be only referred to the artefact, but it depends also on the use the subject makes of it (Meira, 1998). Our aim is to rend the two artefacts more and more transparent to children during the activities of the teaching experiment, in order to let them reach some competencies in using them.

Analysing the data (videotaped activities of group sessions and class discussions, together with written protocols) we observe students' processes towards the construction of meaning of mathematical objects, starting from perceptuo-motor experiences. In this analysis, the interpretation of the students' words becomes the fundamental element for arguing about their cognitive activity. According to the semiotic approach introduced by Radford (2000) we will describe the students' dialogues distinguishing between the *deictic function* (as here, there, where, …) and the *generative action function* of language (as take, put, open, move, …), the former used for indicating something or somebody, the latter for describing an action. These two functions have a semiotic role in the process of objectifying knowledge, together with the so-called *logic function* (Ferrara & Robutti, 2002), which refers to functional relations (as if … then, therefore, …). Another theoretical element for our analysis deals with gestures that accompany a speech (Goldin-Meadow, 2003), considered an essential element for thinking, as well as words (Kita, 2000).

The teaching experiment: aims and methodology

Our research is aimed at studying the cognitive strategies of primary school pupils when involved with problem solving situations aimed at constructing models of motions they have performed. The concept around which the teaching experiment is constructed is the direct proportionality between quantities (here, space and time, but generally every kind of variables). This concept is explored and constructed in various environments: numerical, graphical and symbolic, both with paper and pencil and a graphic calculator (TI-83Plus). In order to mediate the perceptuo-motor experience with the symbolic-reconstructive learning the pupils use a pedometer for collecting data. The reason to involve pupils in this kind of experiences has been given them by an occasion inside their town context: the library has been moved to another place, and they are asked by the director of the library to measure the distance between their school and the new library.

In the following excerpts[2] the students are discussing among them and with the teacher in order to understand how the pedometer works. It is interesting the use of 'human' actions referred to it: in this first approach, the pedometer is far from being transparent to them.

4.Livia: …yes it maybe counts the steps, but if I do one longer and one shorter and if I do one a bit longer maybe it put it together with another a bit shorter …
5.SILVIA: and then?
6.Livia:and then you can see that … maybe more.
7.Federica: for every step … maybe it counts … it sees how many centimetres, for every step it counts …
21.Enrica: what we want to see how many steps are there. For me if we have done few steps it is useless that it gives us in kilometres, because if we have done few it would be a number with … not integer, on the contrary if we have done more steps, as we will do to go to the library, it will give us a number in kilometres.

Pupils must go short routes wearing a pedometer and they collect data, complete tables, analyse them, make hypothesis and try to check them:

45.ORNELLA: so, are the data we found satisfying from the point of view of the hypothesis?
46.Alessio: we made right observations, because I should have done less steps than Filippo and Elisa because my average [step] was 60 [cm], while Filippo was 55, and was in the middle between Elisa, who was 49, and I.

50.Alessio: I was saying that from the hypothesis we did before, more or less they were right, because I had a step shorter … longer than everybody, in fact I was the one who did the least number of steps, Filippo was the one …

51.ORNELLA: are we able to say a rule? On the same distance, if a child has a longer step, he does fewer steps.
52.Davide S.: on the contrary, the child with the shortest step does more steps, but on the same distance.

TEMPO	PASSI	----- 2
0	0	
10	13	
20	28	
30	37	
40	55	
50	63	
60	81	

PASSI(1)=0

Figure 1

Finally they obtained a Cartesian graph that represents their walk. The most significant moment of the activity is in fact the motion representation through the Cartesian graph, which represents a particular relations between space and time:

74.ORNELLA: here we are, these are the data of Eleonora. Can you observe anything about these data?

75.Elisa: it becomes larger and larger, it becomes always bigger (Elisa makes the gesture of keeping something in her right hand, then progressively closes her thumb and her index finger, then opens her hand again, while she refers to the straight line).
76.ORNELLA: what?
77.Elisa: the number … of steps.
79.Federica C.: bigger … is the time, more steps one does.
80.ORNELLA: yes, the bigger the time, the more steps one does.
84.Giorgio: I have an idea, that if everyone really does a normal pace, for every interval of 10 seconds there should be the same number of steps.

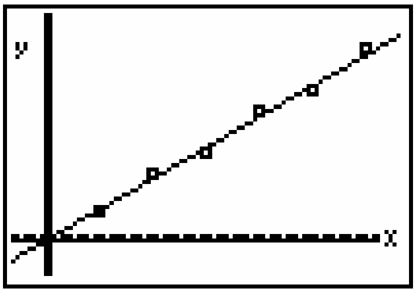

Figure 2

Our project has promoted various competencies: to explore, to solve problems, to propose strategies of solution, to formulate conjectures and hypothesis, to evaluate, to discuss, to collaborate and interact in the small group, to operate with different registers of representation: tables and graphs, compare data and anticipate. We have created a rich and stimulating learning environment, in which children have been able to operate in adherence to visual and motor experiences. The mediation of technologies has characterized all the learning process.

Preliminary results

The analysis of this teaching-learning experiment we are doing is far from being concluded, so our results are preliminary and open to other studies. From the didactical point of view children have developed a meaning of direct proportionality in a social context, with the use of technological artefacts, and the results of assessment confirm the construction of this meaning. In particular, we

[2] In capital letters the name of the teacher or of the observer guiding the discussion.

observed not only the students' final products, but also their processes during the various activities, and as in other researches, we noticed that the students not particularly motivated or high achievers showed a real interest, participation and personal development in conceptualisation. From the research point of view, the protocols listed above are divided into three parts, referring to different discussions: the first deals with the way the pedometer works, the second with the relation between the length of one's average step and the number of steps he does on a fixed distance, the third with the relation of direct proportionality between time and number of steps (the two figures are the table and the graph of this situation represented on the screen of the calculator). In the first one, we note the students' effort of understanding the way of collecting data of the pedometer, progressively obtained with the help of the teacher in reading the instructions of the artefact and trying to use it in groups. Two main ideas come from this discussion: the process of counting the steps, and the use of a measurement unit, to let the data be interpreted by the user. After having introduced the average length of a step, students used the pedometer (become transparent in the meanwhile) and discussed around the number of steps made by children with different average step. The introduction of a rule came from the children through action words for explaining the idea, and the final formulation comes from the observer[3] who was guiding the discussion in that moment: *On the same distance, if a child has a longer step, he does fewer steps.* The very interesting moments is reached in the third protocol, where students clearly perceive not only the shape of the graph (linear), but also the phenomenon it describes, both linked to quantity variation and its mathematical description. The direct proportionality, expressed first by gestures, then through words becoming more and more specific, is explained finally through the increments of two variables, distinguishing between the approximation of the experiment and the perfection of the model: *I have an idea, that if everyone really does a normal pace, for every interval of 10 seconds there should be the same number of steps*. The progression in approaching a meaning of the relation not only lets students acquire an awareness of what constant speed means, but also gives us information about their cognitive steps. The sign of an increasing straight line, introduced by a gesture, is strongly grounded on the referent, the perceptuo-motor activity and the class culture (related to the use of the pedometer). It is immediately shared among the students, thanks to the social activity, and it becomes progressively independent from the context, to be recognised and used in a more general way, as synonymous of a relation between variables. Our main result is that the use of technological artefacts not only favours the perceptuo-motor learning, but also a symbolic-reconstructive learning.

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[3] In the whole experiment, from the planning to the development in the class, the role of the teacher and of the observer (the two authors) was the same: they planned together the activity, video-recorded them or guided the discussions.

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