What subject matter knowledge about the concept of function should the teacher have?

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In the face of changes in how mathematics is perceived by the general public, the need for a better, more comprehensive and meticulous training of mathematics teachers becomes increasingly important (Z. Moszner, 2004). In particular, the teacher should have profound and thorough subject matter knowledge, by which I understand indispensable for teaching knowledge of abstract mathematics, its methods and history. R. Even (1990) put forward a general introductory conception of teachers' subject matter knowledge concerning a given concept. I have tried to apply R. Even's theoretical framework in order to analyse data from my research. It turned out that many substantial modifications, extensions and specifications had to be made. After implementing these changes I have distinguished 6 components of teachers' subject matter knowledge of a given concept. Below I include an overview of the components of teachers' subject matter knowledge related to the concept of function.

1. THE ESSENCE OF FUNCTION

General knowledge of the origin and historical development of the concept of function, being familiar with modern different definitions of the concept, knowing *essential features* (R. Even, 1990, H. Freudenthal, 1983) and understanding the notion of function in the selected aspect (e.g. on the *defining level* (Z. Dyrszlag, 1978), function as a fully shaped object and not only as a process (A. Sfard, 1991), on the levels of *abstraction* and *formalisation* (J. Bergeron&N. Herscovics, 1982) and possessing the *deep idea* of the concept of function (Z. Semadeni, 2002a, 2002b)).

2. DIFFERENT REPRESENTATIONS AND LANGUAGES RELATED TO FUNCTION

(a) Knowing and understanding different representations of the concept of function and its *surface forms* (Z. Semadeni, 2002a, 2002b), being familiar with connections between its different representations, ability to interchange them flexibly and to choose an appropriate representation depending on the context and needs,
(b) Being aware of the fact that representations are secondary to the concept of function (M. Klakla, 2003b, A. Sierpinska, 1992),
(c) Knowing different "languages" used for particular categories of functions, being able to use these "languages", being aware connections between them and knowing advantages and disadvantages of these connections.

3. BASIC REPERTOIRE OF FUNCTION DESIGNATONS

Understanding thoroughly the designations of a function, which are determined in the curriculum (R. Even, 1990; Z. Dyrszlag, 1978) and having at one's disposal a set of function designations much wider than the one determined in the curriculum.

4. ANALYSING FUNCTION DESIGNATIONS

Being familiar with different *ways of approaching functions* (R Even, 1990) while examining them in a given representation, being able to make the right choice in a given context, being able to examine designations of a function thoroughly (Z. Dyrszlag, 1978; J. Konior, 2002b) and constructing examples which are or are not a function designations or which fulfil given requirements (Z. Dyrszlag, 1978).

5. THE STRENGTH OF FUNCTION

Understanding and ability to use the strength of a function in mathematics, which is rooted in:
(a) operations of composing and inverting functions which create new functions and new possibilities for the development of an analysis (R. Even, 1990; H. Freudenthal, 1983),
(b) possibility to side-step a problem by mapping into an image set and solving an easier problem instead, i. e. *the transport of a structure* (A. Z. Krygowska, 1977; R. Skemp, 1971),
(c) generalisations of the function of one variable.

6. MATHEMATICAL CULTURE

The following aspects are considered in the context of solving problems related to functions:

(a) knowing the elements of a mathematical method (R. Even, 1990; A. Z. Krygowska, 1977),

(b) mastering basic approaches and behaviours unique to mathematics i.e. *mathematical mental activities* (A. Z. Krygowska, 1986), e. g: generalising, specifying, defining, deducing, reducing, the role of examples and counter-examples, *creative activities* (M. Klakla, 2002a), *transfer of a method* (M.Klakla, 2002b), etc.

(c) mastering approaches and mental activities, which can be developed through mathematics and transferred to every day life situations outside the mathematical context, for instance: *discipline and critical thinking* (M. Klakla 2003a)

(d) possessing the ability of *self-observation of teachers' mental activity* (J. Konior, 1993).

REFERENCES

(see the references attached to the paper: M. Sajka, *Functional equations as a new tool for researching certain aspects of subject matter knowledge of functions in future mathematics teachers*, in this book.)