Influence of Inquiry into Mathematics on the Student-Teacher's Professional Development

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Abstract: The paper deals with the education of teachers to be prepared for the demands of the educational reform in our schools. The tool we use, among others, is engaging student-teachers in their own mathematical research thus providing them with experience of 'doing' mathematics. This is hoped to influence their future teaching towards using more investigative ways of teaching. A long-term case study of a university student is presented which looks into the influences investigations have on a solver.

1. Introduction

In many countries in Europe including the Czech Republic, an educational reform is currently under way which stresses a humanistic aspect of the teaching of mathematics at the expense of mastering procedures of calculation, skills and algorithms. Teachers are expected to apply investigative ways of teaching, so that students experience mathematics as a living subject in which they can discover some things for themselves. However, we feel that teachers are often not sufficiently prepared to meet the new demands, not only in terms of the teaching strategies, but in terms of their mathematical knowledge, too. Many of them learnt mathematics only in a transmissive way[1] without really 'doing' any mathematics at all.

2. Inquiry into Mathematics and Student-teachers

From the literature on the education of future teachers, we will mention Schifter (1998) who proposes two avenues for teachers' mathematical development – engagement in inquiry into mathematics itself and investigation of children's mathematical thinking. Here, we will focus on the first avenue.

Lachance and Confrey (2003) describe how a professional community was started among a group of secondary mathematics teachers through in-service work on mathematical problem solving and technology. Knoll et al. (2004) describe a project in which future elementary teachers were given experience of mathematical research at their own level and how this affected their approach to mathematics and its teaching. Hejny (2004) stresses that by engaging future elementary teachers in doing mathematics, however trivial, profoundly affects their self-confidence in mathematics, which is usually the most pressing problem. Kratochvilova (2002) provides a case study to support this claim. Zhouf (2005) proposes a mathematical topic accessible even to secondary students, which is currently being used with some future mathematics teachers.

Here, we will investigate how a (long-term) mathematical inquiry affected one student-teacher.

3. Methodology of Research

[1] Transmissive teaching is based on the assumption that it is possible to transmit knowledge by telling students what they have to know and how to understand it. The term constructivist teaching (e.g. Jaworski, 1994) will be used for the teaching based on providing students with challenges to construct knowledge by themselves. It is above the scope of this article to go into more detail.

The case study below is a part of long-term research on university students' ability to structure mathematical knowledge (Stehlikova, 2004). The *mathematical research tool* is a finite arithmetic structure created originally by M. Hejny and called restricted arithmetic (or RA). RA is congurence modulo 99 in 'disguise'; it means that it is not immediately apparent to students. RA is based on an analogy with 'ordinary' arithmetic, which enables students to use their existing knowledge for developing solving strategies and posing problems.[2]

The focus of the case study is Molly, a future mathematics teacher. Our cooperation started in her second year at the university and spanned 3.5 years until the defence of her diploma work. She was one of the volunteers taking part in the above research. However, unlike the others, she wanted to continue after the interviews ended.

The *research questions* were formulated as follows:

- How will Molly's responsibility for her learning and her self-confidence in her mathematical abilities be influenced by the constructivist style of work and the particular mathematical topic?
- How will her views of mathematics and the teaching of mathematics change?
- To what extent is RA appropriate for the above purposes?

3.1. Case Study Database and its Analysis

Molly's case study database includes copies of her work, transcripts of interviews, her concept map of RA, the experimenter's field notes and the Final Interview (or FI). In FI, the experimenter asked Molly questions concerning her views of mathematics and mathematics teaching, her experience with mathematics learning, her experiences from the university and the work on RA. Indirect rather than direct questions were used not to influence Molly's answers in any way.

For the analysis, we used the first strategy, based on convergence, proposed by Yin (1998) for a case study database – the same question is asked of different sources of evidence so that a fact which emerged from one source is confirmed by other sources of evidence.

Our work with Molly can be divided into 4 non-sequential phases. A deeper comparative analysis of the case study database revealed 8 situations describing it more fully.

Phase 1 (*Interviews*) includes interviews (recorded and later transcribed) during which Molly solved tasks given by the experimenter and later posed her own partial problems. First, they were Semi-structured Interviews (S1), later Teaching Interviews (S2) (i.e. the focus was Molly's development rather than getting data for the original research).

Phase 2 (*Investigations*) began when Molly started proposing own problems and comprises Molly's (mostly self-directed) investigation of RA. It was initiated mostly by Molly, the experimenter's role was reacting to Molly's needs. It comprises Discussions (S3) and Impromptu Discussions (S4) (sessions not prepared in advance).

Phase 3 (*Writing*) includes Molly's Mathematical Writing (S6) (including her diploma thesis), and also putting down her Results for Herself (S5). She produced several subsequent versions of her description of properties of RA, of her investigations of general powers and of Pythagorean triples in RA.

[2]For a detailed description of RA and possible problems to be solved, see Stehlikova (2004). Problems Molly, the student presented in this article, solved and her solutions will be presented during the presentation.

Phase 4 (*Didactic Research*) concerns the second avenue proposed by Schifter (1998). Molly prepared, carried out and analysed her Teaching Experiment (S7) with another student.

A special position is held by Individual Work (S8), which appears with a varying intensity in Phase 1 through to Phase 4.

By a detailed analysis of the eight situations, we addressed the research questions above. The analysis was based on the grounded theory approach (Strauss, Corbin, 1998).

3.2 Some Results

It is not our purpose to present the development of Molly's understanding of RA (see Stehlikova, 2004). Rather, we will summarise the influence of the above process on her professional development.[3]

Attitude towards her Learning

Towards the end of the work, Molly showed a growing responsibility for investigating RA, autonomy in determining solving strategies (from strategies based on ordinary arithmetic to new strategies) and in problem posing. We could see a development from the external to the internal sense of authority, relying more on her judgement of the validity of a mathematical statement rather than on the judgement of others, and growing self-confidence in her mathematical abilities.

We believe that this was caused mainly by the suitable context of RA (adequate complexity) and constructivist approach (suitable initial problems, the experimenter's reluctance to provide solving strategies and to indicate correctness of what Molly said, etc.), which Molly welcomed.

Attitude towards Mathematics and Mathematical Knowledge

Molly's development in this area could be described as a shift from being a consumer of mathematics to being a creator of mathematics. By experiencing joy from a mathematical discovery, albeit of minor mathematical importance, she glimpsed what mathematics was really about as a science.

Mathematical Competencies

On a general level, we can see the improvement in Molly's ability to analyse a mathematical area and structure and restructure knowledge about it, develop new mathematical concepts, formulate and test hypotheses, and present mathematical results (more specifically, the ability to experiment, look for relationships, create solving strategies, formulate and check hypotheses, look for suitable organising principles, present knowledge in a succinct way and symbolise it, etc.).

Approach to the Teaching of Mathematics

We hoped that being introduced to RA in a constructivist way and probably connecting this experience to that of one university course also run in this way, would influence Molly's approach to the teaching of mathematics which tended to be rather traditional. Molly valued her independent discovery of some parts of RA, however, on the other hand, she also appreciated those courses in which "they [teachers] proceeded systematically and explained things clearly. All parts are logically ordered." (FI) She understood the relationship between the constructivist and transmissive ways as complementary, not antagonistic.

[3] The results below will be illustrated by Molly's work and utterances during the presentation.

Molly stressed that only some parts of mathematics should be presented in a constructivist way and only for older students: "... at the secondary school, it is too early, they do not have the basic knowledge."[4] Her opinions are mostly based on her own experience. She had never had an opportunity to experience constructivist teaching prior to university.

4. Conclusions

We have shown that a long-term mathematical inquiry can have a positive influence on a student-teacher's professional development in several ways. The biggest influence can be seen in their attitude towards mathematics and learning of mathematics. We agree with Schifter (1998) that "... many teachers become aware, for the first time, that they, themselves, can be initiators of mathematical thought – that they can offer conjectures, test them out, become curious about mathematical questions, and make their own way through problems they did not expect to be able to solve." Student-teachers must have an opportunity to feel like 'initiators of mathematical thought' as they should pass a similar message to their future students.

We have been interested most in the influence of one's own mathematical inquiry on the attitude towards the teaching of mathematics. We have seen that Molly transformed her experience into the belief that even her students can learn mathematics in a similar way only to a certain extent and only after some time. At first, she thought that an independent discovery could be made by students only when sufficient knowledge of mathematics was gained. She did not associate this gaining knowledge with the process of discovery itself. However, this changed after some time when she acquired more practical experience with teaching mathematics and tried to introduce some concepts in a discovery way.

To sum up, we are not advocating the opinion that all mathematics should be presented to all students in a discovery way. We are saying that even less able students deserve to experience joy from mathematics (provided by reaching a mathematical result, however trivial). We contend that engaging in one's own mathematical inquiry (combined with practical teaching experience) can help to influence student-teachers and teachers towards constructivist approaches to their teaching.

5. References

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[4] She teaches at a secondary school now and she sometimes applies the constructivist style.

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