CIEAEM SICILE 23 / 29 juillet 2005
Résumé de la communication de Gérard Vergnaud

Field of experience and conceptual field

Can these concepts help us reflect on the aims of mathematics education ?

I can see three main social aims for mathematics education in our modern societies :
-formation of a variety of professionals, whose use of mathematics is different from a profession to the next
-transmission of an admirable cultural heritage : the discipline of mathematics
-development of certain specific characteristics of the mind

To illustrate the first point I will take examples of professional mathematical skills which are not well represented in the curriculums, and could be better taken into account. The concept of field of experience is relevant to introduce children and adolescents to meaningful situations and organize valuable transpositions. This is true for the primary level but also for the secondary and the university levels. By transposition, I mean not only transposition of professionals'mathematical situations for future professionals, but also transposition for general education, in some of their characteristics at least. The reference to the mathematics of mathematicians is of course unquestionable, except that it is too exclusive of other references : the mathematics of farmers, shop-keepers, accountants, architects, engineers, nurses.
However, the concept of field of experience is not self-sufficient, as it does not offer, by itself, a description of the conceptual organization of the mathematical domains that are essential in the discipline. Therefore, unless one gives up helping students catch at least part of the mathematical heritage, it is necessary to establish connections between the professional and ordinary life situations one may introduce in the class-room, and the underlying mathematical concepts that can be useful to analyse and classify them. This is the very reason for my endeavour to analyse the conceptual fields of additive structures, multiplicative structures, ordinary algebra, or elementary geometry : for instance it is essential to analyse simple and multiple proportion problems as examples of linear and n-linear functions. One is then faced with the problem that students have to deal with physical quantities and magnitudes, and not with numbers only. The concept of number is a drastic and excessive reduction, and the theory of numbers does not allow us to understand well enough the long term conceptualizing process that students need to accomplish.
Finally mathematics as a cognitive activity, offers some characteristics that are specific of the discipline. Far from me the idea that mathematics would play a more important role in the formation of the mind than other disciplines : like text comprehension, history, biology, music or football. But it is interesting to try to characterize the cognitive benefits that students withdraw from mathematical activities. The concept of formal proof is certainly specific of mathematics, but it takes its roots in the concept of argument and the process of argumentation, which are not specific of mathematics. It could be useful to relate the ideas of sufficient condition and necessary condition to the general ideas of debate and controversy . Mathematics uses more systematically definitions and precise statements like theorems. But at the same time, a concept

cannot be reduced to its definition and a theorem to a formalised sentence. By showing the existence of concepts-in-action and theorems-in-action, one may have better possibilities to make a connection with the idea of field of experience.