Examples of junior secondary school students' attitudes toward mathematical problems

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I was observing and analysing the behaviour of students who were solving mathematical problems. I noticed that apart from a skilful application of methods, patterns and strategies there are also other components which during the process of problem solving may influence the final result and make it either a success or a failure. If a teacher's estimation of students' work is based solely on observable activities (in my research on solution in a written form) it may happen that this teacher will never reveal the true reason of students' failures and therefor will not be able to undertake suitable re-educational actions. If a teacher's task is not only grading but first of all re-education the reason why students behave in some particular way should not be disregarded. To illustrate this issue I will present an example from which my interest in this problem originated.

EXAMPLE

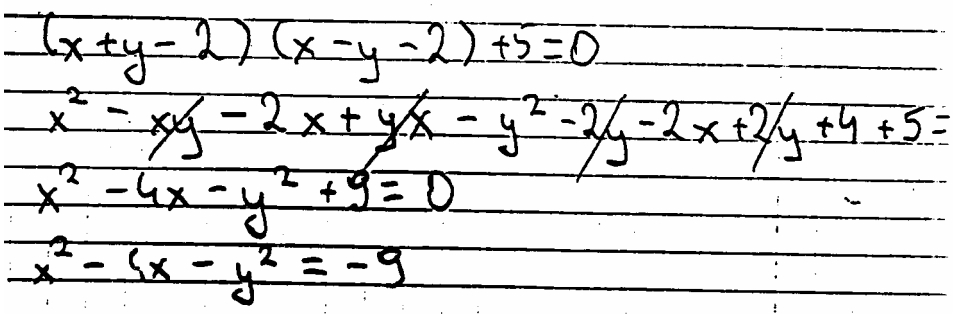
Two students are solving the following problem:

Find all the pairs of integers which fulfil the following equation

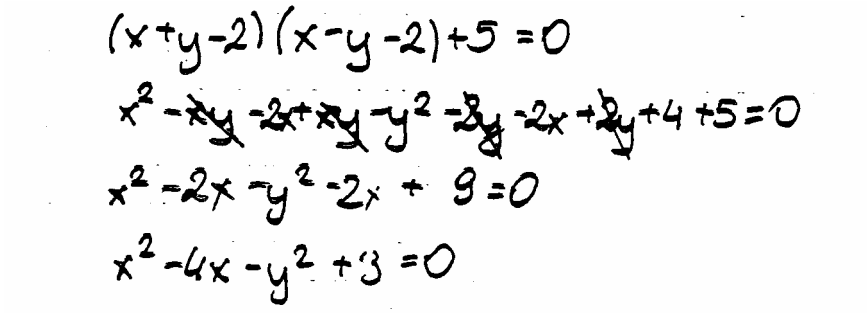
$$(x + y - 2)(x - y - 2) + 5 = 0.$$

These are the solutions produced by them:

Solution 1:



Solution 2:



Student's S1 proceeding (solution 1)	**Student's S2 proceeding** (solution 2)
The students start working on the problem and copy its contents: $(x + y - 2)(x - y - 2) + 5 = 0$.	
oral comments: ***'this is about solving an equation'***	oral comments: ***'What can I do... I may multiply everything out'***
The students multiply out the algebraic expressions in the parentheses and write: $x^2 - xy - 2x + yx - y^2 - 2y - 2x + 2y + 4 + 5 = 0$.	
notation: $x^2 - 4x - y^2 + 9 = 0$ $x^2 - 4x - y^2 = -9$	oral comments: ***'Now I have to simplify this expression and perform all the operations.'*** notation: $x^2 - 2x - y^2 - 2x + 9 = 0$ $x^2 - 4x - y^2 + 9 = 0$
Both students obtain the equation of the second order in two variables, which is a type of equation they are not able to solve yet	
oral comments: ***'Usually it was possible to solve equations in this way. Seems that this one is somewhat different, I cannot solve it.'***	oral comments: ***'I've no idea how to do it. I don't like such problems.'***

The students' remarks:
S1: *'As a matter of fact, I quite like solving algebraic problems. I usually manage to solve them. Only this one was kind of strange.'*
S2: *'I'm no good at solving problems. I've never been.'*

Let me draw your attention to the fact that the notations made by both students are almost identical. If a teacher were to grade these two solutions the marks would be the same. However, the results of the observation of these two processes reveals basic differences in the students' approaches.

- Student S1 (without any noticeable consideration) states that he has to solve this equation. The transformations he performs later on are a consequence of a conscious decision to solve the equation. If the student had completed the solution this attempt would have been crowned with success.

- A different approach was taken by student S2. After a moment's consideration, he performed, most probably, the first action which he associated with the form of this algebraic expression. Two pairs of parantheses with the multiplication symbol between them forced him to perform the transformations. It's difficult to decide whether the student speculated if performing such an action would lead to the solution of the problem; it is possible that he just did the only thing he associated with the form of the equation . This proceeding may be named – after M. Hejný (1992) – the *strategy of the first signal*. It is highly probable that this student's action was not directed at solving the problem. This hypothesis is backed up by his later proceeding.

Another difference between these two reasonings lies in the comments both students make when faced with an obstacle in the form of the equation of the second order in two variables.

- Student S1 (who consistently aims at solving the equation) groups the unknowns on one side of the equation and the given values on its other side. Then he expresses his astonishment by saying: *'Usually it was possible to solve equations in this way'*. What is clearly visible here is the fact that the student is directed at making use of well-known schemes. He does not, however, make another attempt to solve this problem, but he is sure that such a possibility exists.
- Student S2 performed all the possible algebraic operations which he associated with the form of the given equation. He forcefully declares: '*I've no idea how to do it!'*. This remark shows that the student is probably helpless in this situation.

The most significant difference between the methods of reasoning of the students probably lies in the emotional sphere as shown by the students' remarks.

- Student S1 said: *'As a matter of fact, I quite like solving algebraic problems. I usually manage to solve them. Only this one was kind of strange.'* One may infer that the student's attitude toward solving problems (in this case algebraic ones) is positive. It is true that he is quite strongly inclined to using schematic methods in these solutions but nonetheless he displays a certain active approach. If he had related some other familiar pattern with this problem he might have made another attempt.
- The discussion with student S2 revealed that he was not too eager to make an attempt to solve this problem. The only reason why he actually did it was the fact that he was staying alone with the observer and felt obliged to do something. Maybe in a different situation he wouldn't have even tried. He said: *'I'm no good at solving problems. I've never been.'* It may be therefore inferred that the student approached the problem with a deep conviction of inevitable failure. He aimed at aborting his work.

<u>Conclusions from the presented example:</u>

- -If a teacher's estimation of students' work is based solely on observable activities (in this case on solution in a written form) it may happen that this teacher will never reveal the true reason of students' failures and therefor will not be able to undertake suitable re-educational actions.
- If a teacher's task is not only grading but first of all re-education the reason why students behave in some particular way should not be disregarded.

Area of my research situate themselves on the border line between didactics and psychology. In psychology there exists the term *attitude*, which describes the type of phenomenon close to the one I try to investigate. For that reason the object of my interest is *a students' attitude toward mathematical problems*.

I focused on two possible types of conclusions that may be drawn from my observations:

1) The behavioural-cognitive component of *students' attitudes toward mathematical problems*,

2) The emotional-motivational component of *students' attitudes toward mathematical problems*.

With such a research perspective in mind I drew two types of conclusions regarding the cognitive and the motivational sphere of students' behaviour. As a result of the analysis of students' cognitive behaviour the list of phenomena was created, the phenomena which for the observer may be a source of information about basic differences in the course of action performed by various students' during specific phases of the problem-solving process.

Similarly, the investigation of students' motivational sphere resulted in creating a list of certain types of motivations which induce students to studying and which therefore may influence students' actions.

Obviously, I am aware of the hypothetical character of the diagnosis concerning students' motivations. It was based on students' remarks made during and after completing the process of solving problems as well as on the answers given to open questions formulated in interviews constructed for this study. I am also aware that when asked about something a student does not necessarily give a fair answer. This is why the questions I asked during the interviews were very general and did not include questions like 'Do you like solving mathematical problems?' The main objective was creating the atmosphere of an unreserved discussion enabling students to talk about themselves, their classmates, their families.

CONCLUSIONS

- Formulating diagnoses concerning the emotional-motivational sphere is a very difficult task, which, at the same time, is necessary for putting into practice the aims of teaching mathematics of a level which is higher than mere practising basing mathematical skills and knowledge.
- One may doubt the existence of a invariable mental structure known as *a students' attitude toward mathematical problems*. The character of their cognitive behaviours may rapidly change in time as influenced by changes in the emotional sphere (which was backed up by my investigations).

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