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THE APTITUDE OF MUSICIANS TO GEOMETRICS TRANSFORMATIONS

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Abstract

This research studies the interaction among the following contexts: natural language, geometrical language and musical language and it can provide new instruments to accord didactical situations and for a deeper understanding of communication processes. The theoretical reference is the theory of didactical situations by G. Brousseau through the systematic study of problem /situations. From a quality and quantity analysis of the sub-group of pianists has come out that the "misconcept" concerning the translation-reflection decoding is less present and this is due to the characteristics of the piano. In fact, it is an instrument which naturally obliges the performer to use both hands symmetrically effecting both translations and reflections which are learnt in an unconscious way through the neuro - tendinous receptors of the upper limbs. Besides the reading of the score takes place in a polyphonic way and this helps the global vision and perception of the musical language, differently from other musical instruments (winds and strings) which, being monodic, develop in the pupil a vision and perception of the musical language of a punctual type, i.e. as a linked sequence of sounds, which excludes the understanding of the organizational criteria of the sound material.

INTRODUCTION

This article describes a part of the experimental work done for my doctoral thesis.[1] The research has for subject the relationship between mathematics and music as these, born as intellectual and creative activities since the origins of mankind are in close relationship.[2]

Since always mathematics has presented and still presents close ties with the real world. We can say that historically most mathematical theories have taken their origins from the need to quantify, measure, describe, schematize and rationalize particular aspects of reality. Recently, besides, mathematics has taken a constantly growing relevance not only in the field of physics, of engineering, of economics, but also for many other disciplines, once considered "far" from mathematics, such as, for example, chemistry, biology, medicine, social sciences and art disciplines. The relationship between mathematics and music has far fetched roots and geometrical transformations have played an important role and for certain aspects essential in the development of the language of western music. In fact, geometrical transformations have been used in composition techniques since the twelfth century when polyphonic music took its first steps; the affirmation of the tonal model, together with the development of counterpoint, favoured in the seventeenth and eighteenth century a further affirmation and spreading of the composition process based on the principles of geometrical transformations. The counterpoint art, and consequently the employment of geometrical transformations, of the main theme, went through a period of decline from the second half of the eighteenth century to the end of the nineteenth century and apart from some sporadic case, we have to wait for the twentieth century to find again some examples of geometrical structures applied to music. The use of geometrical transformations is resumed in a completely different way by the dodecaphonic school of Vienna and by the vanguards that followed it. The aim of this research is to verify if the constant study of a musical instrument creates unconscious potentialities which are translated into strategies and methodologies for the solution of problems related to isometries. In fact, among the principal functions that the study of music is able to perform, besides the mere knowledge function, the linguistic-

[1] Doctoral Thesis, University of Bratislava (Slovak Republic), Advisor: Prof: Filippo Spagnolo

[2] The epistemological and historical – epistemological reflections are part of the thesis work, they will be taken into account according to the discourse context of this treatment. The thesis will be presented within next summer.

communicative function, the cultural, critical, aesthetical and affective function, a cognitive one is recorded because music exercises and develops the capabilities of thought: the productive-imaginative thought in the first place (in the activities of sound production) but also the analytical, logical and inferring thought (in the activities of reflection and interpretation). This experimental research is based on a comparison between secondary school students that study music at a conservatoire and secondary school students that don't study music at a conservatoire but which, anyway, have a basic knowledge theoretical-musical. This research has focused on spontaneous conceptions concerning geometrical transformations in general and their connection to music. The didactical experimentation was effected at Liceo Statale "Regina Margherita" of Palermo where two different samples of students were chosen:

Students of a music-oriented section of the same school which is connected to the state music conservatoire "Vincenzo Bellini" of Palermo. Number of students involved 70 between 14 and 16 years of age.

Students of a social- psychological-pedagogical section of the same school: number of students involved 70 between 14 and 16 years of age.

On the basis of these considerations the two following hypotheses of research were formulated:

H1 *What are the spontaneous conceptions of the musicians students (musical section, Conservatoire) regarding the geometrical transformations, compared with not musicians (pedagogical section).*

H2 *The students possessing a knowledge of rhythmical structure of music have a greater capability in recognizing the rhythm of geometrical forms for the reconstruction of objects compared with those lacking such a knowledge.*

METHODOLOGY AND THEORETICAL REFERENCE

The study of situations/problem gets into the theory of didactical situations by G. Brousseau.

The experimental stages are:

- Formulation of the didactical problem;
- Formulation of the objective of the research;
- A priori analysis of the problem/situation which should take into consideration
  - The epistemological representation of both mathematical and musical concepts;
  - The historical-epistemological representation of the same concepts (variations which have interfered in the course of time);
  - The foreseeable behaviours of students towards the situation/problem.
- Research hypothesis
- Construction of the instruments for the falsification of hypotheses which consists in devising of an experimental apparatus through the preparation of:
  - Questionnaires;
  - Interviews to couples with the task of writing their common considerations written down after a common agreement (registration of interview protocols).
- Analysis of experimental data: correlation of experimental data in function of an a priori analysis.
  - Quantitative analysis about the problems of the questionnaires

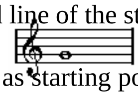
    Application of:
    - Descriptive statistics;
    - Analysis of implicative statistics by R. Gras (1997,2000) with the help of software CHIC 2004;
    - Factorial analysis with the help of software SPSS 9.0 and others;
  - Qualitative analysis of the related protocols of the interviews of couples.
- Documentation and communication of the results of the research.

THE TESTING

I proposed four sets of questions to both samples examined: the first two are about classical exercises on geometrical transformations present in any textbook for the first two years of upper secondary school; the third one is a problem regarding the reconstruction of a mosaic through the identification and iteration of geometrical figures and finally a last set of exercises regarding the application of the geometrical transformations in melodic tune bits. I would like to concentrate my conclusions above all on the pupils' behaviour adopted towards the last set of questions. It is to be précised that both samples hadn't yet carried out in the classroom the study of geometrical transformations and this allowed me to pick their spontaneous conceptions on the subject.

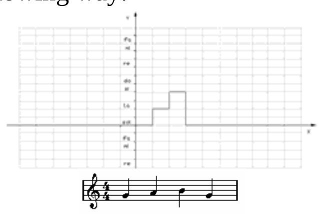
The set of questions proposed is the following one:

"Let's consider a plane (x,y) and put the time on the x axe, which corresponds to a sequence of beats which have constant intervals (for example those ones produced by a metronome) and on the y axe the height of sound from the lowest to the highest. In this way any melody can be represented by a law f so that $y = f(x)$. After that let's choose as unit of measurement the second and match it to the musical crotchet figure (metronomic speed ♩ = 60) for the x axe and the semitone[3] tempered for the y axe; in this way we can have the graphic representation through little squares which simultaneously indicate the duration of each sound that is how they flow through time (on the x axe) and the height they have according to a tempered scale (on the y axe). Moreover, in musical writing notes written on the stave receive their names and indicate their height thanks to the use of the clefs[4]: for example to the treble clef corresponds the G note in the second line of the stave.

Treble clef or G clef 

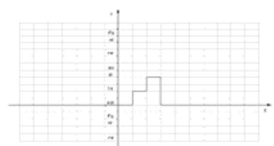
As consequence, if we consider as starting point of our system of reference, that is y = 0, the height of the correspondent sound to a G, the following melody: 

Is represented in a Cartesian plane in the following way:



On the basis of these suggestions try to complete the following charts.

A) In the following Cartesian plane you see the original melody represented.



Draw, in the same Cartesian plane, this melodic tune bit:

Identify if there is a translation or a reflection of the original melody in relation to the x or y axe or to the origin of the axes. Give reasons for your answer.

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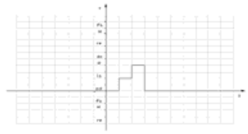
[3]It's the distance between any sound of the tempered scale and its immediate subsequent, either in ascending sense or descending one. It is the shortest interval of our musical system and it corresponds to the half of a tone.

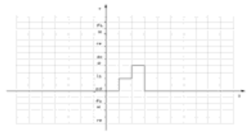
[4]They are graphic symbols that fix the position of all the sounds in a stave related to a sound fixed before them..

B) In the following Cartesian plane you see the original melody represented.
Draw, in the same Cartesian plane, this melodic tune bit:



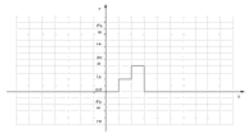
Identify if there is a translation or a reflection of the original melody in relation to the x or y axe or to the origin of the axes. Give reasons for your answer.

C) In the following Cartesian plane you see the original melody represented.
Draw, in the same Cartesian plane, this melodic tune bit:



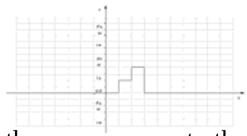
Identify if there is a translation or a reflection of the original melody in relation to the x or y axe or to the origin of the axes. Give reasons for your answer.

D) In the following Cartesian plane you see the original melody represented.
Draw, in the same Cartesian plane, this melodic tune bit:



Identify if there is a translation or a reflection of the original melody in relation to the x or y axe or to the origin of the axes. Give reasons for your answer.

E) In the following Cartesian plane you see the original melody represented.
Draw, in the same Cartesian plane, this melodic tune bit:



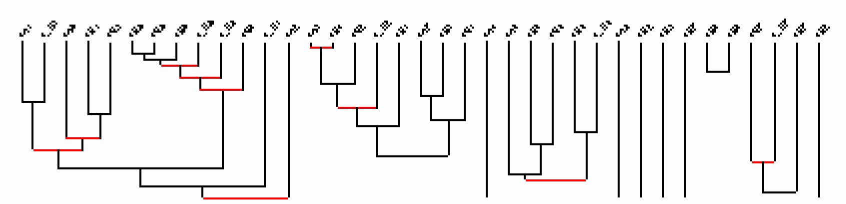
Identify if there is a translation or a reflection of the original melody in relation to the x or y axe or to the origin of the axes. Give reasons for your answer.**"**

ANALYSIS OF DATA

The set of questions was met with great interest and enthusiasm by both samples of pupils because they were made curious by the matching of geometrical transformation with music. The students who have elementary music knowledge[5] preferred to look for solutions in the field of music rather than in that of geometry, for example in the first exercise they said there was a translation because there is a pause. The sample of the musician students, in particular, used the term transposition to indicate the translation because in music translating a melody means moving it in time and height, therefore these students identified correctly the term transposition as a synonym of translation. In a double-entry pupils-strategies chart, for each student I have indicated with value 1 the strategies used and with value 0 the unapplied strategies. The collected data were analyzed in a quantitative way, using the implying analysis of the variables of Regis Gras by means of the Chic 2004 software.

Observing the following Chart of Similarities regrouping the two samples examined

[5] In the Italian school system music theory is studied in junior middle schools and in the high school pedagogical section , while professional study of music is entrusted to state music conservatoires.

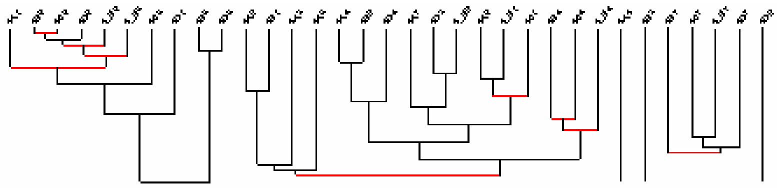


and analyzing all the data collected from the two samples, four typologies of main strategies emerge:

- Identifies the translation in relation to the x axe and the translation in relation to the y axe (i.e. answers the A) and B) questions correctly) but confuses the concept of *translation* with *reflection* in C) and D) questions although answers the E) question correctly.
- Identifies the reflection in relation to the x axe and the reflection in relation to the y axe (i.e. answers the C) and D) questions correctly) but confuses the concept of translation with reflection in A) and B) questions .
- Draws the chart but does not say if there is a reflection or a translation;
- Does not draw the chart of the tune bit but affirms there is a reflection

From this first quantitative analysis I have stressed that in general a concept mistake is present between the terms translation and reflection both for musicians and non musicians.

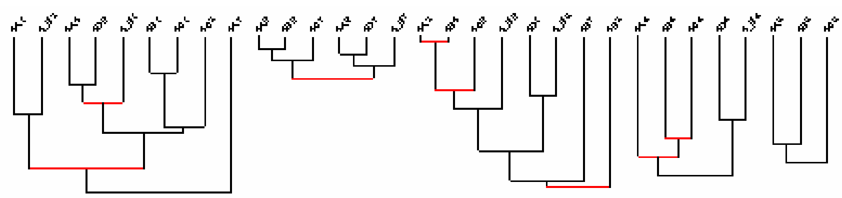
To trace possible different behaviours I analyzed both samples separately and from the analysis of the following chart of similarities came out that the non-musicians sample



chose three typologies of main strategies:

- Identifies the translation in relation to the x axe (4A1) and the reflection in relation to the y axe (4D1), but considers the other charts as *identities*, that is neither *translation* nor *reflection*;
- Identifies the translation in relation to the y axe (4B1) and the reflection in relation to the x axe (4D1) and the reflection in relation to the origin (4E1) and is anyhow able to draw the chart but confuses the concept of *translation* with *reflection*;
- Does not draw the chart of the tune bit but affirms there is a translation.

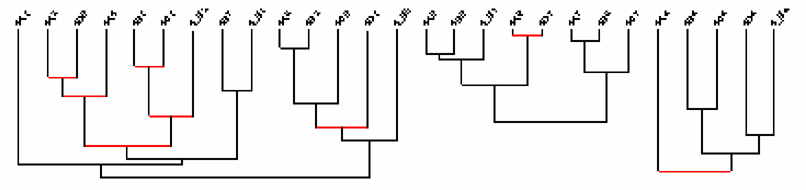
From the analysis of the chart of similarities of the musicians sample five typologies of main strategies emerged:



- Identifies the translation in relation to the x axe (4A1), the translation in relation to the y axe (4B1), the reflection in relation to the x axe (4C1) and the reflection in relation to the origin and is anyhow able to draw the chart but confuses the concept of *translation* with *reflection*;
- Always confuses the concept of *translation* with *reflection*;
- Confuses the concept of *translation* with *reflection* but is able to identify the reflection in relation to the y axe;

- Draws the chart but does not say if there is a reflection or a translation;
- Does not draw the chart of the tune bit but affirms there is a reflection.

Since the musicians sample is formed both by instrumentalists (winds and strings) and pianists I analyzed the sub-sample formed by pianists only and from the analysis of the following chart of similarities we can see that they chose three typologies of main strategies:



- Identifies the translation in relation to the x axe (4A1), the translation in relation to the y axe (4B1), the reflection in relation to the x axe (4C1), the reflection in relation to the y axe (4D1) and the reflection in relation to the origin (4E1) but confuses the concept of *translation* with *reflection*;
- not draw the chart of the tune bit but affirms there is a reflection or a translation but confuses the concept of *translation* with *reflection*;
- Draws the chart but does not say if there is a reflection or a translation.

CONCLUSION

From the analysis of the answers given to the set of questions proposed I have been able to find out a different behaviour, between the two samples taken into consideration, in facing the solution of problems concerning the geometrical transformations. In general, both for musician students and non-musician ones, a concept mistake between the terms translation and reflection is present (which we can hypothesize is a "misconcept") and this is found also in the first two sets of strictly geometrical questions.
From a quality and quantity analysis of the sub-group of pianists has come out that the "misconcept" concerning the translation-reflection decoding is less present and this is due to the characteristics of the piano. In fact, it is an instrument which naturally obliges the performer to use both hands symmetrically effecting both translations and reflections which are learnt in an unconscious way through the neuro-tendinous receptors of the upper limbs. Besides the reading of the score takes place in a polyphonic way and this helps the global vision and perception of the musical language, differently from other musical instruments (winds and strings) which, being monodic, develop in the pupil a vision and perception of the musical language of a punctual type, i.e. as a linked sequence of sounds, which excludes the understanding of the organizational criteria of the sound material.
Theoretical-experimental research in this field might in the future allow a curriculum organization aware of mathematics of music in music high schools and in music conservatoires.

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