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SENSING MATHEMATICS IN THE CLASSROOM THROUGH THE USE OF NEW TECHNOLOGIES

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ABSTRACT

New technological tools currently available in schools enable approaching and experiencing mathematics in a dynamic way, within environments which integrate numerical, graphical and symbolic aspects. This paper will describe and analyze an example of didactical activities which integrate the use of new technologies aimed to support the construction of meanings and the approach to theoretical thinking. The analysis sits within a semiotic-cultural approach to learning. Finally, some reflections about the risks of considering tools as simple add-ons used to overcome lack of ability will be proposed.

RÉSUMÉ

Les technologies aujourd'hui disponibles en classe, offrent des ressources autrefois impensables. Elles permettent de réaliser de nombreuses et riches expériences dans un milieu qui présente à la fois des aspects numériques, graphiques et symboliques. Dans ce travail, je décrirai et j'analyserai un exemple d'activités didactiques réalisées dans des environnements d'enseignements - apprentissage, qui font un large emploi des nouvelles technologies, dans le but de favoriser la construction des signifiés et d'orienter les élèves vers la pensée théorique. L'analyse didactique sera conduite dans une perspective culturelle -sémiotique à l'apprentissage. Je proposerai aussi quelques réflexions sur les risques qu'on court quand on pense aux instruments comme à de simples prothèses utilisées pour remédier à une carence d'habilité.

INTRODUCTION

The tools offered today by modern technologies to the teaching and learning of maths were only a short while ago unthinkable. They have been brought about by cultural more than technological changes and in order to be efficiently managed they require investigation and speculation. As Jimmy Kaput wrote,

> "We are early in an exciting new era for technology in Mathematics Education. Both the representational infrastructures are changing and the physical means for implementing them are changing. We are seeing new alphabets emerging, new visual modalities of human experience are being engaged and new physical devices are emerging – all at the same time. Much work need to be done" (Kaput, 2002).

New technologies enable the teacher to plan, suggest and carry out tasks which give the students the opportunity to experience mathematical environments and objects. The title I have chosen should recall the "*sense-experiences*" ("sensate esperienze" in Italian). the Italian word "sensate" is used with two different meanings: linked perception and ruled by the intellect and specifically by a theory. As I shall examine later, some ways of handling new technologies make these "*sense-experiences*" come true and clear the way for:

1. building up meaning in what is matter of study;
2. starting theoretical thinking.

This paper is based on two main assumptions.

The first one is about the role and function the education system should have in modern society. The school main duty should consist in helping young people to acquire critical and aware consciousness. This is vital in a society in which advertising campaigns generate dreams but reduce creativity and imagination, which are essential to face the difficult challenges brought about by political, social and economical problems.

The second one deals with one of the main problems in maths teaching: building up meaning. Where does math knowledge come from, how is it built and organized, how is it reminded? Neuroscience researches and some cognitive science points of view, suggest the existence of a biological component in the building of knowledge: ideas, and mathematical ideas particularly, are grounded in the sensory-motor system. For example Lakoff and Nunez wrote:

> "The detailed nature of our bodies, our brains, and our everyday functioning in the world structures human concepts and human reason. This includes mathematical concepts and mathematical reason" (Lakoff & Nunez, 2000).

As Seitz wrote,

> "the motor system including related structures is a self – organizing dynamical system contexted among musculoskeletal, environmental, and social forces. We do not simply inhabit our bodies; we literally use them to think with" (Seitz, 2000).

Other researches call attention to cultural and social components- besides the personal ones - in building knowledge:

> "In opposition to the objectivist referential approach, the semiotic cultural-approach that I am advocating does not claim a transcendental status for mathematical objects. [The semiotic cultural approach] conceives of mathematical knowledge as the result of a cognitive praxis: ideas and mathematical objects are conceptual forms of historically, socially and culturally embodied reflective mediated activity" (Radford, 2006).

In short, the second assumption is based on the idea that it is possible to identify a wide part of maths knowledge which may be built by promoting the integration of biological/cognitive roots with the cultural ones of mathematical ideas. This integration may be favoured and made real thanks to the strategies chosen by the teacher. For example by using instruments as mediators in the learning processes and supervising social interactions in the classroom while building up and sharing knowledge. As far as the latter are concerned, recent researches in the psychological field suggest paying attention to students' gestures is highly important as these play a significant role in the conceptualisation processes (Goldin Meadow, 2000). The teacher, in other words, has the task to lead the integration between the individual perceptive/physical level and the socially shared appreciation of maths as cultural heritage historically significant.

Inferences drawn from these assumptions have the direct consequence to affect the teaching/learning environment and the planning of teaching activities. The main features of the teaching – learning environment in which the activities here presented have been realized, are sketched in the following points:

a) a long-term didactics with particular attention to the construction of the meaning of the mathematical concepts;

b) the attention given in the students' problem solving activities not only to their products but also about their thinking processes;

c) the attention given to social interaction among pupils (collaborative or cooperative group work, mathematical discussions orchestrated by the teacher, observing and evaluating students' argumentations)

d) the use of tools as semiotic cultural systems in the teaching learning processes;

e) the attention given to the role played by cognitive roots in the construction of mathematical meaning, and to the role played by theoretical thinking in the passage form implicit to explicit knowledge.

Before examining a class activity I would like to point out two possible risks related to the use of new technologies. The first one is that the teacher may refuse using them, basing his assumption on the idea that the use of new technologies may impede the acquisition of the competence necessary for learning maths. The second one refers to a purely technical use of instruments, seen as simple 'prostheses' used to replace lacks of abilities. Both situations must be overcome by considering technological instruments as products of culture, useful to help promoting individual progress towards socially shared knowledge. As Rabardel wrote,

> "Les approches historico-culturelles de la psychologie ont mis en évidence que les fonctions psychiques supérieures se forment, chez l'homme, dans l'histoire culturelle et sociale. Les artefacts, les outils, les signes contribuent à la formation des fonctions psychiques et des connaissances. Les instruments constituent des formes qui structurent et médiatisent nos rapports aux situations et aux savoirs" (Rabardel, 1999).

ANALYSIS OF A DIDACTICAL ACTIVITIY

I will focus on a real class situation referring to the use of new technologies aimed at acquiring the knowledge of the concept of function. The analysis is based on the film of an activity carried out in the 9th grade of a scientifically oriented high school in Italy (student aged 14). The goal of the activity is to study the relationship between the graphic property of position as a function of time and speed as a function of time. I used graph calculators connected to a motion sensor. With this instrument the students can watch the time changing tracks of a classmate's position while he's moving according to a reference frame originating from the sensor. The students had already used the motion sensors in the past while performing activities, with the aim of experimenting the possibility of modifying the graph of position as a function of time while changing their movements. For example, in a previous task, the students had realised that getting closer to or moving farther away from the origin of the system frame had the consequence of either increasing or decreasing the graphs; or that speed growth generates 'steeper' graphs, that is it raises the absolute value of local slope. I will now shortly describe the two-hour activity performed by the students and then I will examine in depth two stages which will be videoed during the oral presentation.

First the students were divided in small groups and then the following activities took place:

1. one student from each group was told to move in front of the sensor in order to make the graph of position as a function of time look very similar to the one drawn on the blackboard by the teacher. His group mates could repeat the movements to correct any great differences between the graph on the blackboard and the one generated by motion. At the end of each movement the other students were encouraged to take part in the class discussion led by the teacher in order to spot any errors and their reasons.

2. Again at least one student from each group was asked to move in front of the sensor. This time though, he had to move without seeing the graph which was projected on the wall behind him by a view screen linked to an overhead projector and which was visible to the other students. While moving the student had to describe the main features of the graph of position as a function of time. When he stopped moving the other students were invited to point out the mistakes, if any, he had made in his description and try to explain the reasons.
3. At least one student from each group, especially those who had not taken part in the activity yet, had to move in front of the sensor in order to generate a graph of position as a function of time projected on the wall. At the end, all students were invited to take part in a class discussion with the aim of identifying the main features of the speed graph.

In a short film referring to activity 1, we can see the student M. who, by moving, has the task of generating the graph drawn on the blackboard. He moves looking both at the blackboard and at the screen in order to coordinate his movements (see figure 1).

Fig. 1

He manages to move effectively and he generates a graph which is very similar to the one on the blackboard, showing good coordination and knowledge. M. knows *how* to move to duplicate the graph on the blackboard. In the following discussion one of his group mates, E., describes M's movements with the help of gestures which show he has good understanding of what he has been watching. These are his words :

> "At first he moves slowly" [he moves his right hand horizontally towards the right], … "then faster" [he quickly moves his hand up, fig. 2a], … "then he immediately comes down" [he quickly moves his hand until he touches his desk, fig 2b], … "then he slows down" [he slowly moves his hand to the left, drawing a convex curve, first going down and then up, fig 2c] … "then fast again" [his hand goes up to the right, quickly, fig. 2d] … "and at last he stops" [he waves his hand horizontally to his right].

a

b

c

d

Fig. 2

E.’s gestures show that he has clearly understood what he has been watching, especially the relationship between the graph and motion. We can say that his gestures embody all the characteristics of the laws of motion. In fact, when he wants to describe a speed increase his hand moves quicker; on the contrary, he moves his hand more slowly when he wants to describe that speed is reducing. His hand draws lines in the air very similar to the graph on the board and the one duplicated by the view screen on the wall. However in Cartesian graphs the information concerning quantity variation (is it increasing or decreasing and how?) is coded in a single sign, i.e. the graph itself. In this case, understanding information like the relationship between the graph concavity and speed variation between the dependent variable and independent variable may be quite difficult. On the contrary, in E’s gestures there are two things easily recognizable - the first relates to the movement of his hand, which reproduces the graphics of position as a function of time; the second one to the hand speed, which recreates the speed variations of the moving body. E’s gestures are a sort of semiotic mediation between the observed phenomenon (M’s movement) and the cultural sign used to reproduce it (the Cartesian graph of position as a function of time). These gestures suggest there has been an internalization process of the relationship between the increase and concavity of a graphic to speed and speed variation in a body. At the same time they can make understanding easier for the class. And in fact E’s gestures and words are used by other students in the following discussion to make things clear, first to themselves and then to their class mates.

In another short film taken from activity 2, the student G, while moving with the sensor and the graph projection behind him, describes the graph of position as a function of time. The words he uses are those linked to his daily experience of motion. Referring to a graph similar to the one in picture 3 he says:

> “Horizontal line… it slowly increases… it quickly decreases…horizontal line… it quickly increases… still..”

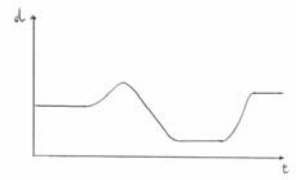

Fig. 3

As we can see the words he uses are strictly linked to his personal sensations – in fact they do not allow him to distinguish among the different types of graph increase. The words *it slowly increases* and *it quickly increases* find meaning only in their relationship and they do not let G. appreciate speed change while he describes the lines with one single word - *slowly* or *quickly*. What a difference it would make in meaning and subsequent developments if he had used words like "*it constantly grows*", " *it increases less and less*", " *it increases more and more*", which would respectively define linear, concave and convex increasing functions. Of course the role of the teacher is essential in the learning process. He must provide the students the opportunity to extend their current skills and knowledge in order to grasp the meaning of what they are studying. More evident in G.'s case, but also in E. and in the other students', the words they use are still linked to their sensations and their daily experiences while some gestures and movements are already the outcome of their learning process. They are in the stage that Vygotsky called 'The Zone of Proximal Development' (Vygotsky, 1978). The teacher should help them develop and acquire the relevant vocabulary to describe what they have started to understand, in order to move from tacit to explicit knowledge and to get closer to the goal of their learning.

The third activity, the aim of which was to learn the relationship between the graph of position as a function of time and the one of speed as a function of time, shows that at this point the students have understood how to control a graph and how to reproduce it with their movement. They also know that speed is linked to graph slope and concavity (but only some of them use this word) to speed variation, but the words they use are still linked to perceptive aspects. This is evident when they face some difficulty in understanding negative speed, i.e. that speed graph goes below time axis. Negative speed is a senseless notion: its meaning requires the use of theory, thing that the students are still unable to do by themselves and here the role of the teacher is essential.

SOME CONCLUSIONS

It is evident that the choice of using new technologies in teaching-learning maths does not limit the role of the teacher – just the opposite. It is the teacher who must detect and concentrate more on information that has instructional significance, in order to help students approach theoretical studies which are the base for rational thinking. He must help his students acquire the skills they need to use the tools aimed at achieving the cognitive goals, and then the teacher must favour the so called *instrumental genesis*. As already said the tools are not neutral but they develop from a culture and they must be properly used in the class activities. This has some consequences also on the meaning of the objects of study. Maths history is full of instructive examples. For instance Descartes' compass, built starting from the ideas of motion and the need to see curves while being generated to understand their meaning, gradually shifts attention onto aspects of algebraic curves. The mathematicians who came after Descartes happily worked on equations and paid less and less attention to the way curves are generated (Shea, 1991). Similarly, the use of technologies, experiences and metaphors linked to movement, conveys meanings different from those passed on by functions defined by points, finite sets or seen as subsets of a Cartesian plane. Besides, as Radford write, the system sensor – calculator,

> "is more than a gadget to economize actions. It carries in itself, in a compressed way, socio – historical experiences of cognitive activity and scientific standards of investigation. However, by taking over some of the human actions, certain aspects of the socio – historical experiences that the system holds remain 'hidden' from the individuals using it" (Radford, 2005).

The last thing I want to point out is that is not sufficient the teacher give the students the occasion to make some experiences: every day the students have many experiences based on motion. The difference with the situation previously described consists in the environment in which such experience takes place and the presence of an adult, the teacher, who suggests, organizes and analyses these experiences to achieve a didactic aim. As Radford wrote,

> "Cultural conceptual objects are lighthouses that orient navigators' sailing boats. They impress classroom interaction with a specific teleology" (Radford, 2006).

Of course the teacher's awareness of the historical and cultural dimension of mathematical objects is crucial in the social interaction in the class and in the way meaning is built (Furinghetti, 1997).

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