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'THE PRINCE AND THE MESSENGER' STORY: FROM A TALE TO MATHEMATICAL THINKING

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ABSTRACT

'The prince and the messenger' story by Dino Buzzati (a famous Italian novelist) has been used as a problematic situation that requires mathematical modelling to be solved. 6th grade students at the beginning of the school year still hold primary school knowledge, and are not yet used to model situations using mathematics. This is why we deemed them as a good population for the research presented here. The students solved the problem working in small groups. We filmed them during both the group work and classroom discussions. In the research, attention is drawn to the strategies the students followed to make sense of the story and transpose it in mathematical language. To this aim, we observed students' gestures, drawings, sketches. The use of some configuration to represent relevant information from the story seems to be a crux to the understanding. The configuration plays a functional role in relation to the passage from the story to the mathematics of the problem.

INTRODUCTION

This research study evolved as part of the Comenius 2.1 European Project entitled *DIAL: CONNECT* (*Using DIALogue to CONNECT learning minds*). The philosophy of *DIAL: CONNECT* encouraged the use of dialogue to connect learning minds, construct understanding and initiate creative activity based on analogy and metaphor. The project involved university academics, and in-service and pre-service teachers, working in the province of Torino. Its philosophical underpinning was useful to challenge the teachers' views and conceptions of teaching and learning, with a consequent need for rethinking and critically reflecting on their classroom practices. Especially, the project forced them to shift from teacher centred/content oriented classrooms to more students centred/learning oriented classrooms (in the sense discussed by Kember, 1997). The message for school is clear. The aims of the project are: to determine proper contexts, contents and approaches that stimulate thinking and empower learning; to build on the natural curiosity and creativity of the child through exploratory and reflective action; to adopt an interactive teaching style and learning ethos that motivate pupils to take ownership and responsibility for their learning. Using a unique expression, the project wants "to develop a learning culture and a learning orientation in schools".

The project concerned the role of dialogue and problem solving in the generation of scientific and technological knowledge, a topic that is very important to the issue of the cultural value of mathematics. One question, relative to the topic and mainly interesting for the research described here, is: what content and activities are most appropriate to let students approach the cultural dimension of mathematical knowledge and making mathematics. Of course, it is not a minor question. It can be faced by both a didactic and a research point of view. Indeed, on the one side, it implies attention to the curriculum and the teaching materials, involving choices in the didactic practice. On the other side, it entails a feedback in terms of the students' reactions and ways of acting with respect to the kind of activities they are engaged in. These reactions and ways of acting are the object of the research. In such a perspective, we are going to discuss about both sides referred above. In so doing, we present the story of 'The prince and the messenger' (from a famous tale by the Italian novelist Dino Buzzati) as a rich content and a suitable context for the creation of a learning situation, which fits for the development of mathematical reasoning at the end of primary school (e.g. 5^{th} grade) or at the beginning of middle school (e.g. 6^{th} grade).

THEORETICAL FRAMEWORK

Our research takes into account recent Mathematics Education studies that have shown the relevance of perceptual and motor activities in mathematics learning. Nemirovsky (2003) stresses that "the understanding of a mathematical concept rather than having a definitional essence, spans diverse perceptuo-motor activities, which become more or less active depending on the context. For instance, seeing a trigonometrical function as a component of circular motion or as an infinite sum of powers may entail distinct and separate perceptuo-motor activities. Learning a different approach for what appears to be the "same" idea, far from being redundant, often calls for recruiting entirely different perceptuo-motor resources" (*ibid*.). The role of the body in cognition is studied within a multi-disciplinary effort involving many disciplines: Experimental Psychology, Cognitive Science, Linguistics, Neuroscience, Semiotics and Philosophy. These fields furnish complementary tools and results that give insights to mathematics educators who are interested in a better understanding of the underlying principle of cognition (for learners). Nemirovsky's viewpoint recalls the earlier idea from embodied cognition theory that sensory and motor resources are at the ground of mathematics thinking (Lakoff & Núñez, 2000). In this case, "rather than the mind operating to serve the body, we find the body (or its control systems) serving the mind" (Wilson, 2002). Building on this kind of results, Arzarello *et al*. (2005) claim that acting is learning, pointing out the significance of relating action and language to mental activity. Bodily actions, gestures, manipulation of materials or artefacts, acts of drawing, but also eye motions, gazes, voice tones, facial expressions can be seen as examples of perceptuo-motor activities. They are those elements Radford (2003) analyses as semiotic means of objectification: that is, those "key elements in the organization of mental processes as they are used to reflect and objectify ideas in the course of the individuals' activities" (*ibid*.). Results from Neuroscience give us even deeper clues on the fact that the whole body 'takes part' in thinking (and thus learning) processes. Gallese & Lakoff (2005) analyse the role of the sensory-motor system in conceptual knowledge. In so doing, they outline that "Multimodal integration has been found in many different locations in the brain […] That is, sensory modalities like vision, touch, hearing, and so on are actually integrated with each other *and* with motor control and planning. This suggests that there are no pure "association areas" whose only job is to link supposedly separate brain areas (or "modules") for distinct sensory modalities". As a consequence, language seems to exploit this inherent character of the sensory-motor system, being "inherently multimodal in this sense, that is, it uses many modalities linked together—sight, hearing, touch, motor actions, and so on" (*ibid*.). Based on research conducted in the last years, it clearly appears that, during mathematical activities in the classroom, students have recourse to many perceptual, motor and sensorial elements, other than to written and oral language, which deeply take part in meaning making. A semiotic lens is then appropriate in order to analyse the students' mathematical thinking that we assume to happen and develop in a multimodal and palpable manner (Arzarello, 2006; Radford, 2009).

The considerations above are interwoven with the didactical perspective of our study, in a dialectic relationship between theory and practice that implied both practical efforts and theoretical reflection (as Bartolini Bussi & Bazzini, 2003 point out). This didactical viewpoint embraces the discussion and the interaction among pupils as a methodology of work. A quotation suitably comes to mind at this point of the argument, because it sheds light on the importance of a social nature of learning: "Any function in the child's cultural development appears twice, or on two planes. First it appears on the social plane, and then on the psychological plane. First it appears between people as an interpsychological category, and then within the child as an intrapsychological category" (Vygotsky, 1981). Besides, from a cultural point of view communication is not seen as "a disinterested communication. The individuals communicate between themselves to carry out goal-oriented activities having culturally motivated goals" (Radford, 2001). In Mathematics Education, the role of the discussion in relation to the construction of mathematical knowledge has been studied for a while now (see e.g. Bartolini Bussi, 1996). In a Vygotskian perspective, the mathematical discussion is considered as a crucial means of communication to make meanings in a social way. It is defined as "a polyphony of articulated voices on a mathematical object (either concept or problem or procedure or belief) that is one of the motives of the teaching-learning activity" (Bartolini Bussi, *ibid*.). That research is then relevant for the study presented here. In fact, the possibility of discussing and interacting with each other is a fundamental feature of the activity of our students. Mathematical thinking happens and develops in a multimodal and palpable manner, but has also a social dimension that allows learners to overcome obstacles and share the solution processes, meanings and understanding. The activity's pedagogical design ensures the social space of discussion and interaction. Actually, there are two moments of social interaction: a first moment of group work when the students work with some peers; a second moment when the teacher leads a discussion the whole classroom takes part in. This type of methodology affects the traditional course of teaching, making the students the protagonists of the learning processes, through their active sensuous participation.

Within this frame, the fact that a story is the motive of a problematic situation given to the pupils is a relevant feature of the present study. Using a story allows for a non-standard involvement of the students: in fact, they are forced to interpret a written text and to transpose it using the language of mathematics, e.g. through numbers, patterns and operations. The transposition is not at all simple: what appears in natural language has to be matter of mathematical reasoning, but variables and data are kind of hidden in the text of the story and, as a consequence, they can be easily misinterpreted. To make sense of a text from a mathematical view is a high level competence, which requires the ability to understand the text itself, and to 'see' the hidden variables and data before expressing them with mathematics. Then, the activity we present is pretty far from a learning discourse whose aim is that of a purely technical preparation of the students. Though, such an aim is widespread in mathematics teaching at primary and middle school.

In this broad theoretical perspective, our study focuses on the strategies the students followed to make sense of the story and transpose it in mathematical language. These strategies are observed through students' actions during their group work. In the present research, in particular, students' gestures, drawings and sketches are considered as actions significant to discern different strategies.

THE STORY

The classroom activity we examine in this paper concerns a fantasy situation based on the story of 'The prince and the messenger'. The students were given the following text:

> *'Quite a man, a prince was curious to visit the realm of his father. Everybody was telling that the realm was broad, full of woods, lakes, rivers, villages, lands and scented green countryside. One day, the prince decided to leave with all his attendants. Once a distance of 50 kilometres was covered, the convoy stopped and camped for the night.*
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> *Soon the day after, the prince called the most faithful knight and he said him: 'Go back to the castle: you have to take some health herbs and bring me news from my parents, telling me what happens there. In the meantime, I keep on'. So, they moved in opposite directions: every day the prince covered a 50 km distance, the knight covered a 100 km distance.*
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> *The third night, the knight came back with herbs and news. They ate all together. But the*

day after, the prince told the knight to go back to the castle again, in order to take jewels for the women met along the way. So, the prince and the knight moved in opposite directions: the knight back toward the castle, the prince forth to cross the realm. The convoy covered 50 km a day; the knight covered 100 km a day. These speeds remained the same for the rest of the journey.

Some days later, the knight reached the convoy for the second time and gave the jewels to the prince. But again, the day after, the prince called him and he told him of a dream made during the night. In the dream, he saw a witch who threatened him: "If, by the end of 30 days from your departure from the castle, you will not carry to me the medal your mother had given you, I will curse you and you will die soon!". "Where will I be able to find you?", asked the knight. "I will be on your path at 1500 km from the castle. If you do not have the medal, you will die", said the witch.

"Go immediately – the prince said to the messenger – and bring me the medal. I left it over my bed in the castle. Go and come back as soon as possible!" So, the prince moved on with the convoy, while the knight went back to the castle. They both kept the same speed as they had until that moment.

Was the prince able to give the medal to the witch in the exact date and place, so to avoid the curse?'

In order to answer the question, the story requires the students to model the fantasy situation using the language of mathematics (numbers, patterns, operations, etc.). As might be expected, the original text of the story was adapted so to match our pedagogical aims. This choice situates in continuity with a previous problematic situation, already designed starting from a story (that is, the historical legend of Penelope: see Barbero et al.*, 2005; Arzarello* et al.*, 2006), which was part of the same Comenius project.*

THE STUDY

From the research perspective, our interests are on the students' cognitive progress in the meaning making about the given situation. The analysis draws attention to the elements the students use in their interaction, as words, gestures and drawings. These elements provide us with means to look at the strategies followed in the course of the activity. In this way, we reflect on the process that leads learners to construct new pieces of mathematical knowledge, starting from their perceptions and actions.

The activity was carried out with 6th grade students attending an Italian school in the surroundings of Torino, in the North of Italy. The period was just at the beginning of the school year, when students' background can still be considered that of primary school. First, the students were given the text and asked to work on it in small groups (each of three or four students). Then, they participated in a conclusive classroom discussion led by the teacher. Both the moments were important with respect to the possibility of sharing and comparing understanding and reasoning, solutions and strategies.

As a research methodology, a moving camera was used in the classroom to film the students, during both the group work and the collective discussion. *A posteriori*, we watched the videos, and we made transcriptions of the recordings. Then, the analysis was conducted using these transcriptions, but also field notes and diaries from observations in the classroom. The present discussion is a result of the many talks we, as a research group, had in relation to all the phases of the research.

AN EXAMPLE OF STRATEGY FROM A GROUP

We present an example from a dialogue of some students who worked together in the small group. As a methodology of work, the students are all around the same desk, each with personal papers but sharing the paper with the activity. In the dialogue square brackets indicate a gesture or a drawing performed by the students.

The dialogue shows that students' talk and gestures develop with a need for representing the main characters of the story (the prince and the knight) and their actions. This need entails the difficulty of making visible the situation in a suitable way. A strategy arises at a certain moment. A first representation enters the scene: a picture on paper to represent the castle and the path followed by the convoy and the knight (Figure 1). But the representation is not enough apparent for the students.

Figure 1

Indeed, while the drawing is performed, a sense for the story and a suitable representation for it are still looked for, as Marta's words clearly show:

> Marta: "*They live **from here*** [she draws a small castle] *and they cover 50 km* [she traces kind of a wave line to refer to the path]*… **there** the convoy… and then they cover other 50 km every day… and the knight comes back… and then he covers other 50 km…"* [she draws two arrows indicating opposite orientations: Fig. 1]

The few words above are meaningful in understanding the role the drawing plays for the students since the beginning. The deictic words "from here" and "there" (in bold letters) Marta uses allow her to refer to specific points (or 'places') on the path sketched on paper. These points serve to metaphorically represent, on the sketch, some spatial positions reached by the prince and the knight in the story. The castle is a reference point, for it stands for the starting point of the movement of the convoy. In Marta's words, 'distances' are also present: "50 km", "other 50 km". Distances are different from places. Places are specific positions with respect to the reference point; and they are identified according to their distance from the reference point. This identification is marked on the drawing, at the end of the wave line, by writing "50 km" (see Fig. 1).

The drawing is already a strong tool to make visible first relevant information given in the text. Even the two arrows, which indicate opposite directions (accompanied by expressions like "the knight comes back" and "then he covers"), introduce a new variable: the fact that sometimes the directions followed by the prince and the knight in their movements are opposite. Drawing the arrows can be considered as gestures performed by Marta, gestures with opposite orientations. From the mathematical point of view, the change of orientation for the movement of the knight entails the fact that some distances covered by the knight have to be subtracted and others have to be added. On the other side, the distances covered by the convoy of the prince have simply to be added one another. This variable is crucial to understand the mathematics of the story.

However, only in a second moment, the need for distinguishing between the two movements of the prince and the knight becomes apparent. At this point, two different colours (blue and red) are used to indicate the two characters and to speak about the distances they covered (see Fig. 2). Now two students play the role of the characters, each following with his fingers the positions (places) the corresponding character reached on the path.

> Umberto and Daniele: *"While he is going back… the other goes **there**… and he goes **there**… **here**… **there**… **here** they are..."* [they count the days by means of the configuration; they move together the fingers along the line, simulating the movements of the prince and the knight; Fig. 2]

Figure 2

The strategy of introducing distinct colours on the drawing seems to be very efficient. The coloured paths have the same function as the arrows had at first; but they are now distinguishable. Again, Umberto and Daniele use such deictic words as "there" and "here" (in bold letters), to indicate the specific places reached by the prince and the knight on the line that represents the path they travelled. Those words come together with their deictic gestures on the drawing, pointing to the reached places. The deictic gestures are accompanied from the gestures performed with the coloured pen and using the other free hand. The figure shows the different functions of the two hands for both students. The right hands hold the pen (and they followed the travelled path); the left hands point to the places reached along the path. Umberto and

Daniele have two precise separate roles, in accordance to the blue and red colours, as actors playing each a specific part. The contemporary involvement of the two students in placing and running the fingers along the line allows the group to understand the relative positions of the two characters of the story. Information on both the prince and the messenger was before contracted in a unique global representation of the path, and the only distinctive element was the presence of the two arrows. Through the strategy followed by Umberto and Daniele, the story becomes more and more transparent to the group. As a result, the mathematical meaning associated to the story comes to be discovered, and mathematical thinking grows through a finer and finer understanding of the tale.

In this process, the following step of the strategy (we do not analyse here) is that of making palpable another variable that is fundamental to solve the problem: time. Time is present in the text of the story: e.g. "the day after", "every day", "the third night", "a day", "some days later". The succession of events is certainly involved in Marta's first, and then in Umberto and Daniele's gestural movements from one place to another one, as well as in drawing the arrows. However, time is no longer present as soon as the students complete their gestures. It is hidden in the configuration. That time is involved in the solution of the problem is not a trivial matter, for the students need to understand the occurrence of the spatial and temporal variables. Such an understanding will allow them to discover the covariance of the two variables for both the characters and finally to know the end of the tale (a component which is not secondary at all with respect to the curiosity excited or attention elicited in the students).

CONCLUDING REMARKS

The importance of using a drawing is not limited to one group work, but it appears to be a constant of the cases we have analysed. This happens even when the strategies followed by the groups to make meaning of the story are different. Regardless of the particular strategy chosen in fact, each group has shown a need for representing the situation in a suitable way, and to develop sketches to this aim. As preliminary results, we believe that the use of some configuration for representing relevant information from the story (in the case of our group, the use of a drawing with distinct colours in relation to the prince and the knight) is a crucial element for pupils' understanding. Indeed, the drawing plays a functional role with respect to the passage from the story (written in natural language) to the mathematics of the problem.

A more complex version of 'The prince and the messenger' story has also been introduced in some secondary school classes (as part of the same European Comenius Project Dial Connect). We have observed that a strategy based on a graphical representation is prevailing in upper classes, too. The analysis of these group activities sheds light on the significance of three types of modalities in the process of understanding: that is, gestures, drawings and mathematical representations such as Cartesian graphs. A comparison between primary and secondary situations stresses that the modalities are the same but the third one is much more developed in upper classes.

What we have found is not a new result, but a confirmation of something we have already observed in other situations (see e.g. Barbero *et al*., 2005; Arzarello *et al*., 2006). The drawing represents the very germ of the understanding processes. Of course, the recourse to the drawing comes with other perceptuo-motor activities or corporeal elements (typically gestures), and with speech. All this happens in a unitary space of action, production and communication. Arzarello (2006) introduced this space calling it *APC space*. In it, pupils' learning develops in a holistic and integrated manner, where there is no distinction between what is external and what is internal, but everything is part of the same process: configurations, representations, drawings, numbers, as well as the body, gestures, actions and perceptions. It is a view that certainly agrees with the theoretical perspective of the relevance of perceptuo-motor activities stressed in our framework, and that supports our assumption that learning happens and develops in a multimodal and palpable manner. Paying attention to all these means, to which learners resort in the classrooms, can certainly give us, as mathematics educators, a wider 'eye' that also enriches our understanding of the complexity of learning processes.

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