Proceedings CIEAEM 58 –SRNI, Czech Republic, July 9-15, 2006
"Quaderni di Ricerca in Didattica (Matematica)", Supplemento n. 3, 2009.
G.R.I.M. (Department of Mathematics, University of Palermo, Italy)

AUTHENTIC USE OF COMPUTERS IN TEACHING PRE-SERVICE TEACHERS THE CONCEPT OF THE DERIVATIVE

Liora Hoch Miriam Amit
Ben Gurion University

This study deals with the introduction of computers into the mathematics education of pre-service teachers. It follows a group of pre-service teachers as they use computers to learn the subject of the 'cusp point', which was completely new to them. This newness rendered the experiment an "authentic" example of computer-based learning. The results show that the experiment group displays not only a sound knowledge of the graphic aspects, having freed themselves of their inclination towards the algebraic, but also a greater appreciation for the importance of using computers in math education.

RATIONALE

The development and growing accessibility of technology has created a demand for its effective integration into the process of mathematics education. This requires future teachers to be well trained in the use of computers; however a general methods course on the use of technology does not provide sufficient training. We teacher educators must see to it that future teachers also experience learning with technology. Therefore, we must integrate computers into the mathematics courses for pre-service teachers. The theme of 'cusp point' provides a good opportunity for authentic learning with the aid of computers. This topic is not taught in high school and therefore all pre-service teachers in this study are seeing it for the first time. In this way, future teachers will experience this method of learning as we would expect them to later apply it themselves in their classrooms.

THEORETICAL BACKGROUND

Research on learning the concept of the derivative amongst students found that there are problems in understanding this concept. Students succeed in solving routine problems, working with an algebraic representation of the function, but have difficulty working with a graphic representation (Amit, 1992; Berry & Nyman, 2003; Baker, Coole,. & Trigueros, 2000). Studies testing the contribution of computers to the understanding of conceptual and non-routine problems found that students who study with computers have an advantage (Berry & Nyman, 2003; Amit 1992).

Derivatives are currently taught in high school. Students deal mainly with polynomial and rational function. Learning is based mostly on algebraic tools. The students in high school do not deal explicitly the notion of differentiability or with cusp points.

Our duty as teacher educators is to do our utmost to give pre-service teachers comprehensive knowledge of algebraic and graphic representation of the derivative, to deal with a variety of functions, to learn and experience the interrelations between functions, their first and second derivatives, and to provide not just procedural knowledge, but to make sure that the pre-service teachers have also conceptual understanding.

In light of accumulated information on problems students have with calculus on one hand, and on the advantages of using computers on the other, we wish to determine whether using computers can improve pre-service teachers' understanding of derivatives. In this paper we focus on solving non-routine problems on the connection between "cusp point" and "differentiability".

This subject was chosen because it is authentic learning and non routine problem so it can indicate understanding.

THE STUDY

Research questions:

A. How does the use of computers affect pre-service teachers' understanding of:

1. The connection between a cusp point and non- differentiability of that point?
2. The connection between a cusp point that is also an inflection point and the nonexistence of the second derivative?

B. What impact has the use of computers as students on their attitudes towards the use of computers as teachers?

The Population, the Setting and Instruments:

Seventeen pre-service teachers (pst) who are specializing in mathematics education for secondary school were checked. The experiment group consisted of 7 pst and the control group consisted of 10. The experiment group studied the concept of the derivative and dealt with cusp points, with the use of computers. The control group studied according to the same syllabus, focusing on geometric and algebraic aspects of cusp points, but without using computers.

To answer the research questions, each student received a questionnaire composed of three parts:

1. Questions with graphic representations of functions and questions relating to various kinds of extrema (examples in the appendix).
2. "Prove or refute" questions on the above subjects.
3. An "open" question designed to determine the student's concept image of the cusp point.

The questionnaire was given to all students at the end of the course. Upon completing the questionnaires, students were interviewed, to clarify their responses to the questionnaire.

Some Outcomes

Differences between the experiment group and the control group were found on several issues, some of which are presented below.

<u>The status of graphic representation vs. algebraic representation:</u> A prominent strong point amongst the experiment group was the attempt to dramatically decrease the use of algebraic representation of the function and work mostly with the geometric representation. This occurred, for example, when the pts were asked to check for a function with a cusp point but does not have a derivative at that point. All the experiment group subjects without exception used the geometric representation. None of them attempted to find an algebraic representation of a function with a cusp point. This stands in contrast to the control group where the pts preferred mostly to algebraic procedures. For instance, they used sentences such as: "in order to find a cusp point one must differentiate and compare to zero, etc." The pts in the control group had trouble leaving the algebraic representation. This was also apparent when they were asked to build a function which includes a cusp point. Most of the group tried to find an algebraic representation and only two students gave a graphic one.

Drawing tangents: To identify a cusp point, pre-service students in the experiment group made use of the fact that "we learned that at the cusp point the function is non-differentiable" and therefore turned immediately to drawing a tangent at the point to check if a derivative exists. The tangent drawings were for the most part done correctly. In contrast, the pts from the control group were unsure. Some relied on a "visual evaluation", others offered a variety of solutions which proved inconsistent. Most of the control group had trouble drawing the tangents, for some due to the presence of the cusp point and for others because they claimed that:" it was possible to draw several tangents for each point."

Connecting between a cusp point which is also an extreme point and the existence of the derivative: The experiment group pts all knew how to correctly define an extreme point, but when it came to checking whether a cusp point was also an extreme point, the control group revealed a difference between their theoretical knowledge and their practical knowledge. On the theoretical level – there is no derivative at the cusp point. In practice – the control group had the mistaken impression that at the extreme point the derivative is always zero and that therefore, if a cusp point is also an extreme point, there is a derivative and it equals zero.

Inflection point: This topic proved to be problematic for the experiment group (similar to the findings in Baker et al, 2000). Some of the pts vaguely remembered something about tangents above or below the graph, but did not use this information correctly. Most of the control group knew how to correctly identify an inflection point, but incorrectly used the algorithm for finding the position of the point.

The importance of using computers: According to all the members of the experiment group, the computer helped them learn the subject in general, and specifically helped them understand the application of the theoretical basis. For instance, Ruth (an outstanding student) said: "We can do many things (with a computer), we can see the behavior of the functions and that saves us work…you see things that you can not draw. Pupils can try things by themselves and discover all sorts of things, things that you can not see on the board". Leah (a weak student) said: "It helped me personally a lot. There were things I didn't know; it shed more light, more information. The graphs are more accurate". Only Jenya had doubts, because "in daily life we don't use computers in math classes and this learning was done to enhance (only) the teachers' knowledge". Some of the pre-service teachers felt that they still lacked sufficient knowledge and preparation to use computers in their teaching, but all of them without exception saw the great advantage of learning a new subject with the aid of a computer and – "really feeling like the pupil feels".

Closing comments

Training pre-service teachers consists partly of expanding their mathematical knowledge per-se, and partly of teaching them how to teach. The latter includes teaching how to use computers in their future teaching. We suggest that more computer-based teaching should be integrated in the first part as well, for three reasons: A. The pts will have a better understanding of mathematics. B. The pts will experience learning with computers while "wearing the student's hat" and not just the "teacher's hat". C. Ethically, you should always "teach as you preach".

REFERENCES

Amit, M. (1992). Student Difficulties in Solving Conceptual Problems in Differential Calculus – Diagnosis and Treatment; Doctorate Thesis, Mathematics department, Ben-Gurion University, Beer-Sheva, Israel.

Baker, B. Cooley, L. & Trigueros, M. (2000). A Calculus Graphing Schema. *Journal for Research in Mathematics Education*, 31, 557-578.

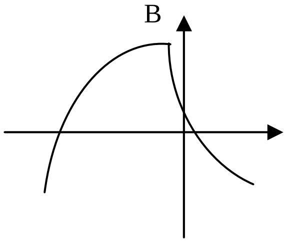
Berry, J. S. & Nyman M. A. (2003). Promoting Students' Graphical Understanding of the Calculus. *Journal of Mathematical Behavior*, 22, 481 – 497.

Ferrini-Mundy, J. & Graham, K. (1994). Research in calculus learning:understanding of limits, derivative and integrals, research issues in undergraduate mathematics learnings. In J. J. Kaput, & E. Dubinsky (Eds.), *Research issues in undergraduate mathematics learning*, MAA notes 33, 31-35. Washington, DC: Mathematical Association of America.

Liora Hoch
Ben-Gurion University
Talpiot and Orot colleges
Israel
liora_h @macam.ac.il

Miriam Amit
Ben-Gurion University
Be'er-Sheva
Israel
amit@bgu.ac.il

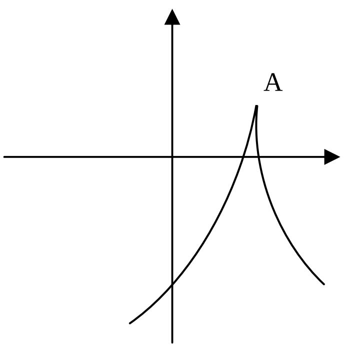
Appendix: Example questions from the questionnaire:



Is B a cusp point? Yes/No

$f'(x_B) = 0$ Yes/No Explain:______

$f''(x_B) = 0$ Yes/No Explain:_________



Is A a cusp point? Yes/No

$f'(x_A) = 0$ Yes/No Explain:______

$f''(x_A) = 0$ Yes/No Explain:_________

Prove or refute:

There exists a differentiable function at x=2 and this point is also a cusp point? Yes/No

Justify: ____