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TEACHERS' EVALUATION ABOUT STUDENTS' ERRORS IN PROBABILITY AND THEIR DIDACTICAL INTERVENTIONS

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ABSTRACT

In this paper research results based on the use of an instrument for students' errors' evaluation and teachers' decision's prevention for primary school teachers are presented. This instrument was produced by the Laboratory of Learning Technologies and Didactical Engineering of University of the Aegean. We investigated the teachers' evaluations about students' errors in a specific probabilistic problem, their proposed interpretations as well as their didactical interventions. The findings showed that the teachers' evaluations differed a lot (as marks) for the students' answers in this problem, but their interpretations and their proposed interventions seemed to converge.

THEORETICAL BACKGROUND

Many educators have mentioned that any change of teachers' beliefs and practices in mathematics education could have as a starting point their behaviour towards students' mathematical errors (Labinowitz, 1987; Wood, et al., 1991). However, in order to change the character of educational process, we have to organize useful tools that can prepare teachers to prevent their decisions about students' errors.

In CIEAEM 57 we have organized a workshop about the use of an instrument for students' errors' evaluation and teachers' decision's prevention for developing ability for educational planning to primary school teachers (Kalavassis et al., 2005). This instrument was produced by the Laboratory of Learning Technologies and Didactical Engineering of University of the Aegean (Kalavassis, et al., 2002) and it can be briefly described in the following stages:

a)Several alternative answers on a specific selected subject-problem are presented. These answers have been systematically observed by different learners on comparable conditions of teaching and learning and they have become subjects of many researches in mathematics education.

b) Participants are requested individually to give a mark to each answer and to interpret each answer. Moreover, they are requested individually to describe their teaching intervention in each case, that is the way they would try to help the students to understand their errors.

c)Participants discuss in small groups of three or four members their grades, their interpretations and their decisions about the ways treating the subject-problem. They chose one representative of the group to present the results of their work.

d)The representatives of the groups present the particular answers of all the participants by discussing and comparing thoroughly each subject-problem as well as the eventual consequences that each didactical decision of the teacher could have on students' mathematical learning.

e)At the end of the discussion for each subject-problem, the interpretations of the alternative answers of the students proposed by mathematics researchers are presented.

In this paper research results based on this instrument referring to a specific probabilistic problem are presented. The choice of a probability task was based on the fact that the interest of mathematics educators in stochastic concepts has increased, as a consequence of the recognition of the role of these concepts in our daily life. Moreover, this interest becomes even greater due to the fact that students have much difficulty with these concepts, as many researchers have shown (cf. Fischbein & Schnarch, 1997; Kapadia &

Borovcnik, 1991; Shaughnessy, 1992). More specifically, by using this instrument, we investigated the teachers' evaluations about students' errors in probability, their proposed interpretations as well as their didactical interventions. This research can allow us to discuss about the differences and the similarities that teachers' answers present when the teachers comment their students' errors.

METHOD

Data was gathered during an in-service teacher training program realized in November 2005 in Rhodes. 72 teachers participated in this program. The problem that was given to them, based on our instrument, was the following:

"Luca and Paolo play with a pair of dice. If the sum of the points is 3, Luca is the winner. If the sum of the point is 11, Paolo is the winner. Which of the following answers seems to you to be the correct one? Why?
Luca is the favorite
Paolo is the favorite
Luca and Paolo have the same chance"
Students gave the following wrong answers:
1."Paolo has a higher chance because he has the bigger number."
2."Paolo is advantaged because with two dice obtains almost always numbers bigger than 3."
3."Paolo is advantaged because there are 5 possibilities to obtain 11, that is 8+3, 10+1, 6+5, 9+2, 7+4 while Luca has only one possibility, that is, 2+1." (Fischbein et al.,1991)
We asked the teachers the following:
I. Give a mark (0-10) to each answer.

Answer	1	2	3
Mark			

II. Which do you think is the origin of each answer?
III. Describe a didactical intervention for each case.

RESULTS

I. The table 1 concerns the teachers' evaluations about the three students' wrong answers in the above problem. (This evaluation -scale is used in Greek primary schools by the teachers.)

Marks 0-10	1st wrong answer	2nd wrong answer	3rd wrong answer
0	3 (4,2%)	2 (2,8%)	2 (2,8%)
1	2 (2,8%)	2 (2,8%)	1 (1,4%)
2	7 (9,9%)	6 (8,5%)	5 (7%)
3	12 (16,9%)	4 (5,6%)	4 (5,6%)
4	11 (15,5%)	7 (9,9%)	3 (4,2%)
5	14 (19,7%)	18 (25,4%)	16 (22,5%)
6	8 (11,3%)	12 (17%)	6 (8,5%)

7	9 (12,7%)	9 (12,7%)	13 (18,4%)
8	3 (4,2%)	8 (11,3%)	11 (15,5%)
9	2 (2,8%)	3 (4,2%)	6 (8,5%)
10	0	0	4 (5,6%)
Answers' total	71	71	71

Table 1: Teachers' marks for each answer

We can observe that: a) the marks that the teachers gave varied a lot for every answer (from 0-9) , b) the majority of the teachers gave: 3 to 7 marks for the 1st answer, 5 to 8 marks for the 2nd answer and 5 to 8 marks for the 3rd answer, c) the biggest percentage in each answer is for 5 marks.

The table 2 presents the differences among the marks that each teacher gave for the three wrong answers.

Marks' differences	Number of teachers
0	14 (19,7%)
1	11 (15,5%)
2	14 (19,7%)
3	11 (15,5%)
4	8 (11,3%)
5	8 (11,3%)
6	2 (2,8%)
7	2 (2,8%)
8	1 (1,4%)
9	0
10	0

Table 2: Each teacher marks' differences

It is obvious that each teacher usually used the same scale to evaluate the different errors. More specifically, the majority of the teachers (70,4%) used marks which differed between 0-3 points from each other.

II. Concerning the teachers' interpretations of the students' wrong answers (9 teachers didn't answer this question at all), we can classify them in the following categories:

Teachers' interpretations	1st wrong answer	2nd wrong answer	3rd wrong answer
Not understandable	13	13	2
Big number is the winner	43	14	
Small number is the looser		12	

Number identification with probability	3		
Dice' amount	2	15	
Experience		4	
Ignorance of dice' numbers			58

Table 3: Teachers' interpretations

In the first category, the teachers mentioned that the students' answers were wrong because they spent little time working the problem out or that they didn't understand the problem at all or a part of it. Other times they mentioned that the students haven't understand the probability theory.

In the second category, the teachers supported the opinion that in the students' mind the bigger number is usually the winner, because it consists of many components or because of their experience from games and from everyday life. This interpretation was mostly connected with the first wrong answer. (For example: "The student compares only the numbers, the number 11 is bigger than 3 and the bigger numbers usually gain", "The bigger numbers have more combinations").

The third category concerned only the second wrong answer. The teachers mentioned that the students believe that the small numbers usually loose. (For example: "As the die has the numbers from 1 to 6, the student thinks that the probability to bring 11 is bigger, because 3 is a small number.", "The student thinks that 3 is a small number and it is difficult for someone to bring such a number with the dice. He thinks that the big numbers come with greater ease".)

In the fourth category, the teachers said that the students believe that we have a bigger probability with big numbers and a smaller probability with small numbers.

In the fifth category, the teachers mentioned that the students believe that we have bigger totals with two dice. This interpretation was mostly connected with the second wrong answer. (For example: "The student believes that the two dice give a sum bigger than 3", "As a number becomes bigger through the addition, so the student considered that the number 11 seems more logical to be constructed").

In the sixth category, the teachers commented that the students were based on their personal experience.

Finally, the last category is connected with the teachers' interpretations for the 3rd wrong answer. The majority of the teachers answered that the students separated the numbers in a correct way, but they didn't have in mind which the dice' numbers are. (For example: "The student did not take into account the numbers of the die, but only the numbers 3 and 11", "The student did not take into account the reality of the problem").

We can observe that for the first and the third wrong answer the majority of the teachers gave the same interpretation. On the contrary, there were different interpretations for the students' way of thinking about the second answer.

III. Concerning the teachers' didactical interventions for each wrong answer (16 teachers didn't answer this question at all), we can classify them in the following categories:

Teachers' didactical interventions	1st wrong answer	2nd wrong answer	3rd wrong answer
Oral explanation	13	8	9
Experimentation with dice	33	23	15

Play with the dice	4	4	2
Attention to the formulation	3	2	1
Dice' construction from students		1	3
Familiarity with the dice			22

Table 4. Teachers' interventions

In the first category, the teachers answered that they would try to teach to the students the 'correct' way of thinking about probability and combinations by the use of examples. (For example: "I would explain the error and I would teach the correct method to find the probabilities").

In the second category, teachers wrote that they would realize experiments with the dice in order to make their students understand what a dice is and to find the probability of having 11 or 3 and 11. Some of them mentioned that they would write on the blackboard all the possible combinations or all the possible sub-totals.(For example: "Experimentation with dice and recording of all the possible combinations on the blackboard") .

A few teachers mentioned games as a means for encouraging students to learn or that they should pay attention at the formulation of the problem(third and fourth category).(For example: "I would propose games with dice, so that the students could discover alone the correct answer" , "The students have to pay more attention at the formulation of the problem. They have to read it again").

Furthermore, a few teachers proposed that the students could construct a die (fifth category). (For example: "Students' construction of a die in order to make themselves the pips and to understand that different numbers have different probabilities").

Finally, in the last category, teachers answered that they should present the dice to their students in order to explain its characteristics. (For example: "Presentation of the die by explaining that the bigger number is 6. So, when we throw the dice the number 11 can be made by the numbers 5 and 6").

We can observe that the majority of the teachers proposed the same didactical intervention for all the wrong answers. For the first and the second wrong answer they proposed the experimentation with dice and for the third answer the presentation of the the dice to their students.

CONCLUSIONS

The above findings showed that the teachers' interpretations and their proposed interventions towards the specific probabilistic problem seemed to converge. Moreover, their interpretations are very closed to the interpretations that many researchers have proposed for students errors' in probabilities, that is many children in primary school believe that throwing a die some numbers are easier to come and some numbers come more difficult based on their personal experiences from their games or their prejudices about some numbers (cf. Shaughnessy, 1992). Furthermore, the availability of the sums for a big number dominates in relation to the restrictions of this problem.

On the other hand, the results showed that the teachers' evaluations differed a lot (as marks) for the students' answers in this problem. Although the teachers agreed about the origins of an error and the way of its treatment, they evaluated differently the significance of this error in relation to the students' knowledge. However, as the assessment of student's mathematical knowledge is an important component of the educational system, what are the consequences of this finding in students' mathematical school experiences? Given that the reform of mathematics education legitimizes students' errors, the question is whether in educational practice there could be a method for the evaluation – interpretation – treatment system, when the teachers are dealing with a wrong answer, so that they could restrict subjectivity during the educational practice. Could we design some kind of educational material that could facilitate this problem?

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