# Social Justice and Equity in Collaboration in Mathematics Education Research and Practice: The Learner's Perspective Study

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### Introduction

Since international comparative studies like SIMS, TIMSS and PISA came to dominate educational and political debates around the world, the fact that East-Asian students nearly always outperform their Western counterparts is a point of repeated concern, particularly in wealthy Western countries that blame the failure of their educational systems and of their teaching force in particular. The immediate political reactions were either strong social demand for new standards of teaching, new curricular provision and regular testing of student performance – or attempts to imitate East-Asian teaching patterns. But success will not occur merely by setting new standards and holding teachers accountable for their achievement nor by "copying" teaching practices that are deeply culturally engrained and not easily transferable. The need to improve the quality of a process like teaching as an essential precursor to the improvement of the teaching product or outcome is well understood in most other professional and industrial fields; the same principle is needed to guide practices in education. Therefore the improvement of mathematics teaching must be founded upon a deeper understanding of both teaching and learning and relating both activities to student achievement. The following discussion aims at enriching this understanding.

Teaching mathematics is an initiation into a practice of mathematics. How students and teachers as actors perceive the practice of teaching mathematics has only lately become a focus theme of research. A wide range of frameworks, notions, metaphors and methodologies is used in research on teaching mathematics, and there is no common understanding of priorities or about the most appropriate approaches. In international comparisons, research has focussed on comparing quantitative accounts of content matter to be taught or the organisational patterns and forms of interaction applied in the classroom in order to define typical scripts (e.g. Hiebert et al. 2003, Stigler & Hiebert 1999). In ethnographic research the analyses of construction and negotiation of meaning as the result of interaction between the teacher and the whole class or among individual students have identified socially framed patterns (e.g. Yackel & Cobb 1996, Bauersfeld 1988; Seeger, Voigt & Waschescio, 1998), Voigt 1996). The difficulty of such studies is the general lack of legitimacy of any comparisons across cultures. If teaching is conceived in culturally specific terms, then a research strategy is required which documents local practice in a form that permits legitimate comparison and guarantees justice and equality to all participants. The standard approach to asking teachers to describe their instructional practices is fraught with difficulties even within a national sample, where terms such as "problem solving" or "learning tasks" are used in very different senses. The lack of a shared language of teaching practice is compounded in a cross-cultural study based on formalised data. The responses are nearly impossible to interpret and legitimate comparisons cannot be made, social justice and equality is very difficult to be established and maintained.

### Social Justice and Equity in International Collaboration

Social justice concerns are no longer seen at the margins of mathematics education practice and research. Issues relating to gender, socio-economic status, multiculturalism, ethno-mathematics, and the effects of ethnic, socio-economic and cultural backgrounds of students learning are regularly discussed in publications and on conferences; many of them have found their way into education policies in countries around the world. Undoubtedly, different writers have used different concepts of social justice – which are leading to contradictory conclusions and demands.



*Gender* in mathematics is perhaps the first area of research and political action to be established as a strong social justice movement within mathematics education. It is worth to mention that CIEAEM never explicitly addressed the issue of social justice or gender deliberately as we thought that we are practicing equality and justice by joining teachers and researchers, in particular the equal representation of female and male very visible in our Commission, and value the contributions and discussions of both equally. Worldwide the foundation of the International Organisation of Women and Mathematics Education as one of the official affiliations of ICMI in 1987 was the result of decades of persistent research and political actions by women mathematics educators from many countries. But in later years we note a widening attention to other claims of social justice to include into research and debates: *ethnicity* and *social and cultural differences*.

My presentation will address issues of social justice related to two areas of mathematics education practice and research: selection of research foci as trends, themes and methodologies on one side, and the forms and practices of national and international collaboration on the other, in particular issues of collaboration between practitioners and researchers as well as teachers and students from the perspective of social justice concerns, which I consider as necessary and more important than ever before.

Questioning social justice in mathematics education research and practice on local, national, international level lead to question if international contacts and collaboration between research and practice can be exploitative or if international contacts and collaboration between research and practice lead to marginalisation of one part. How do we construct power or more likely powerlessness in local, national and international collaboration? How do we prevent international collaboration to lead to cultural imperialism?

While research into marginalised social and cultural groups may give voice to the voiceless, questions of whose point of view and who is benefiting is not always at the forefront of critical evaluation of all academic action. Similarly, international contacts can lead to marginalisation of some participants if their participation is limited on economic and language grounds. Further, if the research questions and methodologies of some countries dominate international research at the expense of issues of concern of other nations, then these are marginalised. In addition to exploitation and marginalisation, economic situations in many less industrialised nations limit the capacity of educators (practitioners and researchers) from those countries to take an active and equal role in (international) academic activities and hence can lead to a sense of powerlessness. And any uncritical transfer of curricula and research results from one country with a certain perceived high status to another can be said to be a form of cultural imperialism. In particular the assumed direct correlation of Western mathematics to economic development and the assumption of the universality of mathematics can lead to imposing certain forms of mathematics that may not be appropriate or relevant to many students around the world. Finally, the tying of international aid and development monies to the imposition of agendas, policies and priorities developed in Western countries can be regarded as a form of violence on less affluent nations.

In this presentation I am going to discuss some findings of the international project "Mathematics Classrooms Practice - the Learners' Perspective" (LPS), which has been initiated in close collaboration by David Clarke (Aus), Christine Keitel (D) and Yoshinori Shimizu (J) in reaction to some of the critical features of international comparative research outlined above (cf. Clarke 2001, Keitel & Kilpatrick 1999), and how we try to establish social justice concerns and equality in an international project with a great diversity of partner groups and countries.



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### The project "The Learner's Perspective Study" (LPS)

The international project LPS joins collaborating countries for investigating "Mathematics Classroom Practice - The Learner's Perspective". LPS has collected a wide range of data to capture the practices and associated meanings in mathematics classrooms in twelve countries: Australia (Melbourne), China, PR (Shanghai), Germany (Berlin), Great Britain (Bristol), Hong Kong & Macao, Israel (Beer Sheva, Tel Aviv), Japan (Tokyo), The Philippines (Manila), Sweden (Gothenburg, Uppsala), South Africa (Durban), Singapore, U.S.A. (San Diego), Czech Republic (Prague), and Korea (Seoul) (recently colleagues from Great Britain (Bristol) have joined).

Each country in LPS uses the same research design to collect videotaped classroom data for at least ten consecutive mathematics lessons and post-lesson video-stimulated interviews with either an individual or a small group of students, teacher interviews and student materials. LPS is guided by the conviction that the characterisation of practices of classroom mathematics must attend to the learner's practice with at least the same priority as that accorded to the teacher's practice. The methodology of data production in the LPS aims at documenting not just the obvious classroom events that might be recorded on videotape, but also the participants' (re)construction by interpreting the classroom events; in general LPS aims at integrating complementary analyses of the substantial international data set generated through the combined efforts of the participating researchers.

### Description of the research methodology of the project

The data gathering or production in LPS is a methodological advance compared to other studies that involve documenting classroom practice by videotaping: e.g. in contrast to TIMSS, in LPS sequences of lessons rather than just one single lesson from each teacher or classroom are documented. It is our conviction that teaching and learning can only be separated analytically, so the study design aims at giving voice to all participants. We also believe that the substance of a social practice like that found in a mathematics classroom cannot be documented without trying to reconstruct the meanings that the participants attribute to their actions. LPS allows parallel exploration of student's practices, of the corporate behaviour of the class as a whole and of the teacher practices, therefore it can address many new research questions not yet dealt with in other studies. The project has just compiled two volumes of books reporting about the first analyses and interpretations, which provide an insider's view as well as comparative accounts under specific themes that had been considered of mutual interest and worthwhile for in-depth collaboration (Clarke, D., Keitel, C., & Shimizu, Y. 2006; Clarke, D., Emanuelson, J., Jablonka, E. & Mok, I. 2006).

#### Research philosophies within LPS: Social justice and equality

Research in LPS is based and deeply depending on an equal collaboration of the members of the research teams from each participating country. Results are negotiated amongst the researchers and interpretations shared and adopted or refused. The project provides in-depth data for various analyses of classroom processes from the varied perspectives of teachers, learners, and mathematics. Thus the process of locating the learner's performance in different classroom cultures hopefully can enable the mathematics education community to interrogate teaching and learner performance in a more in-depth way. The Learner's Perspective Study is guided by a belief that we need to collaborate and learn from each other.

LPS is atypical insofar it is a project without hierarchy: partner groups have equal rights and support within the project, and decisions are only taken unanimously; partner colleagues can decide on their own analyses and interpretations, yet they must ask for reaction and that their data be verified by partners from other countries. Data are accessible to each and every partner country: when a set of data is com-



piled and processed, one can exchange for one data set from another country. It occurred that we have very interesting compilations of countries, although the assembling of countries did not follow any specific or explicit research methodologies, but used friendly contacts. Although the teachers were considered as partners and had access to the data of their classroom videos excluding video-stimulated-recall interviews with the individual students, they also could use some videos themselves to discuss with their students.

The social practices in a classroom can only be comprehensively understood if the interpretations and construction of meaning not only of teachers but also of students are included in the data collection. What is considered as typical is not defined by any kind of sample representativity; it is instead assumed that local criteria of "good teaching" can catch typical cases of classroom practice that may represent some kinds of norms of good teaching. We did not especially search for so-called innovative or especially ineffective teaching practices; instead, we tried to analyse, reconstruct and possibly generate structures, not statistical generalisations.

#### Constraints and chances in LPS

The generating of the video-data has always been enormously time consuming, both materially and personally. But this aspect increased even more for LPS: because our data need to serve for very different aspects and analyses, we chose a very detailed and specified methodology, which is described and justified in detail in Clarke, Keitel & Shimizu (ch. 2, 2006). For purposes of general understanding of this discussion, only few features should be listed here: We used 3 cameras, with one for a changing students' focus group using video-stimulated-(recall-technique) and reconstructive interviews for each lesson (we had 1 to 4 students in a focus group who were interviewed after each lesson); one week of video-graphing was used to acclimate the students and teacher; no extra lessons were to be documented outside normal daily lessons; students' interviews (each focus group) were closely connected to the lesson. All participating researchers can use all existing data from other groups for their own aims and analyses, and compare with other countries. Student material or productions like texts, drawings, tests, and all teaching material like textbook pages and working sheets were collected and documented. All lessons and interviews are transcribed and translated into English as the language of the project.

#### Diversity in structure – structures in diversity?

Our research aims to identify ways in which role-related asymmetries and culturally sanctioned ways of interaction serve as an orientation for the participants in mathematical classrooms, and in particular within certain classroom events and interaction like setting a task or reasoning discourses. Episodes from classroom discourse and student interviews are interpreted in the course of a contrastive analysis. One goal of the ongoing studies is to identify links between similarities in students' agency and structure in differences (Jablonka 2002, 2003b,c,d, 2004, Jablonka & Keitel 2006, Begehr 2003, 2004, Keitel 2003, Keitel 2004). As the project also aims at identifying the ways in which practices of learners both afford and constrain specific teacher practices – including the realisation of the teacher's goals or "scripts", the project also recognized the extent to which teacher practices represent affordances and constraints on the students' practices and goals. Conclusions are drawn as to whether teacher and learner practices are best seen as conflicting or as mutually sustaining – indeed this does significantly inform our theorizing on classroom practice.

When first starting to analyse classroom events or episodes to capture students' reconstruction of teaching school mathematics, we looked for the following themes and possible aspects of teaching and learning patterns without defining them in detail. Preliminary analyses have been reported and pub-



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lished; however, the major task of comparison is in its infancy. Some of the preliminary themes are mentioned below:

- Setting a task (Keitel 2006)

- Reasoning discourses in mathematics classroom practices (Jablonka 2003b,c, 2004a)

- Patterns of student' participation in mathematics classrooms (Begehr 2004, 2006)
- Forms and effects of classroom interactions (Jablonka 2002)
- Values and classroom interaction (Jablonka & Keitel 2006, Keitel 2004)
- Students' struggle for sense making (Keitel 2004)

#### Students' views and goals of schooling

The identification of the students' "culture of schooling" and the evidence (if any) of "student scripts", analogous to the teacher scripts, are taken in the different countries as characteristics of student classroom behaviour that determine the nature of their participation and subsequent achievement. An indication of mutual accommodation or conflicts between and within such teacher and student scripts would be the interpretation of events by which actions of classroom participants (possibly unknowingly) conspire to sustain each others' practices through their subscription to particular, culturally-determined, classroom norms (cf. Jablonka 2004, 2006). The reconstruction of the lessons by the students, and the identification of such a relationship between teacher and student scripts, hopefully improves our understanding of Brousseau's "didactic contract" (Brousseau 1989, 1997).

The absence of the learner's perspective in international comparative research called into question the adequacy of previous research to do more than describe teacher practice, lacking either associative or explanatory potential. The meanings which students ascribe to the actions of their teachers and their classmates are supposed to be as culturally-specific, and as significant for our understanding of classrooms, as the actions themselves. Therefore the research design was developed to support analyses intended to portray, to compare and to contrast teachers and their teaching, not cultures. The documentation of the practices of mathematics classrooms in other countries causes us to question our assumptions about our own practice. Therefore the essential characteristic of our study of mathematics classrooms is the commitment to an integrative approach. Some of the initial questions that guided the design and analyses in LPS will be used to present some findings and snapshots of our data and material.

#### Studying students' perceptions of classroom practices

In this chapter, I want to discuss some findings that indicate differences in students' and teachers' perceptions of classroom practice in our project. This presentation will use only two of the major questions raised at the beginning of our project and provide snapshots as examples of first attempts at analyses and insights (however without final conclusions):

 What are similarities and differences in classroom practices across the analysed classroom cultures? Is there structure in diversity? To what extent are teacher and learner practices in a mutually supportive relationship? Is there a consistency with which teachers conform to a particular cultural "teacher script"? Teachers' reconstructive accounts will be used to augment the researchers' interpretations of classroom teaching practice in all countries.
What are the students' interpretations of mathematics classrooms identified in students' classroom activity and students' reconstructive accounts of their experience of particular lessons? What are students' conceptions of the purpose of classroom activity and of what

lessons? What are students' conceptions of the purpose of classroom activity and of what constitutes success? To what extent are particular documented teacher and learner practices associated with student construction of valued social and mathematical meanings?



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### Lesson event: Reasoning discourse

The German team was mostly interested in looking at any reasoning, and to identify "Reasoning discourses" (Jablonka 2004a), however they were rather rare despite the conviction of nearly all teachers in secondary schools that reasoning is the most important part of classroom mathematics and that reasoningshould be the heart of any mathematics education practice. Jablonka (Jablonka 2003b,c) had chosen 60 lessons and scripts for a preliminary study on "Reasoning discourses", and identified them by looking for all utterances, in which a speaker is giving a reason or a justification for something that has happened or should happen. Her special interest was to find out: Who are the speakers, which are the themes, in which frame of interaction processes or forms of interactions reasons are given or justifications are offered? Are there different styles or types of reasoning in the classrooms? In her study "Structure in diversity: Initiation into mathematical practice in classrooms from Germany, Hong Kong and the USA" (Jablonka 2004a,b) she tried to explain the lack of reasoning discourses and mathematical-logical argumentation in the classrooms investigated after having reconstructed specific forms of interaction from the perspectives of the observer, teacher and student. Her findings suggest that selfinitiated utterances of students that are related to mathematical argumentation or reasoning were very rare, not only in Germany and the USA, but also in Hong Kong; in giving reasons, a mixture of characterisations, conventions und mathematical norms come together; and students as well as teachers do not give reasons or justifications, if at all, mostly because of organisational problems rather than for mathematical reasons. Often the question is not: What is the reason, or why? But: Why have you done something in a certain way? This means that students might usually give justifications for their actions, but no mathematical-logical reasons. Because of the small number of reasoning events in classrooms, the research question changed: Does the culture of mathematics classroom practice have peculiar characteristics that are stronger than the cultural environment in which the lessons and the schooling takes place? Classes from different educational and cultural traditions were chosen in order to separate cultural from structural interpretations. Another selection principle maximised contrasts, as well as the availability of ready data. Most interesting seemed to be the analysis of "Students out at front" or "Students at the board". (Jablonka 2004a, 2004b). When analysing students' perspectives from the questionnaire and interview data, some common themes came up: A most important feature was the simultaneous fear to loose face, yet also some chances:

#### Lesson event: Students out at front

Some perceptions of students of this event are listed below, as snapshots from different classes and student groups, very similar in experience and judgment.

Int:	It's all right. Do you think the way you learn mathematics like in the last lesson is the
	best?
Peter:	It's okay.
Int:	Well Why do you say so?
Peter:	Because Mr Mr. M. gave us the questions and put them on the projector. Then he let
	us do them. I think that's quite good. And then, he calls us to answer them (on the
	blackboard). If you don't understand, he would teach you. That's quite nice

#### An opportunity to get taught "extra" (Hong Kong)

#### Showing results (USA)

Lindsey:	Um usually. Cuz I think, like, we are getting to- we- we learn about something, we
	get into groups and we do something, display it in front of the class.



#### Comparing ways of getting results (USA)

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Abbie:	In class I think something that really helped me learn was when um, we do things on the
	board where we show our different work, and how different people found it.
Abbie:	Because if I don't understand how the teacher does it, somebody else does it on the
	board-
Int:	Mm-hm.
Abbie:	Then I can look at their thinking, compare it to the teacher's and see how they're similar,
	but how they're different. And I can make my own way of figuring it out based on
	theirs.
Abbie:	If you only have one way, then you don't know any other way to do it, and you can't fig-
	ure out your own. Cuz it's too solid, there's nothing to compare, contrast. Abbie

#### Understanding criteria of doing school math (USA)

Int:	What were your personal goals for that lesson?
Autumn:	This lesson?
Int:	Mm-hm.
Autumn:	To understand it and get through it.
Int:	Okay, and what does understand mean for you? When do you know you understand?
Autumn:	When I can- I can like, go up to the board and do a problem and get it right.

An inconvenient situation (Shauna, USA; Angie, USA; Peer, GER; Mona, GER; Diana, GER)

Shauna:	I don't like to look like a jackass in front of the class.
Int:	Oh, tell me more about that.
Shauna:	It's pretty self-explanatory. I don't want to look like a jackass in front of the class.
Angie:	I don't like to show my answers. I don't want to be wrong and have everyone in the
	whole class see, and then
Peer:	No idea because () when you calculate something at the blackboard, then you make a
	mistake you always think that the others will laugh about you but actually it isn't like
	that in this course.
Mona:	Because then everyone can look at me, and I don't know, then they know exactly how
	I'm calculating or what I, what I'm thinking or
Diana:	I don't know. It's somehow embarrassing if you made a mistake when calculating or
	something like that.

Jablonka (Jablonka 2003b,d; 2004a,b) identified some similarities among the countries investigated: 'Silent' presentations are a form of activity that occurs in all the classrooms. It is peculiar to learning and teaching mathematics in classrooms: students seem not to be used to speaking mathematics. Teachers appropriate students' work on the board and incorporate it into the lesson plan, while students perceive of being at the front in terms of interaction rather than of knowledge development. Solving a mathematical task in public carries a high risk of vulnerability, especially if the task is not open-ended or a new task (a "Learning Task"), so many students from the lower achieving classrooms do not like to go to the front independently of the function of this activity. Mathematical practice. Chattering or talking to each other is a must: Talking to each other during the lesson does not mean a lack of participation or engagement. Students change most quickly from talks about tasks to talks about private themes and back.



### It is necessary to have someone to ask

Before asking the teacher, it is preferred to turn to classmates, friends or relatives. Most often one only asks and wants to know if the result is right or wrong. Most often there is a complete absence of criteria for checking, controlling or proving. The judgement of the teacher is more important for students with a low level of achievement. Only in Hong Kong do some students report that they look into the textbook at home before they address relatives or friends for problems and their solutions. There is a common conviction that the teacher has to explain *"how it works*": students do not perceive mathematics as a domain of knowledge, which they could acquire or appropriate by themselves. Most prefer detailed "step-by-step"-instructions or prescriptions. When those types of tasks or problems are presented, in which mathematical connections have to be explicitly established or constructed by themselves, most students feel lost. Students do not represent or hold learning theories which ask for discovery learning or problem-solving approaches. It depends on the teacher if students like math or not; only one single student described the whole social system of schooling and outside as connected.

# Fear of "loosing face"

The atmosphere in the investigated classrooms was perceived as very diverse. Mostly students from San Diego emphasized the danger of "loosing face" when being called at the board. Anxiety to loose face referred more to the other students than to the teacher and often experiences from very early school years or from other school subjects played a major role.

Amiri :	Cuz some questions that I'm not sure of, I don't really want to raise my hand cuz I don't want to //ya know (USA)
Mona:	It was always () when I put up my hand, it was always wrong(Germany)
Alfonso:	Cause I was tr- I was thinking about how how I already did it on my paper, but I was thinking about how I can do it like in front of everybody cause you get nervous if you get it wrong, but they said um, X equals is equal to five (USA)
Int:	Yeah, she has people stand up a lot. Do you like to- to share your answers or do you.
Angie:	I don't like to show my answers. I don't want to be wrong and have everyone in the whole class see, and then (USA)
Amiri:	Cuz some questions that I'm not sure of, I don't really want to raise my hand cuz I don't want to ya know (USA)

### Students values and mathematics

One very important aspect in our studies was students' views of mathematics and their values: Why do we learn mathematics and what is important? What is math about? However, during the course of studies students learn to abandon the question 'Why' and 'For what' and have nearly no ideas *about mathematics* or where to apply math beyond the shopping mall. Their knowledge about application and applicability is very limited, mostly wrong.



	possibly do with it?
Steffi:	It's not clear to me at all.
Sharon:	Well, it just belongs to basic knowledge. You just have to be able to do it, and then you're allowed to forget it.
Int.:	But what's the use of knowing it?
Sharon:	Well, I don't know if//
Steffi:	//Perhaps if you want to become a mathematician
Sharon:	Or working in a bank. (GER)

Int.:	Uh, you like this lesson?
Peggy:	Uh, a little bit. I can, I mean, I can discuss with my classmates.
Int.:	Uh, what do you think is the most important thing to learn in this mathematics lesson?
Peggy:	The most important? Calculation.
Int.:	Calculation is the most important?
Peggy:	Yep.
Int.:	What have you learnt?
Peggy:	I've learnt(learnt). I don't know. (Hong Kong)

Int.:	Um Do you like this lesson?
Rachel:	like it. Uh, I don't know why, but I am interested in mathematics lessons. All teachers,
	no matter now or before, are quite good.
Int.:	Do you like Mathematics lessons?
Rachel:	Yes.
Int.:	Why?
Rachel:	Well, because, I do not know how to tell you. But I think that mathematics can make
	me think faster.
Int.:	Yep.
Rachel:	That is, I need not to use my brain to think in a lesson usually. That is, I only need to
	memorize all the information in books. Also (in mathematics lessons), it can - that is I
	can find the answer when I see the question, so there is no need to find the information
	in books. (Hong Kong)

Int.:	You enjoy doing (math)?
Friedrich:	Most of the time, yeah.
Int.:	Do you think you have a talent for it?
Friedrich:	I don't think I am excessively talented. Because I enjoy doing it, I have the feeling that
	I get more practice, I don't know. Take these daily exercises that we always do, I usu-
	ally get through them quite quickly and have, well I work out pretty quickly and then
	I've always enough time to check them through again. That is I can go through each
	problem twice and can usually spot most of the mistakes.
Int.:	Is speed then the criterion so to speak, the quickest is the best.
Friedrich:	Not necessarily, with me it's always much better if I finish quickly, as I've then still got
	time to go through it all again, because I've usually got one or two mistakes and then
	I've got more time for them. (Germany)



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Martin:	Math is one of my favourite subjects. It is fun.
Int.:	What is fun about it?
Martin:	No idea. Numbers that nobody understands. You try to understand what you are sup-
	posed to do with all the x's and that's fun, juggling with numbers.
Int.:	Like a sorcerer?
Martin:	Yeah, to do something magical, you get a result, and then someone says that it is right.
	Yeah, but sometimes you get nothing out, just something you can't understand like
	why x equals y or the like. But it's fun anyway. Wellif the maestro says that it's
	sothen so be itI really have to believe what he says. (Germany)

To be recognised as a good and as an active participant is considered as important in Hong Kong schools as in Germany, but there are different perceptions of what might be honoured by teachers and valued by classmates:

Int.:	Martin, I noticed, you went to ask Mr. Reimer if your solution of the given task was
	correct. But you were completely sure already before that you had a correct solution.
	Why did you ask Mr. Reimer then?
Martin:	I have to make sure that he knows and notes that I did it correctly, that I have well
	done. You have to care for that. (Germany)

Michael:	Well, here I think we were not energetic enough.
Int.:	Energetic means
Michael:	You were not answering the question actively. That means not - actually, it is not nec- essary to - not necessary to reach the level of activethe situation was that there was no one put up his or her hand. We only stood up to answer the question when teacher called us. This was not so good.
Int.:	You think the whole class
Michael:	Yep, to see the whole class as a whole. Also including me. (laughing)
Int.:	(laughing) How about you, why don't you put up your hand?
Michael:	Ha?
Int.:	You were shy?
Michael:	No, thinkI think that others would put up their hand to answer. Also, anybody else liked to answer- answer the questions. I don't know why.
Int.:	Some students like answering the questions?
Michael:	Yep, there is one. He would answer, so I gave the chance to him to answer. (laughing) He would like to be recorded. So. (laughing) (Hong Kong)

When we investigated students' values and math, we first compared the differences and similarities in structure and policy of the educational system in Germany and Hong Kong : Both provide a highly advanced education in mathematics, focussing on pure rather than applied mathematics, to an increasingly large group of the school population. One of the most significant differences is the fact that Hong Kong schools are examination driven. Several tests have to be passed in order to proceed to higher levels of school and to university. Although teachers are encouraged to adopt student-centred, activity-based and hands-on-approaches to organise student learning, the atmosphere of examinations and strong competition suggests that teachers and schools consider the attainment of good exam results as a priority. Students are exposed to constant drills on skills in order to secure their abilities in written exams. Mathemat-



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ics is considered a core subject and is taught every day. It is expected that Hong Kong students spend more time out of school doing extra homework, extra lessons or studies in mathematics than in other countries. Teaching and learning are focused on the public exam syllabus and on students' proficiency to work out problems of this syllabus (Wares & Becker, 1983, Wong, 2002a,b; LI, 2006; Seah & Bishop, 2006; Lam, 2003; Lam, Wong, & Wong, 1999).

In Germany even the final exam of the most advanced level of schooling, at the university-bound high school, still follows mostly the teacher-based assessment mode. The syllabus is mostly focused on abstract and technical aspects (e.g. of algebra in our examples), on strict use of terminology, and on following clearly stated, formal rules, although the focus slightly varies across the federal states. A common understanding of what mathematics teaching should be about among teachers is constructed and secured in the practical phase of teacher training, the pre-requisite to the entrance into the profession. Large groups of immigrants from different social, ethnic and cultural groups challenge the system. A new diversity of students is noticeable in many areas, but the university-bound Gymnasium is not yet very much affected (cf. Keitel 1992, 1998).

When we attempt to explain structural similarities, we find that the "community of the classroom" is not a phenomenon detached or separated from the teaching and learning of mathematics; it is connected to the mathematics classroom practice and allows or discourages a willingness to discuss and also to publicly present even provisional solutions of student work at the board. The asymmetrical relationship with respect to accountability for the production and evaluation of knowledge changes the rules and maxims of conversation. Most re-contextualisations of mathematical discourses for the sake of classroom practices effect a transformation of problems into sequences of tasks to be done.

All "meta-discursive rules" remain implicit. This explains why the system of organizing students by ability rarely leads to positive results. Such rules are especially evident in the analysis of the so-called inquiring-developing / question-answer-question type of classroom model that appears in all classes. In the USA it is called "Guided development" or "Guided Discovery". (It should be noted here that this model occurs much less often in Hong Kong, where a regular alternation between teacher presentation and working individually on tasks was common.) The discursive structure is further visible in group-work: a student takes up the teacher's role, imitates questioning-developing classroom talk; the effect is that it is not said what is relevant, but what the teacher might want to hear. An important reference for our main goals is Goodchild (2001), who focussed on students' perceptions of classroom practice and studied their mathematics learning and teaching, as well as individual and collective classroom activities in detail, participating as an observer in a mathematics classroom for one year. The conclusion, which he drew from his analysis of students' values and goals of mathematics classroom teaching, represents a threat and challenge to any research on teaching mathematics:

"The main outcome of experience (of the students) in mathematics classrooms is to learn how to do mathematics classroom practice. In the classroom, mathematics is set within a specialised 'classroom' discourse, that allows students to locate and follow cues and signals, skip over peripheral text and apply a variety of resources to bring the highly stylised tasks to some form of resolution. Activity within the classroom is not mathematics and for all its pretence it is not about the students' current or future experience of the world outside the classroom. Success in classroom practice does not prepare a student for the practice of mathematics or any other activity outside the classroom." (Goodchild, 2001, 227)

Our students' goals and perceptions are not so much different from those Goodchild observed. But we are convinced that the inclusion of the learner's perspective has enriched our portrayal of mathematics class-



rooms and shed more ligth on questions of social justice and equality in mathematics learning. However, our studies indicate how more in-depth investigations can offer additional explanations that provide more culturally specific, as well as differential insights.



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