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DIFFICULTIES OF USING WORDS OF SPATIAL RELATIONS

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In this study we deal with the possibility of development of spatial orientation in mathematics education. We observe difficulties of using words to describe spatial relations and in connection with this the difficulties of describing routes. In addition to analysis of solutions of orientation problems in elementary school, we make suggestion to develop pupil's competence on this area in classroom practice.

CULTURAL AND POLITICAL CHANGES IN PRIMARY SCHOOLING: INFORMATION, KNOWLEDGE, TECHNICAL TOOLS, AND EDUCATION

Introduction

The development of spatial orientation is one of the aims of mathematics education. Spatial orientation describes the visualization of a spatial arrangement in which the observer is part of the situation. (Maier, 1999)

The problem of orientation has a rich context in everyday life, so pupils learn about it not only in different school subjects, but they learn this topic spontaneously and directly too.

The weak spatial orientation often leads to different learning disabilities, e.g. dyscalculia, dyslexia, dysgraphia. Studying the rich literature of the problem of dyscalculia we can establish that spatial orientation is one of the important areas in exploration of the symptoms of dyscalculia. (Dékány, 1995)

Our topic has an obvious connection not only with the mathematics curriculum, but the classroom communication too. The wrong interpretation of the words of spatial relation may lead to misunderstanding of instructions.

In this study we deal with the possibility of development of spatial orientation in mathematics education and with the analysis of difficulties pupils have even in case of solving simple orientation problems.

Studying recent textbooks and curricula in Hungary, and speaking to teachers in elementary schools, we realized that there are only a few tasks on this topic. Pupils only learn about the orientation at the beginning of the first school year, and nevermore. We can not observe the systematic treatment of this topic.

On the basis of mathematical and historical analysis we may divide the relevant mathematics curriculum with respect to the topic of spatial orientation into 6 subtopics:

- Using words to describe spatial relations
- Describing routes (using simple maps)
- Ordering cyclically
- The coordinate system
- Geometrical transformations
- The front-, side-, and top-view of an object

From these subtopics we shall pick the first two, and observe the difficulties of using words to describe spatial relations and in connection with this the difficulties of describing routes. In addition to analysis of orientation problems we make suggestion to develop pupil's competence on this area.

Research questions, hypothesis and methodology

The research questions are the following:

1. What types of difficulties can we find by the use of words of spatial relations studying the thinking processes of pupils of grade 1-4?
2. How can we insert the topic in the recent structure of school-work successfully?

Hypothesis:

1. There are many inconsequent situations in everyday life, and they might be the reason of confusion in problem solving.
2. The systematic and spiral treatment of this topic helps to develop pupils' competence in spatial orientation.

In this paper we shall deal with the first research question only.

Methodology:
Our investigation consists the following phases.

I. An exploratory study
   a. Paper-pencil test (1)
   b. Interviews
   c. Paper-pencil test (2)
II. A teaching experience
III. A post-test

I. We planed an exploratory study connected to the term of orientation with pupils of grade 1-4. Our aim was to estimate the problems of elementary school-pupils in different ages in order to adjust the actual knowledge with the farther experience. We chose three elementary schools in Debrecen, in Hungary (In spring 2005). The first was the school associates to the teacher training college, the second was a school in a relatively old housing estate, were are blocks of flats, and the third was a school in the city. The pupils in the school associated to the college had very good abilities; they had been accepted to the school after a selection. We can say that average pupils attend the school mentioned secondly, and in the third school there are pupils whose abilities are average or below average, and whose social backgrounds are not optimal. We chose also three classes on every grade. The classes were without any specification, their learning based on the normal curriculum of their school. With the composition of the pupils participating in our experiment, we tried to represent the real situation in the grades 1-4 in Hungary.

The following table shows the number of participant:

Grades	Grade 1	Grade 2	Grade 3	Grade 4	All
Number of participants	63	78	73	62	276

I. a. We made a paper-pencil test for these pupils.

I. b. We chose pairs of pupils from every class and made interviews with them after the evaluation of the first test. We wanted to refine our estimates, because the paper-pencil test is not the best way to know more about spatial orientations ability. This topic requires communication and different forms of expressions, like body movement and gesticulation.

I. c. We finished the exploratory with a second paper-pencil test. By composing items we took the results of the first test and the interviews into consideration, furthermore we expanded the area of orientations problem.

II. After this exploratory study we carried out a classroom experience with pupils of Grade 2. (In autumn 2005) We chose pupils who were first graders in the school associated to the college in time of our exploratory study. Our aim was to try our conception and ideas to develop pupil's ability in the field of spatial orientation. The grade 2 seemed good decision in following aspect: pupils are already familiar with school life, reading and writing; we can see the remained orientation problems, which asks for our special attention. The results of the exploratory study in grades 3 and 4 were useful because of identification problems which remained and knowledge which was getting in every day life in this age.

III. In January 2005 we prepared a post-test for our second graders (Class 2B). The post-test was solved by other second graders in the school associated to the college (Class 2D), and by second graders from the school in a housing estate, who took part in our exploratory study as first graders. (Class 2T). We were interested in the development of "our" pupils comparing their results with other pupils, whose abilities are also very good, and with "average" pupils. In the second case we wanted to know also about the spontaneous development of pupils who didn't pay special attention to the topic of orientation.

In this paper we deal with the first two subtopics of the topic of spatial orientation:

- Using words to describe spatial relations, i.e. spatial relations in static situation. The words of spatial relations are such kind of polarities as left-right, front-back, above-below.
- Describing routes (using simple maps), i.e. spatial relations in dynamic situations. The dynamic situation assumes a mental travel on the route. By traveling in their own minds pupils have to imagine the position of the traveler, so they have to relate to the other person who is moving. (Without restricting the topic, we can restrict the word *route* to a sense *street*.)

Connected to the works of Freudenthal and others (Freudenthal, 1983), we distinguish three levels of problem solving.

In static (S) situation:

Level S1: Pupils distinguish polarities (left-right, above-below, front-back) relating to their own body.

Level S2: We transfer the distinction left-right from own body to other persons bodies.

Level S3: The most difficult level is the distinction of polarities relating to other non-living objects.

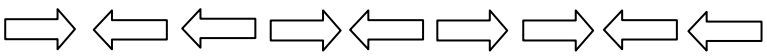
In dynamic (D) situation:

Level D1: One can distinguish the two sides of a street if he or she knows the direction of walking on the street.

Level D2: One can interpret expressions turning left or right well, which are a problem linked to cyclic orientation on the plane. The concept of left-right turn is closely connected to the concept of the left-right side.

Level D3: Pupils task has two parts: understanding a sketch of a part of a city, and after a mental image verbalizing the travel. Verbalizing and especially writing brings new difficulties for them.

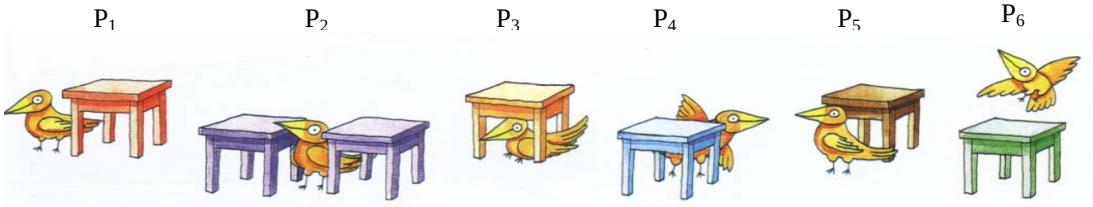
We show some exercises about these two subtopics to give an overview of the whole experimental process. Problems shown below represent the three levels. The tasks were parts of the exploratory study (I.a.) and the post-test (III.).



S1: *Colour the arrows showing right!* (Level S1, in I.a., Grade 1-4)

S2: *Which hand holds up the boy?* (Level S2, The first and third pictures in I.a., Grade 1, the five pictures together in I.a. and III., Grade 2))

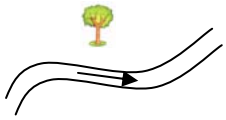
S3: *What is the position of the bird in relation to the chair? Make connections between the pictures and the adequate words!* (Level S3, in I.a., Grade 1-2 and III.)



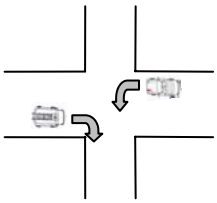
BELOW	ABOVE	IN FRONT OF	RIGHT	LEFT	BETWEEN	BEHIND

We show three problems representing the three dynamic levels.

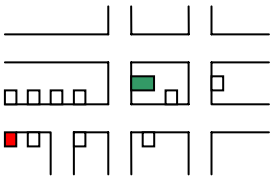
D1: *On which side of the street is the tree, if the arrow shows the direction we walk on the street?* (Level D1, in I.a., Grade 2-3)



D2: *Colour the car which turns right!* (Level D2, in I.c., Grade 1-2, and III.)



D3: *On the picture there is a part of a city-map. How can you go from the red house to the green school? Write a few sentences, like Go straight!, Turn left!, Turn right!, On the right-left side of the street …* (Level D3, in I.a., Grade 2-4, and III.)



Teaching experience:

Before referring to the results of these six problems, we give an overview of the experimental course for second graders of the school associated to the college (In autumn 2005) with respect to these problems.

We planed a ten-week course, with twenty special orientations problems. The pupils had 4 mathematics lessons every week; we took part on two lessons per week with two exercises. This way the teacher didn't need to modify her curriculum, we could insert our program into the accustomed teaching-learning process. The exercises took 8-12 minutes out from the lessons. Between the twenty orientation tasks, there were 3 for using words to describe spatial orientation, and 3 for describing routes, furthermore tasks for cyclic order, coordinate-system, geometrical transformation and front,-side,-top-view of an object.

By composition of tasks we took the results of the paper-pencil tests and interviews into consideration.
By design of the experimental phase we respected the representation theory of Bruner. (Ambrus, 1995) Bruner suggested three ways of transforming experiences into a model of the world: the enactive, the iconic and the symbolic representation. The thinking process can go
- enactively: through concrete actions with concrete objects,
- iconically: through pictures, diagrams, imagined situations, or
- symbolically: through mathematical symbolisms or words.

In primary school the enactive phase is very important. In case of polarities it means, that pupils must be confronted with real situations not only at home but in the classroom too.

To clarify the inconsequences with respect to the non-living objects which have or don't have a "face" we used three geometric shapes (circle, triangle, square), objects, which don't have a face, then we play the bird-chair situation, also with objects which may have a face. In the third case pupils must form a figure from geometric shapes based on the instructions of the others (Like *Take the square below the circle!...*)

We are convinced that the main problem is not to distinguish left and right side, but to know the relating object or person. In the course we had discussions with the pupils about this question, and agreed about the fact that in every case we shall tell the relating point. So in the post-test we completed tasks **S3** with the following sentences:
S3: *Decide the left/right directions relating to own body!*

We dealt with the dynamic situations in the following way:

First we played a robot-game. A pupil, the robot, went out from the classroom and the others hid an object. The robot came in, and followed the instruction to find the hidden object. We recognized two important problems. Pupils changed the left-right turn very often, and their estimations regarding the number of steps necessary to reach a corner or the object were wrong. Fortunately the robot was very capable and performed exactly the given instruction. This was a good exercise to understand the meaning of the left-right turn related to another body. We practiced the mental travel in a well-known context, in the building of the school. Pupils had to explain the route for example from the classroom to the school-library. Finally they practiced the orientation, the description of a route with the help of a top-view of a little town. They answered questions like *How can I go from the post to the supermarket?* and so on.

Some results:

We show and compare the results of the exploratory study and post-test.

The table shows the ratio of the good solutions, regarding tasks S1-3 and D1-3.

		S1	S2	S3	D1	D2	D3
Expl. study	**Grade1**	81%	45%	51%		35%	
	Grade2	92%	67%	33%	68%	63%	38%
	Grade3	96%			81%		41%
	Grade4	97%					45%
Post-test	**2B**		92%	84%		84%	64%
	2D		42%	42%		83%	54%
	2T		55%	25%		65%	30%

Task **D1** was quite successful in the exploratory study, so we left it from the post-test. **D3** was a very unsuccessful complex task in the first pre-test, so we picked a part (**D2**) from it and took it into the second pre-test. **D3** was the same in the post-test as in the pre-test, pupils in class 2B was managed such a kind of verbal situations, but the others did not.

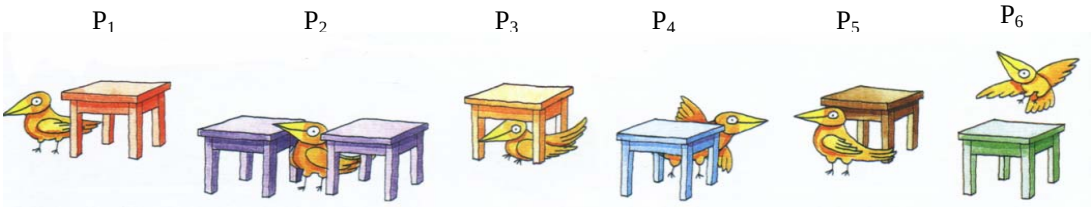
Interpretation of the results:
S1: The task was given only in the exploratory study. At the end of the school year we see a relatively big difference between Grade 1 and Grade 2. This year is the time of the consolidation of reading and writing. These activities assist the identification of the directions on the paper. We mean left-right and above-below directions.
We can see also, that after Grade 1 pupils can distinguish left-right very well, but a little hesitancy remains every time.

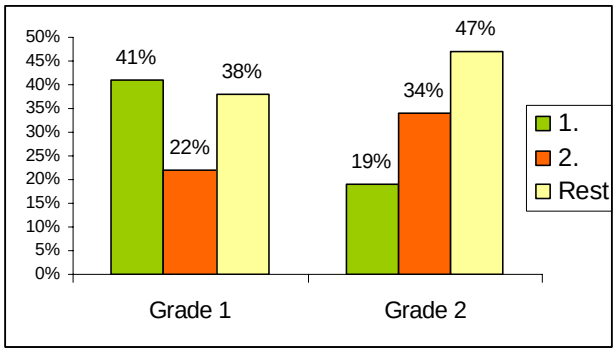
S2: For Grade 1 the exploratory study contained only the first and third pictures. The table shows only the results of the first and the third picture, because we can compare them only in this case with Grade 1. We can establish that the experimental course (2B) was successful, the solutions of class 2B were almost correct. In addition to the whole task we can say that ratio of the good results was almost the same, the classes 2D and 2T were below the level of Grade 2.

Studying the mistakes we can't observe any consequences; we assumed the forth and fifth pictures are easier then others, but the solutions didn't confirm our hypothesis.

We have possibility to compare the own development of classes 2B and 2T. The ratio of the good solutions in the case of class 2B, in the exploratory study was 68%, in the post-test 92%. In the case of class 2T 24% and 55%. It seems to be, that the spontaneous development of class 2T is remarkable, but the percent of the wrong answers is nearly 50 %, by Class 2B only 8%.

S3: In the post-test we inserted the *Decide the left/right directions relating to own body!* instruction. After evaluating the exploratory study we could establish that two most frequent answers were, in order of the pictures:





1. P_1: *left*, P_2: *between*, P_3: *below*, P_4: *behind*, P_5: *in front of*, P_6: *above*
   P_1: *right*, P_2: *between*, P_3: *below*, P_4: *behind*, P_5: *in front of*, P_6: *above*

We can nevertheless see that the ratio of other answers is quite high. We can not highlight anymore concrete answers; they showed a very great deviation. Pupils did not have problems with Picture 2, 3, 6. More than 92% of them gave the correct answer for the polarities above-below, and between. Picture 1, Picture 4 and Picture 5 were not so easy for them. The table shows us the case of Picture 1. There are no significant differences between answers right and left. The rest of the answers were mainly *in front of* (approximately 10%).
Picture 4 and 5 showed us not so big differences, but we can see more hesitancy then in case of Picture 3 and 6. The "wrong" answer was mainly *left or right.*
We also saw this left-right hesitancy in the experimental course as well, so we made the conclusion, we have to determine the position of the observer in every case, if we want to get an unambiguous answer.

		Left	Right
Expl. study	**Grade1**	51%	32%
	Grade2	33%	52%
Post-test	**2B**	84%	12%
	2D	42%	50%
	2T	25%	45%

The result of the class 2B is very good; the result of the class 2D and 2T is the same as the average result of Grade 2. It seems that the reason of the solution of 2B is not only what we determined as the relating point, but the practice.

D1-D2: The results illustrate well the distinction between the two levels, and between the static (**S1-2**) and dynamic (**D1-2**) situation too. Comparing the results of **S2** and **D2** in the case of class 2D and 2T, we can note that we need more exploration to establish which task is easier.

D3 was a complex task; we could evaluate not only the left-right answer, but the competence of writing a route. The solutions show the expected results: With the help of practicing this competence is can be improved.

Conclusions

We came back to our research questions:

1. What types of difficulties can we find by the use of words of spatial relations studying the thinking processes of pupils of grade 1-4?

There exist at least two different situations. First is the orientation in the space, second is the transferring the spatial orientation-problem into the plane. We investigated the three main polarities: *left-right, above-below, in front of-behind*. The *above-below* caused no problems, thanks to gravitation. The other two pairs, especially the *left-right* direction is more problematic. Our experiences show us that the main problem is choosing the relating point in every situation. It is not enough if we say how we can relate to our own body or to the object, without practice this information means nothing.

The problem solving in a classroom situation (e.g. on the enactive plane) was more successful, then in a paper-pencil test (iconic plane). In our opinion in a real classroom situation we can ignore the problem of transferring the space into a plane, and we can concentrate on the spatial situation.

Studying results of "our" second graders we can conclude that communication, we mean verbalizing, giving or understanding instructions, speaking about a route is developed through practice.

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