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CONTEXT CALLS ON AXIOLOGY

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The crisis of universality has caused a diffuse bewilderment among the teachers of mathematics, because it questioned the value of axiomatisation. We use the “fragment” constituted by paper folding in order to propose an axiological perspective, in which the traditional inventiveness of Italian school meets the “optimal” formation of Gal’perin and Davydov.

Our work is a reflection that comes into being from the perception of a diffuse bewilderment among the teachers of mathematics, a disorientation that we believe is engendered by the challenges that the changes of all-engaging society pose to teaching mathematics. By means of the “fragment” constituted by paper folding activity we intend to open a path that allow to go beyond this bewilderment that influences negatively the teaching.

PAPER FOLDING

Since the year 2000 we are carrying out activities of paper folding in some classes from sixth to tenth degree. We already have presented a report about these activities at the CIEAEM57 in Piazza Armerina and now the same activities collected in a book (Locatelli and Rottoli).

The particular type of paper folding we experienced in our classes goes directly by the book of Sundara Row “Geometric Exercises in Paper Folding”, published, in XIX century, in India. This book possesses the important characteristic to plunge the western, substantially Euclidean, geometric knowledge, in an oriental way of geometrising, based on transformation of geometric figures.

At the same time we believe that this our proposal fits into the Italian tradition that started already in the first part of the last century, thanks to Emma Castelnuovo, and then developed by contributions of many other researchers and teachers, as to as acquire the properties of a specific school of didactics of mathematics.

Paper folding reveals the rich potentialities of language of body, that strengthens the Emma Castelnuovo’s idea that “the geometry is something that is made by hands”. Gestural expressiveness of paper folding involves listening and respect of the rhythm of body. It permits to disclose and to recover through the rhythm of gestural representation, the modalities of thinking according to their time of learning, that is to historicize the procedures, to bring back the attention on single moments of the process and so to put in evidence those steps and those references that form its texture.

Paper folding must be seen as an instrument that broadens the semantic registers that are associated to geometrical knowledge (Duval): it is poor in comparison with the enormous possibilities that technology puts at disposal; but differently from the hurry and from the instability sense that sometimes technology seems convey, it distinguish itself for the peculiarity of its specific temporal scanning. This latter has a strong didactical meaning because it permits to rediscover the rhythm of corporeity.

Here we outline the value of rediscovering the rhythm of corporeity in a teaching that aims to guide students in approaching the culture of scientific argumentation. It is important that student is brought “to formulate conjectures, to investigate their contents, to discover their limits of validity, to identify possible arguments of validation” (Boero), by using the exploration that is made possible by programs of dynamical geometry.



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At the same time the slowness of manual exploration may become the opportunity to mark by the actions the times of the procedures, setting so a rhythm that gives form to argumentation. In doing this, it is important to make use of open discussions and to organize written texts, discussion and text that had a central role within Emma Castelnuovo's proposals.

THE AXES OF A QUADRILATERAL

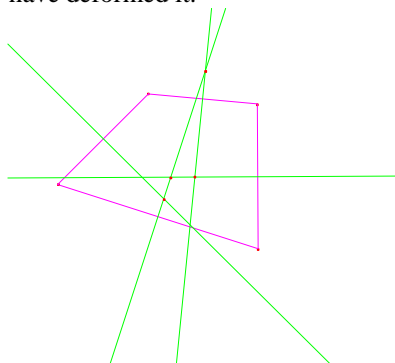
As example we briefly present one of the made activities. It concerned with the axes of a quadrilateral and it has been proposed after that students have found that axes of a triangle intersect in a unique point.

- The activity started from the following question, posed by teacher: *“Do the axes of a quadrilateral intersect themselves too in an unique point?”*

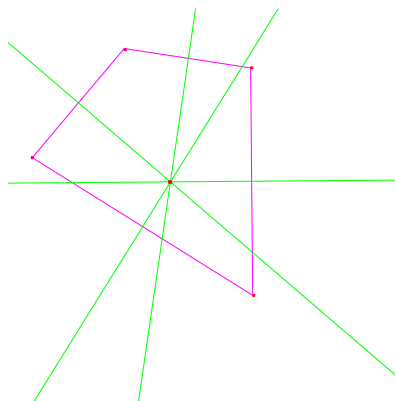
Students fold different quadrilaterals (square, rectangle, rhombus, generic quadrilaterals, etc.), and discuss the behavior of axes in each figure.

- Then teacher posed an another question: *“You have seen that in some quadrilaterals the axes intersect themselves in an unique point, while in other quadrilaterals this fact doesn't happen. Which property must a quadrilateral possess in order to their axes intersect in an unique point?”*

To answer this question, students made use of the dynamical exploration that is made possible by a program of dynamic geometry: after to have represented a generic quadrilateral, they have drawn the axes of its sides and then they have deformed it.



In this way they obtained some quadrilateral in which the axes intersect in an unique point: in a rectangle, in isosceles trapezium, for example, but also in many other generic quadrilaterals.



- “Which property must a quadrilateral possess, in order to their axes intersect in an unique point? Sign the amplitude of angles of quadrilateral”

By deforming the quadrilateral and discussing the forms in which axes intersect in an unique point, students easily discovered that the property that these forms have in common is that the sum of the opposed angles is 180° .

- Now students reconsidered the proposed questions and elaborated a written text in which they had exposed the argumentations that justifies given answers.
- The teacher had cure about the all practical activities, made both by paper folding and by the program of dynamic geometry, should be used in scanning the “rhythm” of the argumentative procedures. (Locatelli and Rottoli)

BEWILDERMENT

Two circumstances are characterizing actual Italian situation of teaching mathematics

- *An exciting experience is exhausted: during last decades of XX century, the great project of enforcement of compulsory schooling was accomplished in relatively brief times, even through many contradictions. Certainly the Italian school of didactic of mathematics have contributed in a determinant way to carry out this project. After that, it is as if a true unifying project is lacking.*

- *At the same time, deep transformations that are crossing society have generated the crisis of universality, a crisis that seems influence in a destabilizing way the process of determination of a unifying project.*

The crisis of universality involves closely mathematics, because the link between mathematics and Western society resides just in the idea of universality: since the beginning, mathematical truth becomes necessary for the “universe” of considered objects, a prototype of rationality and a chance of discovery.

This crisis questions the original value of the axiomatization (Longoni, Riva, Rottoli); it interlaces with the difficulties of science in front of the complex systems and, moreover, it must reckon with technology, the main product of interaction among Western society, modern science and mathematics, that is pushing to globalisation. So the idea of universality of Western society has to face the active presence of other cultures that makes relative reason, criteria of truth, values and rights: “the loss of safety”.

The crisis of universality produces a sense of bewilderment in teachers of mathematics too, at least for two reasons:

- Teachers perceived the loss of centrality of axiomatisation. But the centrality of axiomatisation has turned the abstraction in priority with regard to any context: “In mathematics, context is the part of an unimportant detail, that must be made evaporate upon the flame of abstraction, so that the crystal of the pure structure is made evident.” (Moore and Cobb). So, in teaching mathematics, axiomatisation had formed a sure reference. The loss of centrality of the axiomatisation, brings teachers to feel the nature of taught mathematics as fragmentary.
- In actual domination of technology the crisis is the more deep the more the technical use of mathematics is lived as obvious. This obviousness is dragging the teaching of mathematics to standardization, with the risk that, in Italy, the inventiveness and the more innovative and meaningful aspects of the “Italian school”, sparked by Emma Castelnuovo, are sterilized.

CRISIS AS POSITIVE OPPORTUNITY: AXIOLOGICAL ATTITUDE

“Where hazard is, there salvation” (Hölderlin). All crisis is ambivalent. It is a loss but also an opportunity of revolution, that is, a possibility that the unforeseeable, the unheard, the unhoped-for comes on stage. (Longoni et al.)

Paper folding is a “fragment”. But “the whole stays in the fragment”: fragments represent the beginning of new courses, the possibility of new choices, the opening of new directions.

The specific values of paper folding, that is, to reconsider and to bring back the attention on single moments of the process of learning, to rediscover the rhythm of corporeity, to recover modalities that are proper to thinking, transform it in a didactic process that put attention to directions and to meanings that are inherent in acts. For this reason we speak of axiology.

The term “axiology” shares the root with the terms axiom, axiomatic, axiomatisation. Their etymon lies in the Greek term “*axion*”, whose meaning is : *dignum, id est worthy, what is of value.*

Axiological attitude acquires sense to the extent that it permits to open the fragment to a wider breath. It is distinguished from the axiomatic attitude.

- In axiomatic attitude, choices and acts are referred to a well defined whole. They characterize themselves by presumed possess of totality, by referring to the “One”, upon which the entire didactics must be structured.
- Axiological attitude is rather characterized by the sharing of the idea of “uni-versality”, understood as aspiration towards a not-possessed horizon and, then, opened to the different and to the new. The idea of approach to a culture is intrinsic in it: a culture that grounds its strength upon its scientific character and, at the same time, opens itself to different contexts, living the care of daily practice.

AXIOLOGY AND “OPTIMAL” FORMATION

A first attempt to achieve an axiological attitude in Italian teaching of mathematics, is, in our opinion, to try an integration between the inventiveness of Italian school of didactic of mathematics and the Gal’perin’s idea of “optimal formation of concept and mental actions” (Haenen). This idea of optimal formation found its more meaningful application, as far as the teaching of mathematics is concerned, in Davydov experimental school that operate in Moscow during sixties. These two schools, the Italian one of Emma Castelnuovo and the Russian one of Davydov, both possess different but meaningful, innovative elements. They operated at the same time but they don’t “contaminate” between them in an adequate way, perhaps because in one the geometric aspects were prevalent, while the other has privileged the arithmetic and algebraic aspects.

An adequate contamination between these two schools should consist in renewing the proposals of Emma Castelnuovo by directing them in a perspective of optimal formation. As the paper folding is concerned, we identified the optimal formation in giving it in an unitary and coherent organization by tracing the teaching back to “five fundamental facts”. (Locatelli, Rottoli).

Paper folding is a fragment; but it opens “directions of sense” concerning axiological perspective.

Paper folding holds a particular form of “historicity”, when it runs through past moments of Italian teaching, in order to rediscover what has been neglected, forgotten, or underestimated. Here “to historicize” means to recall moments and tools of recent past didactics, in order to recover didactical values that an excessive hurry could make forgotten and that new contexts may point out.

In paper folding we propose a solution to the question of “optimal” formation. But “optimal” formation has different answers for different fragments; besides, answers must be conjugated with specific contexts. Axiological perspective acquires a particular meaning as search of “optimal” formation in real contexts. So it needs that mathematics education research recovers a firm relation with the teaching in “real” classes.

RESEARCH AS “AUTHOR”

Sometimes research forgets real classes. This is perhaps one the most important causes of the detachment between mathematics taught and research: «... Claims could be heard time and again that mathematics education research is “not very influential [or] useful”...» (Sfard).



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Many activities that are brought back to Emma Castelnuovo's proposal, are suitable for recovering, in each context, opportunities for curiosity, exploration, choices. But this attitude risks to scatter in manifold "monological voices", that doesn't interact.

A relevant duty of research is to have the same function that "the author" has in Bakhtin, that is to put in resonance these "homophonic" voices and to generate a dialogical action, a polyphonic orchestra that opens new meaningful horizons in teaching. Teaching and research can find an "optimal formation" that accounts for the complexity of contexts and make "the texts spoken among themselves" (intertextuality or, better, interdiscursivity), in "of a great time", as Bakhtin calls a work that absorbed the problems of its contemporaneity and of its past. The texts, that is, classes, teachers and researchers meet themselves through the principle of cooperation, that is the reciprocal disposition and assumption among interlocutors, in a dialogue that overcomes closing and one-sidedness.

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