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MULTICULTURAL MATHEMATICS CLASSROOMS: WHEN THE DIFFERENCE CHALLENGE WELL ESTABLISHED IDEAS

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In Srni, July 2006, I presented a reflexion on the work that the EMiCS[1] group (*Educació Matemàtica i Context Sociocultural* – Mathematics Education and Sociocultural Context) had been developing until then in multicultural mathematics classrooms. Even if it included some of the ideas that we had already published by then, it was intended to be more than a summary. It was my purpose then, as it is now, to use our own work to show how the reality of the multicultural classroom has been a challenge to the theoretical ideas that were underlying our approach to the research domain. Again, I want to thank the organizers of the conference, and in particular Paolo Boero and Juliana Zsendrei, for having given me the opportunity to share my ideas with you. I also want to thank Filippo Spagnolo for the opportunity to have my thoughts written down in the proceedings of the conference and for his insistence, and patience until I finished the writing.

Since 1998, then still under the acronym MUMA (*Multiculturalitat i matemàtiques* – Multiculturality and mathematics), the group has been researching multicultural mathematics classrooms with the support of different institutions, administrations and research agencies. Many changes have taken place since then. The group has evolved as have the ideas, since they are constructed in interaction with people. The social and educational contexts in which the ideas have arisen have also varied. The legal regulations for immigration in Spain, as well as the educational framework in Catalonia, have been modified. Nevertheless, in this paper, I will mainly keep to my notes from Srni with the conviction that the reflexion that was the thread of the presentation then is still valid today.

The focus of my paper will be on: a) how some students in multicultural mathematics classrooms challenge 'normality' by challenging well established norms; b) how multicultural mathematics class-

rooms challenge the idea of norm in itself; and c) how our research challenges the well established understanding of the construct 'norm'.

There is a basic assumption underlying the ideas presented in the paper. It is through classroom practice that the opportunities for immigrant students to participate are opened (or closed). Therefore, classrooms may become both a space for inclusion and exclusion. Representations play an important role when norms regulating participation are at play. Therefore, some dominant representations, in some way or other well spread among educators, should be questioned. It's the possibility of changing exclusion into inclusion that leads our research forward.

Our efforts to understand the complexity of the multicultural mathematics classrooms in our studies lead us to reconstruct the idea of norm, upon the basis of social representations of those involved in the action. Researchers' representations also shape which reality is researched and how. In particular, therefore, reflexion on our own research process leads us to propose some issues for a new research agenda.

Challenges for schools

Rapid changes in society inevitably have an impact on school. Globalization comprises migration and people mobility. The complexity of a multicultural classroom raises many questions related to issues concerning equity and justice. Everyday reality shows the inadequacy of the educational provision in schools and classes which could be thought of as highly multicultural. In particular, it is not clear whether mathematics education is changing as fast as the students' population evolution would require in adapting itself to the new reality.

There is a discontinuity between the new demands from society on schooling and what school may offer to society, and this is particularly true with regard to students that are 'different'. To promote educational changes based not only on social or individual commitment and political good will, but developed on a solid basis, these new demands should contribute to establish the research agenda for mathematics educators and mathematics education research.

Immigration into Spain is a new, intense and growing phenomenon. Until recently, when the immigration rates seem to have stabilised, there had been an increasing immigration into Catalonia which has

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led to significant changes in the school population. Moreover, one has to take into account the demographic characteristics that increase significantly the immigration impact. While the Spanish birth rate is decreasing significantly, the children of immigrants fill the baby units in Spanish hospitals.

At present, more and more communities and schools are, and will be, facing a multicultural reality. Even when the immigrant students do not join the regular classrooms until they master a minimum of the Catalan language, the cultural difference within a class attended by immigrant students is still a reality.

What's a multicultural mathematics classroom?

The ideas that we may all have of a multicultural mathematics classroom are diverse. They are linked to our own images of culture, mathematics and learning. However, one clear way in which we can detect cultural diversity in classrooms with immigrant students is through a consideration of the use of different cultural artefacts, such as culturally distinctive algorithms or symbolic representations of numbers that mediate students' cognitive processes.

Nevertheless, when thinking about culture, one cannot forget about other symbolic artefacts that mediate the learning process such as meanings and values associated to the physical artefacts and to the learning process itself. Moreover, the culture of an individual has not only to do with their ethnicity, language, country of origin or social background, even though these are elements that are commonly understood as constitutive factors of the individual's culture. The culture of an individual has also to do with the many different smaller groups to which he or she belongs. For instance, cultures based on age, on place of living, on activities –both work and leisure time–, may crosscut with cultures related to gender, race or social class.

Therefore, from that point of view, any mathematics classroom is a multicultural classroom. However, in this paper, I wish to focus on classrooms that most schools and teachers would label as multicultural, classrooms that have students with a background distinct from the mainstream in terms of culture of origin. This includes classrooms serving recent immigrant communities and established minority ethnic groups.

From now on, I will use the term "immigrant students" to refer both to them and to the children born into immigrant families. Moreover, I will improperly employ the adjective immigrant by making use of a dominant representation. By "immigrant", I will not strictly recall the meaning that could be given by a dictionary as "a person who comes to live permanently in another country". The "immigrant students"

in the schools in our studies are not the children of the wealthy families that came to Barcelona because the parents work in big international companies. These would be named "foreign children" making use of another dominant representation. By "immigrant children" I will refer to the children of those that came from abroad as part of a working force in search of a better life. Therefore, sometimes in the scenarios of our studies the cultural and social distances are difficult to disentangle.

Our representations of a multicultural mathematics classroom may also be different from those of the teachers serving them. For example, in our investigations we often visited mathematics classrooms where teachers have not thought of them as being multicultural, despite their being multiethnic. For them, one of the reasons was that since mathematics is 'international' (as it was said to us by one of the teachers during an interview) the cultural background of the students was not important. This teacher thought of the students' diversity in the learning processes only from a cognitive perspective.

Challenging 'normality'

Probably, the most significant impact of immigration into school has to do with differences in the social dynamics of the educational situation, both at the school level and at the classroom level. Mathematics classrooms do not escape these differences. When talking about differences in a social situation, one means differences from "normality", where normality is defined according to the assumptions and expectations of the individuals concerned. Moreover, "normal" often does not even correspond to the idea of statistically frequent.

Thus, mathematics teachers find immigrant students to be "different" from what they expect their pupils to be. Immigrant students find their mathematics teachers "different", as they find different the dynamics of the mathematics classroom and the school they are in. The interactions among students and between students and teachers are also culturally "different" as are their interpretations of the needs and reasons to learn mathematics and the way it has to take place.

First of all, the different cultures of the students within the multicultural mathematics classroom become visible through the use of different cultural artefacts, such as different patterns for the written algorithms. Different school traditions, for example, present the written algorithm of the division in different ways. This is true not only among countries with different cultural roots, such as The Netherlands and Spain (see figure 1), but among Spanish speaking countries, such as Chile, Ecuador, México and Spain (see figure 2).

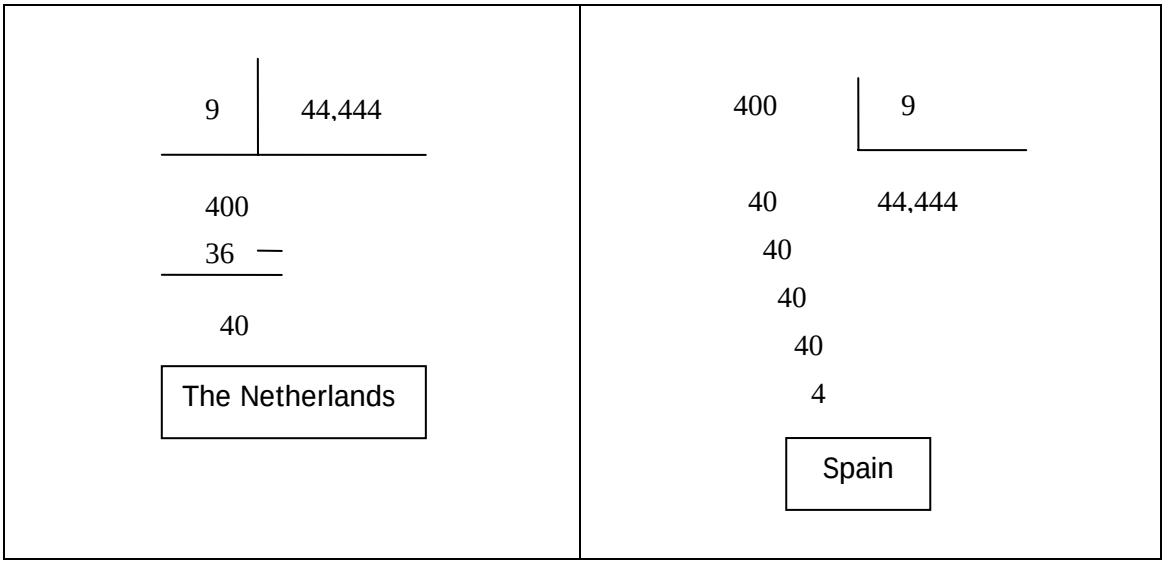

Figure 1: The division 400:9 done by someone from The Netherlands and someone from Spain

The fact that divisions don't look the same in various parts of the world could be considered as anecdotic. However, the difference between divisions with decimal numbers in Equador and Spain was the origin of David having problems during his mathematics lessons.

David is a boy from Ecuador that arrived in Barcelona at the age of 10. He was a good student there, had no difficulties with the basic algorithms, and had already learnt decimal divisions. When joining his new class in Barcelona the teacher was showing the students how to work out divisions with decimal numbers. Since he had already learnt it otherwise, he did his homework his own way. However, his teacher marked all his divisions as wrong, without even paying attention to the details of his homework.

The issue here is not whether divisions look the same in different national school traditions, but how teachers react to these differences and how this has an impact on the students' learning opportunities –for a full account, see Gorgorió and Abreu (2009) or Gorgorió and Prat (2009). The challenge has more to do with meanings, values and attitudes than with the actual use of a particular artefact. Teachers' representations on mathematics shape their understanding of their mathematics teaching. The representation of mathematics as being 'universal' is so strong among some teachers that they are not challenged even by strong evidence.

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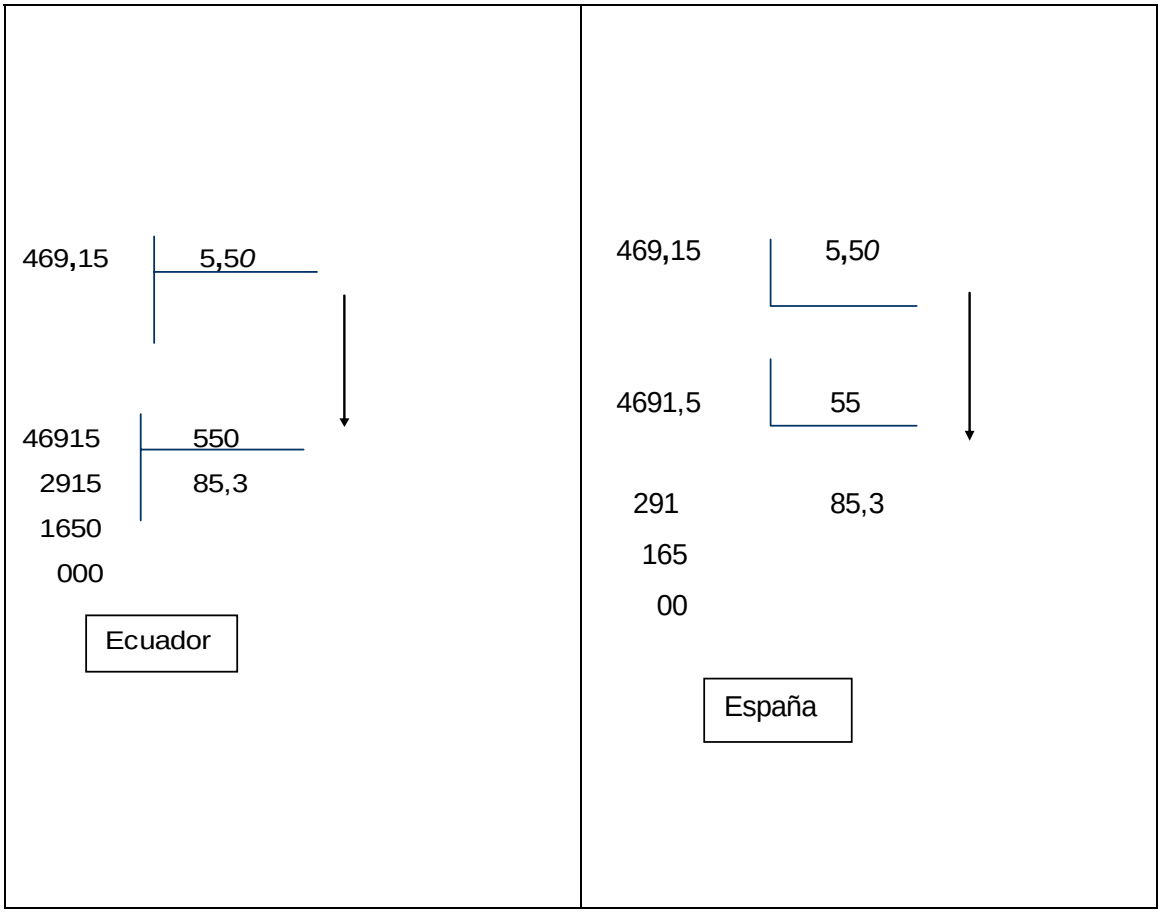

Figure 2: The division 469,15: 5,5 done by someone from Chile, México, Ecuador and Spain

The focus in David's case must go beyond the simple consideration of the fact that mathematics is a cultural product and that existence of different cultural artefacts, like number, symbols or algorithms must be taken into account. David's case show us that there is an issue linked to people's perception of what is or should be the proper way of doing mathematics, of acting and interacting within the mathematics classroom and to their different expectations regarding their mathematics learning.

In front of a multicultural mathematics classroom, an observer may possibly be challenged by unusual –different from their own usual– forms of algorithms, or by the many languages spoken in it. However, the co-existence of different ways of understanding the teaching and learning of mathematics,

and how it should take place, are more subtle and more significant ways for the diverse cultures to be present.

Challenging 'norms'

When beginning our research, more than a decade ago, we approached the multicultural mathematics classroom using the construct norms as it had been presented by Cobb and Yackel and other colleagues and has been widely used since. Cobb, Yackel and Wood (1992) introduced the idea of social norms as social constructs that involve a taken-as-shared idea of what constitutes an appropriate contribution to a discussion. The term socio-mathematical norm was coined by Yackel and Cobb (1996). Socio-mathematical norms have to do with the actual process by which students and teacher contribute to a discussion. They designate the classroom social constructs specific to mathematics that individuals negotiate in discussions to develop their personal understandings, and are the result of legitimating explanations and justifications. Socio-mathematical norms are also understood upon a taken-as-shared basis.

However, we soon realized that norms did not fully allow us to interpret what we were observing: different understandings of the same norm within a mathematics classroom were difficult to reconcile, and could certainly not be taken as shared. Norms and practices within the classroom were in conflict with those of the students' immediate contexts. The immigrant students in our classrooms had a different way –different from that of the local students and teachers– of understanding, valuing and using mathematics, differences that gave rise to obstacles to their opportunities to participate in the classroom activities.

In the multiethnic classrooms we studied, there were different perceptions of a particular contribution as 'being acceptable or desirable', a fact that was causing obstacles to communicative processes. The main issue was not to reach consensus on, for instance, what constitutes mathematical evidence, a good hypothesis, or a good explanation. In the classrooms of our study, where immigrant children from different parts of the world were expected to work together with local children, there were other more basic, or prior, issues, also related to mathematics learning, on which agreement was needed. The meanings and values associated with mathematical knowledge and who is mathematically knowledgeable, the expected role of a mathematics teacher, the working organisation within the mathematics classroom, or the idea of learning mathematics itself, were at the basis of the difficulties in the interaction processes and were not in the least 'taken-as-shared'.

Norms have arisen to develop teaching experiments whose goal was to institutionalize social and socio-mathematical norms characteristic of inquiry instruction (see, for instance, Cobb, 2000 and Yackel,

2001). We were using them to analyze multicultural mathematics classrooms as 'natural settings', just like external observers that were trying to understand the classroom micro-culture as it existed. This could be a first explanation why the classrooms we were observing through the theoretical lens of norms were seen as 'distorted'.

Since the immigrant students in our classrooms had a different way –different from that of the local students and teachers– of understanding, valuing and using mathematics, we could not longer think of norms upon the basis of taken-as-shared. We were in search of constructs that explained the complexity of the multicultural mathematics classroom, but that at the same time understood it.

Why do I make this distinction between 'explaining' and 'understanding'? To study multicultural mathematics classrooms, we could have used any of the available existing theoretical constructs that focussed on the aspects we were interested in. In fact, this is what we initially tried. However, we soon discovered that under some of the existing theoretical lenses our object of study was seen as 'distorted'. From our perspective, it was not that the reality we were studying was 'abnormal', but that the theoretical lenses we were using were not adequate to look at it. Therefore, we needed to adapt the lenses to the reality we wanted to scrutinize. That's why I refer to constructs that were helpful 'to understand' the multicultural mathematics classroom and not only 'to explain' it.

The classroom micro-culture developed under certain socio-mathematical norms has been fully studied in teaching experiments. However it was not clear to us how norms were established, how they could be agreed on, negotiated or changed or how the 'desirable' or 'acceptable' was established in 'natural settings'. We needed a construct that not only explained what we were observing but also that could 'understand' and 'accept it' as it was taking place. The issue then, for us, was that the meaning of the word social in the social and socio-mathematical norms needed to be revisited.

From a socio-cultural perspective the learning of mathematics is affected by what takes place within the classroom and in the nearest contexts. The mathematics classroom is a social micro-context, which is embedded within a social-macro context. We could no longer understand the word social as simply 'being conjointly constructed by the different participants in the classroom', without considering that all participants were, in turn, social individuals, with their own social and cultural experiences and expectations. Could we then still regard norms upon a 'taken-as-shared' basis?

Reconstructing norms

The idea of 'social representations' (Moscovici, 1983) plays a significant role in our reconstruction of the concept of norms. We understand social representations to be an interpretative framework (Zittoun et al., 2003) that allows people to make sense of and organise their reality, both social and physical, and to relate to other people and groups. They are reconstructions of reality, arising from communication between individuals; reconstructions which, in practice, regulate behaviour between and within groups.

Social representations focus on, select and retain certain relevant facts of reality, according to the interests of the individual as inserted within a group. Selected aspects of the object of the social representation develop into an implicit theory that allows individuals to explain and assess their contexts. They constitute an operational guide to understanding complex or difficult situations, to facing problems and conflicts, to coping with unexpected realities, to justifying actions and to maintaining differences between groups when these differences seem to be fading. Social representations are neither directly based on scientific knowledge, nor necessarily verified by means of empirical facts.

Our reconstruction of norms focuses on their social weight. The group's social valorisations shape the values, expectations and emotions of the individuals who identify themselves with it. When the teacher calls on a certain norm, and the students tackle it, they all bring to the process their own interpretation of a social understanding about mathematical knowledge and mathematical knowledge ownership, and a social valorisation of mathematical practices. Broader social structures, like the educational system, impact on classroom interactions through implicit messages about what are the legitimate norms within the classroom.

In our reconstruction of norms we refer to socio-mathematical norms and to norms of the mathematical practice as regulating actions and interactions within the mathematics classroom: socio-mathematical norms when taking into account the individuals' and groups' social understanding and valuing of mathematical knowledge; norms of the mathematical practice when considering the individuals socially interacting with specific mathematical knowledge.

We refer to socio-mathematical norms as the explicit or implicit regulations that influence participation within the mathematics classroom and the interactive structure of the development of the mathematical practice. They have to do with how the different participants value mathematical knowledge, and value and position themselves, the others and their group(s) with regard to mathematical practice(s) and knowledge.

A sociomathematical norm explicitly stated by a teacher could be, for instance, *'In this class we work collaboratively and people must help each other'*. When stating it, the teacher resorts to her under-

standing of an appropriate way of working in the mathematics classroom, which may come, for instance, from the collective image of a particular school culture. When putting into action this norm, the teacher has to take decisions about how to organise the students in small groups, and in doing so, s/he is borrowing meanings and values from the cultural scripts and social representations of a particular group. Too often, the fact that a student identifies himself with the group where he has been assigned, or the fact that the group accepts him within it, has little to do with his real mathematical abilities, but with their interpretation of the social representations of what constitutes 'doing mathematics'.

Note that we consider as socio-mathematical norms some of the norms that in Cobb's system would be regarded as social norms. We agree with Cobb and Liao Hodge (2002) that mathematics teachers, as well as history teachers and science teachers, may want students' participation. However, norms about participation in the mathematics class have other meanings and consequences than the same norms in history or science classes. The way teachers conceptualise the learning of mathematics constrains the prevalence of one norm over another. When establishing, for instance, *'who needs to work with whom'* or *'who can benefit from a particular participation structure'*, mathematics teachers base their decisions on their conceptualisations of what teaching and learning mathematics is about. Their conceptualisations are unavoidably shaped by cultural scripts, social representations and valorisations of mathematical practices and of social groups in relation to mathematics.

We regard norms of the mathematical practice to be the norms that legitimate the mathematical activity, strategies, processes and certain ways of thinking within the classroom. They have to do with the rules and ways of doing of mathematics as a scientific discipline, and with how teachers and students interpret mathematics as a school subject. When teachers decide whether a content, algorithm, procedure, task or strategy is appropriate as school mathematics, they borrow their meanings from the culture of the groups they are part of, be it an innovative association of teachers of mathematics, or a group of mathematicians educated in a certain way. They also borrow their meanings from the culture of the educational system and from their particular school cultures.

The official intended curriculum, the syllabus and the textbook also convey to teachers cultural scripts of what constitutes school mathematics. Students interpret what mathematics is about through the lens of the culture(s) they have participated in throughout their lives, be they the classroom culture(s), the school culture(s), the group culture(s) or, their home culture.

A norm of the mathematical practice explicitly stated by a teacher could be, for instance, *'In this classroom, a visual strategy is also a proper strategy to solve a problem'*. The teacher may have a particular understanding of mathematics, while her students may think that visual strategies are not 'proper

mathematics' because they have never seen such a strategy used before. Another teacher may prefer an approximate solution while her students believe that 'exact' answers are more 'real mathematics', because this is part of their cultural scripts of what counts as mathematics, scripts brought from home or from their previous school history.

Again, note that we would consider as norms of the mathematical practice those relating to what constitutes mathematical evidence, a good hypothesis, or a good explanation, norms which according to other authors would be regarded as socio-mathematical norms. I would also like to make clear that, although it is usually the teacher who explicitly states the norms, very often norms are established in implicit and less clear ways.

The potential gain

We ask ourselves where lies the significance of revisiting a well established construct, such as that of norms. On the one hand, one of the interesting aspects of our reconstruction of norms is that it contributes to enlarging the set of theoretical constructs that not only allows explaining the multicultural mathematics classroom but also takes into account its particular features.

On the other hand, having broadened the empirical domain where the construct may be usefully applied contributes to the consolidation of its theoretical value, even though there are some norms that under the different perspectives are seen as socio-mathematical norms or as social norms.

From the perspective of the practice, using the construct of norms as placed in its social context has led us to understand it as closely linked to social representations. Interpreting norms as social representations explains how difficult it is in certain context to achieve agreement between the legitimated norms and the norms as understood by the different individuals. The sharing of norms is certainly no longer a taken-for-granted starting point, and we may wonder if it should be a 'to-be-reached' aim.

Moreover, the understanding of norms as shaped by social representations allows us to establish a link between the micro and macro social context of mathematics education that contributes to explaining the failure of certain students belonging to minorities. Though issues related to the failure of certain social or cultural groups have broadly explored how people from a certain ethnic group perform in school mathematics when compared with other groups, they have been conceptualised at a group level.

This particular focus has been useful in exposing inequities in the mathematical performance of people from certain backgrounds and these studies are important as tools that provide public information of the access to available cultural capital in a democratic society, despite the risk of contributing to the arousal of certain social representations that can picture the immigrant students as less able. Focussing on

the day-to-day activity within the mathematics classroom is likely to help in the understanding of different chances for participation processes within the multicultural mathematics classroom. Norms, being elements that regulate classroom action and interaction, are at the very basis of classroom discourse. Understanding norms as constituent elements of discourse, as mediators of actions and interactions, they become valuable constructs in our empirical work to analyse participation processes within the classroom.

Should we not all challenge our representations?

Representations coming from the educational institution and from the whole society that host the minority groups shape the norms that regulate the practices within the classroom. Immigrant students, most of them socially at risk, tend to be stereotyped as less competent and their mathematical abilities have traditionally been considered from a deficit model approach. Therefore, immigrant students and their practices are more prone to be valued negatively due to a-priori socially constructed assumptions and this valuing interferes with the orchestration of the norms that should facilitate, or at least allow, their participation.

The difficulties that immigrant students encounter when they are to understand and use 'new' norms may not lie only in their novelty, but also in the fact that norms are not neutral. To what extent do norms, as cultural artefacts shaped by social representations of the dominant group(s), have as a possibly unintended effect the continuation of the culture and social positioning?

We must ask ourselves: in what way research in mathematics education has addressed the multicultural fact? What has been the contribution from the research domain to the understanding of the complexity of the multicultural mathematics classroom?

Until now the existing theories to explain the mathematics teaching and learning processes have been mostly developed within prototypical classrooms and upon the basis of dominant understandings of 'normal children, classrooms and schools'. It could be said that the research done until now is only a partial account of reality. When partial accounts of reality become the rules for normality, one has to ask who is excluded by these rules. It is not only society, educational institutions or teachers that jeopardize immigrant students' learning opportunities but researchers as well, unless the empirical domain is broadened to embrace non-prototypical classrooms and socially constructed representations of 'normality' are questioned.

It is the purpose of our research to understand the complexity of the multicultural mathematics classroom and not only to describe it. Understanding a social situation requires looking at it from a re-

vised theoretical perspective, but should we not also take into account the interpretation of those (immigrant students) involved in the situation?

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