Proceedings CIEAEM 58 –SRNI, Czech Republic, July 9-15, 2006
"Quaderni di Ricerca in Didattica (Matematica)", Supplemento n. 3, 2009.
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THE IMPORTANCE OF MATHEMATICS FOR THE DESIGN SCIENCES

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In the present paper the role that Mathematics play for the Design Sciences is analyzed and the ways and methods of teaching Mathematics in the academic departments of the Design Sciences are discussed.
As a representative example the special factors are examined that influence the teaching of Mathematics in the Higher Technological Educational Institutes of Greece, all the cognitive objects of which belong to the Design Sciences.

THE ROLE OF MATHEMATICS FOR THE DESIGN SCIENCES

The scientific status of the *Design sciences,* or otherwise called *Sciences of the Artificial,* was clearly delineated from the scientific status of the Natural Sciences by the Nobel Prize Winner Herb Simon (1970). Indeed, while the Natural Sciences describe and interpret the structure and operation of natural objects, the mission of the Design Sciences is the design and manufacture of artificial objects, having certain desirable properties.

According to the above delineation it becomes evident that Architecture and generally the Structural Science, Engineering, Electrology etc belong to the Design Sciences, but Medicine, Economics and even Law Science belong also to them. Indeed for Medicine the design of artificial objects concerns the treatment and prevention of the various diseases with medicines, operations, vaccines, suitable diet etc, for Economics concerns the construction of the various economical models, while for the Law science concerns the creation of the laws and of the legal arguments, which are used in the trials.

Simon (1970: p.p. 55-58) explains also the resistance offered to the Design Sciences in academia. It is ironic, he notes, that in this century, while the design (with the wide significance of the term presented above) should constitute the barometer of all kinds of professional training in the corresponding sciences, the Natural Sciences have considerably displaced the Design Sciences from the curricula of the relevant academic departments, with the climax being in the Polytechnics, the Medical and the Economic Schools, that being included in the general culture and philosophy of the academic rendering, have been changed at a big part to departments of Physics, Biology and of Finite Mathematics respectively!

On the other hand mathematics, even if it is not in place to interpret alone the structure of an object, it can however describe it in an explicit and plausible way. Indeed in many cases a mathematical equation is in place to express in an absolutely evident way something that would need perhaps entire pages of written speech to be expressed.

In order to become more explicit let us give an example from the modern Architecture, where the roofs of buildings with wide openings (closed stages, swimming pools etc) are usually designed in the form of a saddle, because such type of surfaces have big resistance to bending (cf., Salvadori & Feller, 1981, paragraph 12.12).

As it is well known from Analytic Geometry such a surface (called a hyperbolic paravoloid) in a suitably chosen system of coordinates has an equation of the form

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=2cz, \text{ with } a, b > 0.$$

Writing this equation in the form

$$\left(\frac{x}{a}+\frac{y}{b}\right)\left(\frac{x}{a}-\frac{y}{b}\right)=2cz$$

it is straightforward to check that the above surface is generated by the straight line with equations

$$\left(\frac{x}{a}+\frac{y}{b}\right)=2\lambda cz \quad \text{and} \quad \left(\frac{x}{a}-\frac{y}{b}\right)=\frac{1}{\lambda}\ ,\ \lambda \neq 0.$$

In fact the above line lies entirely on the surface and generates it for the various real prices of λ .

Rightly therefore mathematics is considered today to be the queen, but simultaneously and the servant of the other sciences and therefore it is necessary to occupy a part of the curricula of the most departments of the Higher Education, either if they concern the Natural Sciences (e.g. Physics, Chemistry, Biology etc), or the Design Sciences.

Moreover in many academic departments of the Design Sciences, which are under our concern in the present case, including those of engineers, of economists etc, it is necessary nowadays for mathematics to constitute a big part indeed of their curricula.

Thus the problem that Simon locates (see above), at least for the case of mathematics, is not focused mainly on the quantity of matter that it should be taught in these departments, but on the way in which it will supposed to be taught, in order that mathematics will be harmonically tied up with the remainder body of courses and become a useful and essential tool for the student (and future scientist) for the study and the deeper comprehension of his science.

The real question therefore is in which way and methods mathematics has to be taught as a course for the Design Sciences and in the next we shall attempt to give a well framed answer to it.

MATHEMATICS AS A COURSE FOR THE DESIGN SCIENCES

The easy solution for the teacher of mathematics in an academic department of the Design Sciences could be to avoid the theoretical proofs by presenting "ready" the corresponding mathematical results to the students and to turn the attention to the examples and applications of these results on the cognitive object of the corresponding department (e.g. Engineering, Economics etc).

However, although according to my almost twenty five years teaching experience of mathematics in various academic departments of Design Sciences in Greece (see the third section of this article), it looks sometimes necessary in practice to apply this method of teaching, I believe that the above manipulation is not the most advisable as a general way of confrontation of the subject .

Indeed, the graduates of the academic departments of the Design Sciences, although many of them continue today their studies in postgraduate level and acquire doctoral degrees even in pure mathematics, physics and other theoretical sciences, are intended mainly to make carriers as scientists of applications (technologists). This however does not mean that they should not acquire the theoretical knowledge that is essential in order to possess their science at least in the depth which is indispensable in order to be able to check the correctness of their actions in the field of applications.

Accordingly the proofs of the mathematical theorems should reach and stop up to the point that is indispensable for the student to consolidate the corresponding matter, thus acquiring the ability, but also the essential dexterities in order to apply it effectively in practice for the study and the deeper comprehension of his science.

For example the presentation of the equations of the various curves without proofs and the turn of the attention to the solution of exercises and applications only could not permit the student to conceive the important role that the vectors play for the study of the geometric properties of the various figures with algebraic methods.

On the contrary, if the lecturer is certified that the student has conceived empirically the notion of the limit, that is to say what means for a variable or a function to "tend" to a certain value, it is not necessary to present meticulously all the proofs of the properties of the limits by making use of the analytic definition of the limit.

Similarly, if the student consolidates rightly the geometric and the natural significance of the derivative, it is not necessary to present all the proofs of the rules of differentiation of functions. Indeed, in both of the above cases it is sufficient to give the proof of a simple property and then, by stating "In a similar way one can prove that…", to present the remainder properties.

Under the same logic, given the continuous in the interval (a,b) function y=f(x), where a and b are real numbers, one may present empirically the mean value theorem of the differential calculus (of Lagrange) by making use of the geometric significance of the derivative and "observing" that there exists a point, say P(ξ,f(ξ)) on the curve y=f(x), a<ξ<b, such that the tangent of the curve at P is parallel to the chord AB, where A(a,f(a)) and B(b,f(b)) (see figure 1), i.e. that $\frac{f(b)-f(a)}{b-a}=f'(\xi)$.

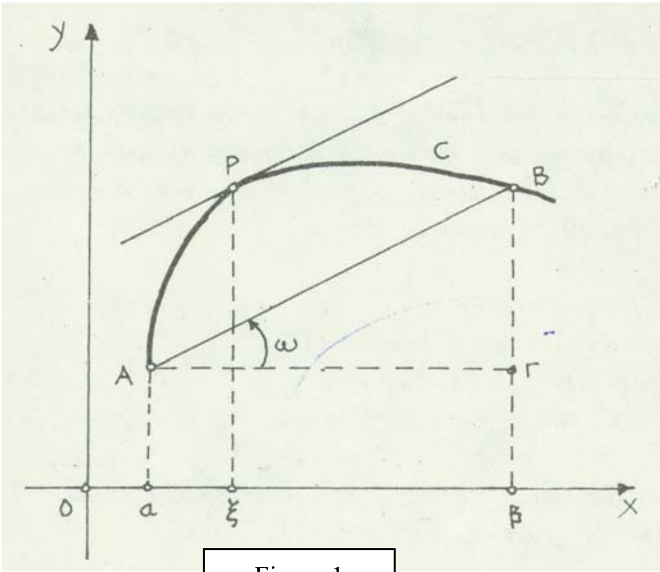

Figure 1

In general, through my long teaching experience of mathematics in such departments, I have reached to the conclusion that the excessive mathematical severity and meticulousness in the proofs disorientate the students, many of which have not the suitable mathematical background for this purpose.

In Numerical Analysis for example, according to the well known method of Newton and Raphson for the approximate calculation of a root of the equation f(x)=0, starting from an approximation, say x_0, of the root we find a better approximation by the repetitive formula

$$x_{n+1}=x_n-\frac{f(x_n)}{f''(x_n)}\ ,\ n=0,1,2,\dots ,$$

stopping when the corresponding value of x_n satisfies the precision, say a, that we ask, that is when two successive values of x_n differ absolutely less than a. For the strict proof of the above formula one has to make use of the Taylor's expression of the function f(x), cutting the terms that they contain derivatives of second and greater order. However a much simpler graphic proof may be presented instead, based again on the geometric significance of the derivative (cf. Tompkins & Wilson, 1988: paragraph 3.2).

It is not however always easy for the lecturer to decide up to which point he should proceed with the relative proofs. This depends upon many factors, as they are the cognitive object of the department where he teaches, the time that he has in his disposal, the level of his students etc. Nevertheless his previous experience from proportional cases in the past can always direct him to the right decision.

Finally it should be noticed that there are topics from applied mathematics with many applications on certain cognitive objects of the Design Sciences that, at least in Greece, they are not taught in the required depth, or even more they are not included in the curricula of the corresponding departments at all (this should be taken seriously into consideration in future for the revision of the curricula).

An example for the first case is the theory of Markov chains, which is a successful combination of Linear Algebra and Probability theory and enables us to make short and long run forecasts for the evolution of various phenomena. In the course of Operational Research for example, which is taught in various departments of the Faculty of Management and Economics, they are usually presented the basic principles of the above theory and the equilibrium situation of a chain, which is in fact in effect for the case of ergodic chains (cf. Kemeny, 1976: Chapter V), but not the case of the absorbing chains (cf. Kemeny, 1976: Chapter III), which also find many applications in Management and Economics (e.g. see Voskoglou, 1994 and 2000).

An example for the second case is the theory of Fuzzy Sets introduced by Zadeh (1965)**,** that gives solution to problems where certain definitions have not explicit limits (e.g. positive development of the stock exchange) and where the classic theory of Probability is not applicable, e.g. decision-making in a fuzzy environment, statistical evaluation of fuzzy data etc, e.g. see Klir & Folger, 1988: Chapter 15, Voskoglou ,2003 etc.

Perhaps even certain basic elements from the Chaotic Dynamics and the Fractal Geometry it would be useful to be included in the curricula of certain departments of the Design Sciences, where they find applications for the confrontation of problems of complexity - non linearity in time and space respectively, that are frequently appear in Physics, Chemistry, Biology, Economy etc.

A REPRESENTATIVE EXAMPLE: THE HIGHER TECHNOLOGICAL EDUCATIONAL INSTITUTES OF GREECE

After exposing my general thoughts about the suitable way and methods of teaching mathematics in the academic departments of the Design Sciences I shall present, as a representative example, the case of the Technological Educational Institutes (TEIs) of Greece, all the cognitive objects of which belong to the Design Sciences.

The TEIs belong to the Higher Education together with the Universities and provide to their students a first degree in higher education (of B.Sc. level), as well as postgraduate degrees and diplomas in cooperation with Greek or suitably selected foreign universities.

But while emphasis is given in the Universities to the basic and (or) the applied research, the orientation of the TEIs is towards the field of applications and technological research, i.e. the use of the existing scientific knowledge to provide new or improved products and services.

The ascertainment of Simon about the significant displacement of the Design by the Natural Sciences in the curricula of the corresponding university departments (see the first section above) is not certainly in effect for the TEIs, where the studies have traditionally applied character. Indeed a particular gravity is given in the TEIs to the laboratories and applications, which are characterized by the term "Exercises of Act".

I have been teaching at the TEIs since 1983 the year of their foundation with the evolution of the previously existing Centres of Advanced Technological and Professional Education (known, with its Greek spelling, as K.A.T.E.E.).

Initially I taught as a part – time professor at the TEI of Patras and next as a full professor at the TEIs of Messolonghj (1987 – 2000) and of Patras (2000 until today).

I have taught in the Faculty of Management and Economics, as well as in the Faculty of Technological Applications, wherefrom they graduate technologist engineers of various specializations, a wide range of cognitive objects, that starts from the, so called in Greece, "General Mathematics" (Linear Algebra, Probability and Combinatorial Analysis, Complex Numbers, Differential and Integral Calculus, Differential Equations etc) and Analytic Geometry and is extended up to Numerical Analysis, Financial Mathematics and Operational Research.

Therefore I consider useful to externalise my reflections as well as the conclusions and beliefs to which I have been led through my almost twenty five years teaching experience, and especially to describe the special factors that prevail in the environment of the TEIS and influence the teaching of mathematics and of other, theoretical mainly, courses.

First, although recently the bases of the Panhellenic examinations for access in certain departments of the TEIs have gone up perceptibly exceeding many of the university departments (for more details about these examinations see Voskoglou, 2004), to the TEIs have in general access students with lower on average grades concerning the equivalents of the relative departments of the universities. This of course means that also the mathematical level of these students is comparatively lower in general.
The problem however is not focused so much on this point, as long as to the fact that the students of the TEIs have actually a non-homogeneous mathematical background.
Indeed graduates from the technical secondary education, that have been taught considerably less topics of mathematics with respect to the graduates of the positive and technological direction of the general secondary education, have also access in a percentage (with special examinations) to the TEIs. Still in certain departments of the TEIs, apart from the graduates of the above directions, have also access and graduates from the theoretical (classical) direction of the general secondary education, a thing that intensifies the problem still more.
This situation is faced casually in certain cases with optional preparatory tuition courses, apart from the regular schedule of teaching, although for arrangements of this type there exist internal disagreements among the teaching stuff of the TEIs (e.g. some believe that in this way the level of studies is degraded).
Therefore in most cases the lecturer of mathematics is compelled to search to find ways to adapt his lectures in order to cover the existing voids of some of his students from the one hand, but also to maintain undiminished the interest of the advanced on mathematics students on the other hand, a combination that it is really very difficult to be achieved.
With regard to this it should also be taken into consideration that in general the graduates of the secondary education in Greece today, although they have been taught considerably advanced topics from Mathematical Analysis (up to the integration of a function in one variable), they have serious voids on issues from elementary mathematics, e.g. in the last three years of their studies (Lyceum) they have not been taught the Geometry of Space at all!
Imagine therefore the position of the mathematician, who wants to teach for example in the department of Renovation and Restoration of Buildings of a TEI (where I am currently lecturing in Patras), or may be in a department of Architecture or Civil Engineering of a Polytechnic School of a Greek university, some, even elementary, topics from the theory of surfaces, that constitutes the necessary mathematical background for the design of roofs in big buildings (cf. Salvadori & Feller, 1981: Chapters. 11 and 12), when his students have difficulty even to realise that a straight line in space can be described, as a section of two levels, with a linear system of two equations in three unknown variables!
Something else, that should also be taken into consideration is that,, since the permanent educational staff of the TEIs is not enough to cover all the teaching needs and the employment of extra staff is also limited (for economic reasons), the teaching of the theoretical courses is performed to a big audience. Thus the audience of theory in the mathematical courses of a big TEI, as it is for example at Patras, exceeds in many cases the 100 students, while in the corresponding tutorials (exercises - applications), where there exists some possibility for separation of the students to teams, it is usually around to 50 students.
This should be connected with the fact that the teaching hours of mathematics in the curricula of the TEIs are relatively limited. For example in most departments of the Faculty of Management and Economics General Mathematics are taught for only one semester, that is a half-year period (several topics from Applied Mathematics are also taught in more semesters, like Financial Mathematics, Statistics, Operational Research etc), while in the Faculty of Technological Applications they are usually taught for two or three at maximum semesters (in the latter case including Statistics, Numerical Analysis or the Fourier Series etc).
It becomes therefore evident that under these conditions it is very difficult for the lecturer of mathematics to come in personal contact with his students, to discuss with them, to resolve their queries, and generally to guide them suitably to their study.

It also should be noticed that colleagues of other specializations (e.g. economists, engineers etc) in many cases avoid, as far as it is possible, the use of mathematics in their courses. This usually happens because, having in mind the weaknesses of their students in mathematics, they try to simplify their teaching object and to present it in a more practical form. In this way however, and despite of efforts of the lecturer of mathematics to enrich his teaching with applications, mathematics is broken away from the body of the other courses and the students cannot conceive as it should the importance that it has for the deeper comprehension of their science.

Finally , but not less important, it would be an omission if we don't turn our attention to some factors, that disorientate in general the students of the TEIs from their studies and decrease perceptibly their disposal and zeal to deepen in their science. The most serious of these factors is the suspense that there exists, especially for the departments of engineers of the Faculty of Technological Applications, with the acknowledgement of their professional rights by the Greek government, which is caused by the reactions of the graduates of the corresponding departments of the universities!

In fact the TEIs are relatively new institutions in the area of Higher Education in Greece and therefore the graduates of the universities have the power in all mechanisms of our society and they want to keep in a distance the graduates of the TEIs. As the result of this situation the graduates of the TEIs have certain professional rights in all the countries of the European Union apart from the country of their origin (Greece), where the graduates of the corresponding departments of the other countries have already these rights!

As Head of the department of Renovation and Restoration of Buildings of the TEI of Patras I had recently a very unpleasant experience with regard to this subject, that prompted me indeed to submit my resignation, when I conceived that, despite my efforts, I was not in position anymore to help effectively towards the resolution of the problem, that has been created with our graduates.

This enormous problem, without the resolution of which the status of the TEIs as higher institutions remains without meaning, could possibly be faced effectively through the creation of a National Frame of Degrees (cf. Kazazis, 2005), which is already in effect in many countries, as they are Britain, Ireland, Australia, New Zealand etc.

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