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EMBODIMENT AND A-DIDACTICAL SITUATION IN THE TEACHING-LEARNING OF THE PERPENDICULAR STRAIGTH LINES CONCEPT[1].

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ABSTRACT

My research was born from the idea that difficulties and problems students of different grades have, mainly come from language and, in general, from the formal aspect of mathematical concepts. It appeared extremely important to me to consider two sides apparently divergent

- *The specific quality of mathematics and its own language*
- *The role of the context (space, time, people) in communicating mathematics*

Do not forget that language is not only a source of trouble; it is also a necessary player in every learning process. My personal idea is that, in order to make easier the mathematical communication, it is necessary to create a proper context.The theoretical idea I go with is the one from Guy Brousseau who in "Theory of Situation" defines the milieu: the environment where the student and his knowledge building process happen. We built an a-didactical situation where the relation learning-teaching into the knowledge-pupil-teacher triangle is controlled and analysed in connection with the outside environment and the emotional sphere of the student. But I did not use only the Theory of the situation; this research aims to link two different theoretical frameworks:

- *The theory of the situation that structures the a-didactical situation and has a methodological control role.*
- *The embodiment theory and neuro-science theory regarding the body experience learning, that leads to the process of creating metaphors and learning into an emotional context.*

It is possible to link these two theories?
We can probably say that the link between the two theoretical frameworks is that the student is the only player of his knowledge process; by the "devolution" act according to the theory of the situation, by his own senses, brain and mind according to the embodiment theory (chapter 4).
The experimental work has been led within the S.P.O.R.A. project, the school involved were at a quite high degree of risk (based on social and economic indicators). The students involved were from 3 to 11 years old (mother school and primary school).

The field research has required some answers on pupils' concepts of perpendicularity.
The quantity analysis has been done with the Chic software, the quality one according to the theory of the situation. Final considerations have been done by the process of metaphors building trough bodily activities that pupils have joined.

THE AIM OF THE WORK

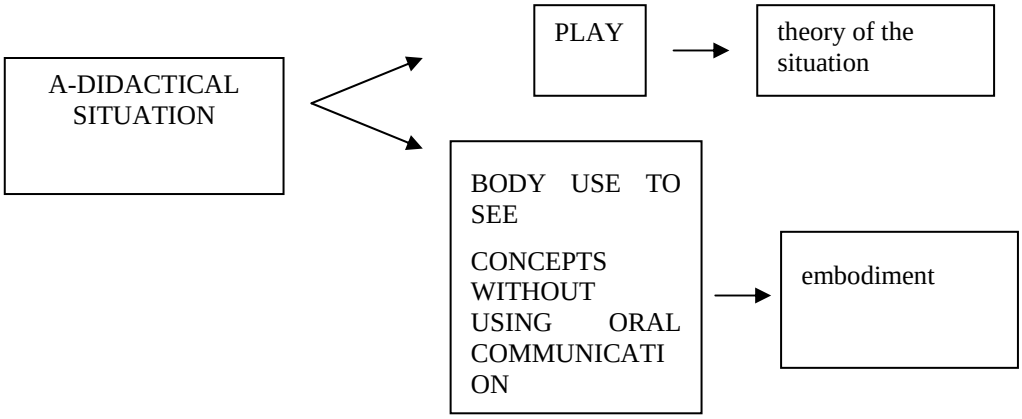
The idea of a research on the development of the concept of perpendicular straigth lines was suggested by the requirement of "looking at the geometry" and stimulated by the result of a test proposed in a secondary school. I have then selected as the object of this research the setting of the concept of perpendicularity, convinced that this simple concept has a fundamental role in the comprehension but also in the build up of geometry as an operation of thougt and language.
We starter from some consideration to delineate the aim of the research:

- Obstacles to the correct setting of the concept are probably active in an early stage of the psycophisical development of the subject;

[1] Doctoral Thesis, University of Bratislava (Slovak Republic), 2006, Advisor: Prof. Filippo Spagnolo.

- Many student understand the concept of perpendicular straigt lines as verticality (language mis-concept)
- Didactical interventions at various levels are generally inadeguate and shallow, and they look like transmissive lessons instead of concept constructions.
  The aim of this research is to find out different notions or ideas of the concept of perpendicularity that students have, whether if they are spontaneous or coming from teachers.
  It's very interesting to analyse these phenomena and search some answers to these questions:
- Have pupils the implicit model of plumb line?Are the difficulties of understanding the concept of perpendicular straight lines related to difficulties of linguistic kind?**Theoretical framework**

The teoretical framework that I assume is the theory of situation and the point of view of the Theory of Embodiment in the particular side of building metaphors, as we can read in this schema:



REALIZATION OF THE AIM

On the first stage of the research we gave students an open test made of four questions. To be sure of the independence of their answers from teaching or research stimuli, students had been told that their answers would have helped teachers throughout their job and would have been evaluation-free. That has been a winning strategy because it has made students free from any emotional impact. All the students involved joined the research, a very positive result itself, considering that schools involved were all at risk (so to say, schools were dealing with a difficult social and economic context). Further more we used cameras to shoot students over the period of research, and this has been fully accepted.
On the second stage we created an a-didactical play situation, called play-path where students had to play Tom and Jerry characters.
Teacher used the gym to build paths and ways as they were shown in the test and asked students to find out where Tom and Jerry should be in order to let Tom catch Jerry.
Students had so been able to experiment that the concept of perpendicular is not linked to the plumb line model but to the concept of minimal distance.
We followed all the stages implied in an a-didactical situation:

1 action situation
2 formulation situation
3 validation situation
4 institutionalisation situation

HYPOTHESIS

The hypothesis I start from is:

H1 Vertical and perpendicular are synonymous to students (language misconcept)

H2 If teachers formalise the concept of perpendicular (in a maths way) students become acquainted to the concept as long as they use in a proper way the concept of the plumb line and are able to adapt it to the new situation. If students do not break this epistemological obstacle (implicit model) they will not be able to solve a given problem where the reference system is not a line crossing the centre of the earth.

H3 To build particular "*milieu*" in consideration od the Theory of can be a contribute to create the didactical way more oppartune to the correct and persistent formation of the concept. Is possible to build a context were the perpendicular straigth lines are conceptualizated as minimum distance?

We can easily think that obstacles hard to overtake are born in a very early time of psychic and physic development, and teaching action produces rather an instruction than a conceptualisation.
Resume of the phases of the research:

- To suggest an open text;
- To realize an "a-didactical situation"
- To analize the text;
- To realize the "Analisi a priori";
- Quantitative (chic software) and qualitative analiysis of the data

EVALUATION OF THE EXPERIMENT

The experimental data of the reseach are interpred by qualitative and quantitative (statistical software chic) analysis. I report here only a resume of the principal results.
Qualitative analysis of the text:
Demand: children generally have been able to answer and give explanations, the analysis of the entire body of tests, so to say their percentage, would let us think that finding the minimal distance between two points is the same that finding a vertical line crossing one of the two given points. After a first glance on children explanations we can assume that the concept of vertical is seen through the lens of daily experience of gravitation. Students consider the vertical position between Tom and Jerry a winning strategy. To enhance our opinion we can look at some children explanations to what they have done:

> "Because the mouse walks and at a point all of a sudden the cat jumps on him", "Because it is easier", "Because Tom was ready and jumped at the right time", "Because tom jumps and catches him", "Because the cat is in line with the mouse and it is easy to catch him";

There have been children that understood that uprightness (vertical) was not the right answers for all the questions. Also very interesting are children explanations about the position choice of Tom and Jerry, they have not been deceived by the idea that if Tom jumps following the vertical line he will go faster and *all of a sudden* he will catch Jerry, children rather have an idea (they are not yet presented the concept of perpendicular) of the concept of minimal distance. Here we have some of the children explanations:

> "because he is closer", "because the cat waits for the best time", "because Tom has been waiting for Jerry to come closer and he caught him", "because Tom wants to catch Jerry and when he is closer he catches him"

Quantity analysis of 1-2 grade primary school (age 5 to 7) tests

The analysis of the similarity reveals that children ideas are split into three directions:

- Able to find out the vertical line in exercise 1 and to guess vertical in exercise 3

- Reveals that the correct strategies uses in the 3-4 exercice is not a general compatence infact the student uses a worong strategies in 1-2 execise.
- The last similarity group shows that when children have fragile correct ideas they are not able to make them general and to use them in different contexts.
  The analysis of the implicative graph reveals that children ideas are split into three directions
- If a child finds out the perpendicular line in exercise 1 he will have some correct ideas in left exercises but not always he will follow the correct strategy;
- If a child follows once a wrong strategy he will follow it also in a different context
- If a child finds out the perpendicular line in exercise 3 he can also understand the minimal distance between Tom and Jerry in exercise 4. If we consider answers children gave in exercises 3 and 2 we can assume that even if they are in front of similar questions they give different answers. To explain this we could say that the problem is in the number of solutions. Children follow the didactical contract and think that it is not possible that two exercises have the same solution, especially if they do not understand that there are two possible choices to solve exercise 3. Why should teachers give two exercises that are almost the same?

Quantity analysis of 3-4 grade primary school (age range: 8-9 years) and 1 intermediate (10-11) tests
The similarity analysis reveals that children ideas can be shared in three groups:

- Many student consider perpendicular straight lines and verticality as the same concept, they stress the verticality for indicated the pependicularity.
- Some students have the correxct idea of the perpendicular line, but it is only an idea no a mathematical concept;
- Some answers given by the students reveals that children draw a line that connects the cat to the highest, and closest to them, mouse point.
  From the graph we can imply that:
- children have the same model of vertical and use it in different exercises. (The model of plumb line is very strong and childrens uses it in different context);
- children that guess perpendicularity in exercise 4 do not get an implicit and inner model of vertical.

To remuse is possible to think that the implicit model of verticality is very strong in the student, and those student that haven't this inner model are not able to identify the mathematical oncept of perpendiculariry.

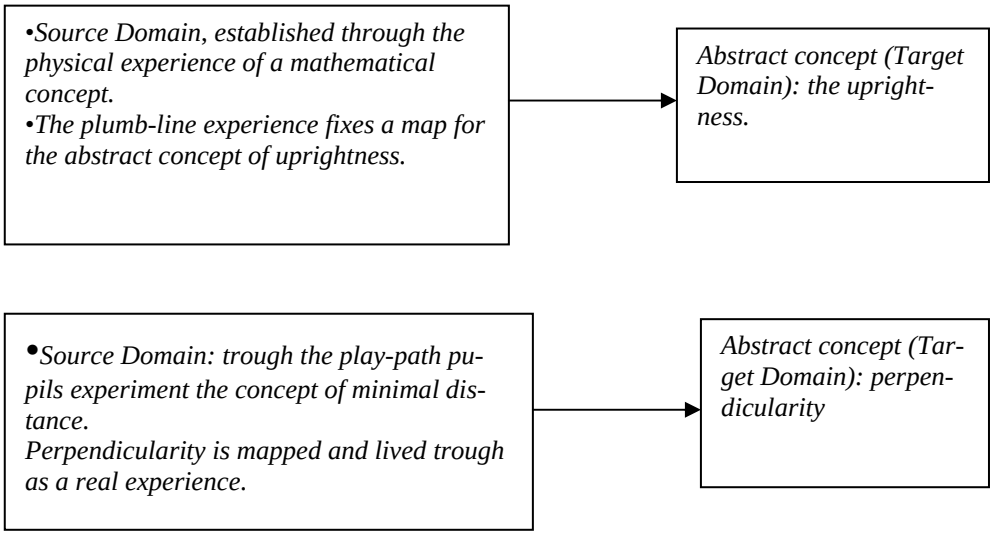
Play-path analysis

During the first path teams have found the "right" point thanks to the concept of vertical.
While doing the second path one team kept following the concept of vertical repeating the word *under*. Other teams, also not forgetting the vertical concept, have found as more relevant the "*in front*" position of cat and mouse, thing that makes easier to see each other (here more easily seen than in a written test): to prove that, they often use the word "*he sees better"* and criticise the curve on other teams paths. The idea of closeness, even if in a certain way guessed, was still not fully expressed and understood in its importance.
During the third path, the idea of closeness as the most important criteria to solve the problem starts to reveal its importance. In order to help students to find the winning strategy they had been given a string without being told of its usefulness. Students repeated their ways using this string. Students have had group discussions about winning strategies (validation) and ended up on an agreement: the "right" point to catch Jerry was the closest and this point was the perpendicular concept itself. The most successful theorem was:

> "the perfect time for Tom to catch Jerry is when the mouse is the closest to the cat not when he is under the cat".

CONCLUSION

Third chapter tells us how the experiment bears pupils to succeed in finding the right theorem; when pupils understand that the concept of "being perpendicular" has to be related to the distance one.
The hypothesis we start from is the one about the uniqueness of mathematics and its language; otherwise fourth chapter describes us the subjective vertical and suggests that difficulties we find in defining the concept of perpendicularity are strictly linked to our body and brain (we do have gravity receptors in our abdomen). It has been really interesting evaluating the results coming from the embodiment experience; this experience has made clear that there are different set of metaphors, some of them are related to the perpendicular concept and some others are related to the vertical concept. The play-path has helped a lot in this achievement.



When using the embodiment theory we also receive a positive feed-back on the effectiveness of the theory of the situation, but these two theories are fully valid alone, they do not need each other positive feedback to show that they are both true.
Pupils have built a mental image that links the concept of perpendicularity to the plumb-line model, there are several reasons for this: the similar meaning of the two words, as we said above, the inner knowledge of pupils, a personal experience of the gravity concept, the way our body works; but at the end this strategy is successful in some particular cases.
The play-path undermines this mental image and requires a new one that is finally successful in every case, not only in some specific cases.
We have seen that after this experience pupils link the concept of perpendicular to the one of minimal distance.
The embodiment theory proves this result and explains it (new metaphors are born), and it is the experience itself that let pupils link the right metaphor to the right mathematical concept.
In my opinion it has been determining the environment we provided in order to experiment these mathematical concepts, it would be really interesting to repeat a similar experience with other mathematical concepts.

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