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CULTURAL DIFFERENCES IN SCHOLASTIC AND NON-SCHOLASTIC ENVIRONMENTS: REASONING PATTERNS AND LOGICAL-LINGUISTIC QUESTIONS IN EUROPEAN AND CHINESE CULTURES

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ABSTRACT

This work analyses some similarities and differences between the different patterns of reasoning in different cultures. The epistemological and historic tools used are logical paradoxes which are traceable in Chinese and western cultures (problems of Aristotelian logic). The investigative tools are quantitative and qualitative. This work is placed within a framework of a vaster plan of research on the problems of teaching/learning in multi-cultural environments which are being confronted within the GRIM.

Our attention was concentrated on thought and the use of natural language which can transmit different ways of reasoning and expressing oneself. The experimental research, which investigates the deep sources of ways of thinking, can open new roads in the processes of teaching/learning of mathematics in every kind of school. This experimental work consists:

1. *analysis of two cases and this analysis is of a type which is exclusively qualitative.*
2. *quantitative analysis of a group of Chinese students from Nanjing and Palermo..*

By means of experimental analysis, the prevalent role of the didactic contract in standardising substantially the behaviours of the students has been highlighted. Comparing the interview protocols with the results of the questionnaires administered in the classes, one instead notes significant differences.

INTRODUCTION

The studies concerning the analysis of the schemes of reasoning are almost always orientated toward the teaching/learning of specific mathematical contents. The goal of this work is to present situations/problems typical of mathematical thinking, but with attention on the logical-linguistic problems.

This work is part of a research project which has highlighted:

1. the role of natural language in the development of mathematics in the history of thought. (Spagnolo, 1986, 2000, 2001, 2002);
2. the role of the history of mathematics as an instrument of observation and analysis of multicultural learning/teaching situations.

The most significant reference is the investigation, with historic epistemological instruments, of oriental patterns of reasoning in relation to European ones. Equally important is the epistemological analysis relative to the use of subtended logics in relation to natural languages.

All of this allows us to form the following hypothesis:

H1: The differences and the analogies in the history of oriental and occidental cultures also have an equivalent in the differences and analogies between the patterns of reasoning found today in situations of teaching/learning of mathematics.

To be able to falsify this hypothesis, we need the following paradigmatic references:

1. Historic and historic-epistemological analysis of mathematical thinking as regards the study of the different patterns of reasoning (deducing, conjecturing, arguing the validity of the statement, proving) in different European and Chinese cultures. This kind of analysis is conducted with the argumentations typical of history and epistemology and will be the basic reference for all the work. In some ways, it represents a possible point of view of ontogenetic development.
2. Analysis of cases. This kind of analysis utilises the methodological instrument of the individual interview. The situations/problems discussed in class are the subject of the interview.
3. Experimental analysis of situations/problems in Nanjing (China) analogous to those tried in Palermo.

THE WORKS OF REFERENCE: NOTES

The principal reference for mathematics in Chinese education is that of the "Nine Chapters on Mathematical Procedures": this constitutes a canon[4] both for the construction of mathematics (1st Cent. B.C. – 1st Cent. A.D.) and for the teaching/learning of the same in the various historic periods. Among the most notable is the commentary of Liu Hui (263 A.D.) presented in the collection of the Mathematical Canons of the Tang Dynasty (618-907 A.D.). This canon of Mathematics was chosen to be included in an even greater reference regarding the revision of the classics of Confucianism.

SOME REFLECTIONS ON "ARGUING, CONJECTURING AND DEMONSTRATING" IN CHINESE CULTURE WITH RELATION TO OCCIDENTAL CULTURE.

This paragraph analyses in a schematic way some substantial differences found in the history of Chinese thought and in the history of western thought.

In the comparative analysis of science in pre-modern China and the west, Geoffrey E.R. Lloyd (2001, pag.574) says that, "The aspirations of ancient Greek tradition represented by Euclid, which proposed deducing all mathematics from a single set of indemonstrable but evident axioms were not shared by the Chinese at least until the modern age. In China, as a matter of fact, the goal was not axiomatic-deductive demonstration, but gathering unifying principles from all of mathematics."

HOW WERE THE SITUATIONS/PROBLEMS CHOSEN?

Each situation foresees a possible reasoning pattern, but does not exclude others. (The questionnaire is in the appendix)

In questions 1 and 4 the term "prove" is used deliberately and in 3 "demonstrate" is used. This is because the first and the fourth questions lend themselves more to processes that induce proofs, than to empirical attempts. The third, instead, foresees reasoning by deduction, in any case, whatever the technique may be (by means of the representation of the possible cases or not) to arrive at the solution. The fifth necessitates reasoning by "exclusion of cases" which comes closest to a reasoning to the indirect proof (reduction ad absurdum). Question 2 is a paradox of the traditional Chinese culture which dates back to the "School of the Nouns" (370-310 B.C.), that plays on the linguistic ambiguity relative to qualities and it lends itself very well to analysing the different oriental (Chinese) and occidental points of view. The argumentations for confronting questions such as the ones proposed are closer to natural reasoning than to mathematical demonstration. The analysis of the different discursive forms and the different levels of organisation of the argumentations produced by the young people in the sample chosen, offer, however, the possibility of distinguishing and comparing different reasoning patterns and making some instructive observations about different behaviours.

PRESENTATION OF THE EXPERIMENTAL WORK IN THE ITALIAN CLASSES

This work was carried out at the state Scientific High School "S. Cannizzaro" in Palermo. The classes involved were: one third year (16 year olds) and one second year (15 year olds).

The young people of the third year had already confronted the question of Aristotelian syllogisms and therefore they expressed themselves more suitably, where the students of the second year were,

in any case, able to solve the questions (the percentage of the questions solved was very similar). Everyone was able to use the language of set theory correctly.

INTERVIEWS WITH TWO CHINESE.

Two interviews were carried out on the sketch of the situations/problems done in class. Tong (born in Canton 1954) went to Chinese schools until the upper secondary experimental school, but did not however complete his studies. He moved to Palermo in 1978 and obtained his Italian middle school diploma in 1985. Currently he manages a Chinese restaurant in Palermo. Jouzou (born in Palermo in 1986?) is currently attending the last year of High School. He has studied Latin, Greek, philosophy, mathematics, etc., and he considers himself to be culturally Italian. His awareness of the Chinese language and culture has been by way of his parents who are both Chinese. Tong is Jouzou's father.

THE EXPERIENCE IN THE CHINESE CLASSES.

The experience was conducted in September 2003 in some classes of the upper secondary schools of Nanjing and with some students of the 1st year of university. The questionnaire was the same one given to the Italian students. The a priori analysis of the behaviours was shared by Doctor Zhang Xiaogui and translated into Chinese.

The exam subjects were 12th grade high school students and first year university students. Students chosen were representative. There were 105 students, 65 high school and 40 university, who joined the exam. The questionnaires received back included 52 from the 12th grade and 30 from the university students.

GENERAL CONCLUSIONS AND FUTURE PERSPECTIVES

Following is a comparative analysis between the different behaviours which emerged from the enquiry of the questionnaire in class in Italy and China and the interviews. The different points of view include all the casuistic of the answers and are reinforced by the historic-epistemological analysis of mathematics in the two cultures with particular reference to deducing and demonstrating.

Questions	**Prevalent behaviours in the protocols of the interviews (PC)**	**Prevalent behaviours in the experimental results in Italy (I)**	**Prevalent behaviours in the experimental results in China (C)**
1	***Heuristic approach for attempts and errors. Research of an algorithm as a tool of formalised demonstration.***	***Inductive reasoning: finite chain of conjunctions.***	***Experiment and induction. Proof and intuition.***
2	***Request of a concrete context to analyse the adequacy of the proposition in hand.***	***Use of Venn's diagrams for deduction (the proposition thus ends up false).***	***Proof of the truth and falsity of the propositions.***
3	***Measure of the conformity of the affirmation at hand with the premises. More care of the analysis of the text. Use of tables or matrices.***	***Use of Venn's diagrams for deduction and a correct interpretation of the syllogisms. Deductive processes in N.L.***	***Correct us of the reasoing patterns of the syllogisms.***
4	***Organisation of the data for the research for conformity with a model (diagrams, previous idea, analogous situations)***	***Organisation of the data for the analysis of all the possible cases. Use of division by distribution. The pigeon hole princi-***	***The problem is not recognised as referable to a known pattern of reasoning. It is not solved.***

		ple.	
5	***Reasoning of the combinatorial type with representations by tables. Analysis of all possible case to encourage the renewal of the model.***	***Use of contrapositives and therefore of reasoning to the impossible in N.L. and with the help of double entry tables.***	***Use of counter contrapositives and therefore reasoning to the impossible in N.L.***

Three important results can be identified:

1. Deductions**,** both for proving that proposition 2 (the Chinese paradox, variable 2c) is true and for proving that it is false, are present in high percentages in the two samples examined (I) and (C). This brings to light that fuzzy reasoning is present in equal measure in the two scholastic populations (Ajello-Spagnolo (2002)).
2. With respect to the question 4 (refered to the so-called "the pigeon hole principle") most of the young people of the Chinese sample did not answer while the percentage of the young people in the Italian sample that did not answer is insignificant. The question is not referable to patterns of pre-organised reasoning (it does not have reference models). From the point of view of classical Chinese mathematics education, this constitutes a problem in the moment in which one looks for the reference to a pre-established model for confronting an "analogous" situation.
3. While numerous similarities are come across between the two samples examined in a situation of stated didactic contract (I) and (C), the answers in the protocols of the interviews (PC) turn out to be more varied where one sees greater cultural influence in the absence of a didactic contract.

The initial hypothesis needs, therefore, to be reformulated in the light of the role that that didactic contract plays in confronting the situations/problem proposed.

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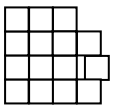
APPENDIX

Questionnaire on the abilities "to argue" problematic situations



1. A chessboard of 2^n x 2^n cells is given. You remove a cell in one of the four angles, for example:

Is it possible to cover the whole chessboard with pieces of this type?
Suggestion. To reason by induction, you put the piece from three in the centre:

How does it proceed?

1a) Solution

1b) It motivates the proposed solution

2. "A white horse is not a horse." Are we able to declare this proposition true or false?

2a) Solution

2b) It motivates the proposed solution

3. The premises are given: "All the adults can vote. Sabrina is of age. All those people who have a driver's licence are of age." It considers the following affirmations:
   a. Sabrina can take the licence
   b. Who is not of age doesn't have a licence
   c. Who doesn't have a licence is not of age
   d. Sabrina can vote

   It shows that three of them are true and only one is false.

3a) Solution

3b) It motivates the proposed solution

4. In a class there are 30 pupils. In the dictation, all have made at least one mistake.
Alex has made 13 mistakes and all the others have done less than he did.
Prove that there is at least one group of three pupils that has made the same number of mistakes.

4a) Solution

4b) It motivates the proposed solution

5. Mario, Benedetto and Giovanna are the first names of three young people aged 14, 16 and 17. Rossi, Bianchi and Verdi are their last names. The order of the first names can correspond or not to correspond to that of the last names, and it is not known to whom the ages belong. Knowing that: the girl Rossi is three years older than Giovanna and the young Verdi is 16 years older, find the complete name of every pupil and his age.

5a) Solution

5b) It motivates the proposed solution