Technology and mathematics in the classroom: lights and shadows

Ferdinando Arzarello
Dipartimento di Matematica
Università di Torino
ferdinando.arzarello@unito.it

RESUME: Les recherches conduites sur les Technologies pour l'Information et la Communication (TIC) ont pour but surtout de décrire les aspects techniques de l'innovation permise par leur usage.
Ici on introduit deux critéres qui considérent l'usage des TIC dans une perspective cognitive et culturelle. Le premier critère considère les TIC comme des *systèmes culturelles sémiotiques*. Le deuxième, comme des *énergisseurs cognitifs intrinsèques*.. On peut ainsi analyser les TIC d'un point de vue culturel et cognitif et les considérer, dans certains cas, comme des *infrastructures représentationnelles*, c'est-a-dire, comme des structures socio-culturelles qui peuvent re-définir notres connaissances, en particulier celles de l'école. Dans l'analyse, on souligne le rôle fondamental de l'enseignant comme médiateur, outre que comme auteur du projet didactique qui organise un milieu convenable pour l'apprentissage, où les ingrédients cités trouvent leur place. Un *espace de action, communication et production* est créé, où l'apprentissage des mathématiques peut avoir lieu. Dans la présentation orale on donnera une description de cet espace dans le cas specifique des TICS: on fera référence à une expérience conduite au niveau du lycèe à propos du concept de fonction et à une approche précoce à quelques concepts de l'analyse mathématique.

A good starting point for studying technology based mathematics learning is the fundamental analysis of 662 papers from the existing literature (written in the years 1994-1999) concerning Information and Communication Technologies (ICT, which include computers but do not coincide with them) developed by Jean-Baptiste Lagrange, Michèle Artigue, Colette Laborde and Luc Trouche (2001). In that paper a survey of literature about educational uses of ICT in mathematics education was done: they produced a quantitative analysis of all the 662 papers and two qualitative analyses of a sub-corpus of them concerning Computer Algebra, in order to establish the various dimensions in the approach to the use of technology. They choose five main "problematiques", according to which the sub-corpus of papers were analysed and could be classified (in not a mutually exclusive manner), namely:

1. The papers with a mere technical approach of possible use of ICT (53% of the papers).
2. Papers arguing in favour of classroom innovations (13%).
3. Research papers starting with assumptions on expected improvements resulting from students' use of ICT (18%).
4. Research papers starting with questions about this use (31%).
5. Papers explicitly addressing the issue of integration, which implied a study of questions like those in type 4 papers, but with a specific approach of an ecologically sustainable use (7%).

The findings of Lagrange et al. are impressive. In fact, they showed that much of the theoretical and empirical research concerning the use of ICT in mathematics education considered mainly the technicalities of an ICT approach and were prevalently concerned with the value-added component provided by the technology (type 1 papers).
Things have not changed so much since the publication of that report. The assumption that ICT adds a benefit in learning mathematics continues to remain a pure speculation in most papers; also today only a few researches show how the value-added component is achieved (type 3 and 4 papers) and only a minority of them (type 5 paper) addresses the issue in a scientifically critical way. Moreover, as pointed out by C. Kieran and R. Herschovicz in the same Research Forum, there are research works that show that this value-added component is not easily achieved and that, in certain

contexts, the use of technology may block learning processes such as problem solving, justifying, and so on.

Hence at the moment, it is not (yet) possible a sound approach to this topic: the findings of research are not systematic and sometimes are also contradictory. For example, as the general document of this Conference sketches, some skills seem enhanced by the simple fact of using the computer, other skills can be enhanced by access to software conveniently managed by the teacher (or another mediator), but other skills and attitudes (e.g. memorising) risk to be damaged by the facilities offered by the computer. A major disadvantage consists in the fact that papers on ICT are more concerned in "learning how to use technical tools incorporated in the computer ... than [in] understanding the theory behind those tools".

On the contrary, the main problem, which should be studied, consists in understanding the ways in which technological artefacts can mediate/support the construction of the student's mathematical knowledge. This issue should be faced from different perspectives: from a *didactic* point of view (e.g. considering the role of the teacher, that of social interactions induced by the used technology, and so on); from a *cognitive* point of view (e.g. considering how technology changes the mental structures of the learners); from a *cultural* point of view (e.g. considering what kind of mathematics is really taught and the framework of rationality towards which the use of technologies may push the student).

Taking into account these perspectives, the talk will discuss how ICT can be analysed as new *representational infrastructures* (a concept elaborated by J. Kaput, R. Noss and others: see Kaput et al., 2002), namely the ways "we use to present and re-present our thoughts to ourselves and to others, to create and communicate records across space and time, and to support reasoning and computation" (ibid., p. 51). As such, their understanding concern relations among people, not only among things; moreover such relations often work in an 'invisible way', so to say, hence it may be difficult to focus them: to make a simple example, consider the ways prices and weights are recorded and processed in a supermarket invoice and the consequent arithmetic of change at the cash desk, compared with the way the same operations were developed in an old fashioned shop.

This implies to analyse ICT at least according to two criteria:

a) as *Cultural Semiotic Systems*, namely as cultural systems which make available varied sources for meaning-making through specific social signifying practises (a concept which is due to L. Radford, 2003); such practises are not (only) to be considered within the strictly school environment but within the larger environment of the whole society, embedded in the stream of its history;
b) as *Intrinsic Cognitive Energizers*, that is as media which have *intrinsically* a logic which may enter (or not) in cognitive resonance with the subjects (an idea which comes in the long run from the notion of *intuitive model* of E. Fischbein, 1987).

Roughly speaking, a particular IC medium (e.g. a micro-world like Cabri-géomètre, but also a physical artefact like a compass) 'passes' criteria a) and b) if it is in *positive resonance* both with the socio-cultural and with the cognitive environment. In the presentation these criteria will be illustrated considering some simple and well known examples (e.g. Cabri I vs CabriII PLUS; Derive; Dienes blocks).

Two main problems will be discussed within this framework.

<u>Problem 1</u>. When an IC medium is a representational infrastructure, it can be the agent of a redefinition of knowledge, in particular of school knowledge. Specifically for mathematics this issue implies the restructuring of the role of notations and symbols and consequently the meaning of concepts (e.g. through the *instrumental genesis* of concepts, in the sense of Rabardel, 1995). This is a delicate issue: generally when an ICT medium does not pass one of the criteria a) and b) above, this may be the cause of pitfalls and misunderstandings of different types. E.g., in case of deficiency as an Intrinsic Cognitive Energizer, maybe it does not produce the mental restructuring for which it

has been built and consequently it does not produce or produces only partially the waited mathematical learning (e.g. this is the case of some structured materials). In case the deficiency concerns the cultural semiotic aspects, maybe that there is a more feeble resonance with the culture to be taught. All this can be the origin of what N. Balacheff (1997, p. 113) calls "a new semiotics of mathematics" that mathematics software creates and poses the problem of the relationships between the 'official' mathematics and the new one which is taught using ICT (e.g. the geometry of ruler and compass or of Logo or of Cabri with respect to the 'official' Euclidean or transformational geometry). The two criteria above can be considered at least necessary to avoid the major pitfalls.

Problem 2. One needs a reasonable balance between the two issues a) and b) above and this must be achieved in the classroom through the mediation of the teacher. To do that in a suitable way, she/he must analyse the mathematics to teach from different perspectives (cultural, cognitive, didactic and others), and design the technological transposition of that piece so that it fits with them as much as possible. Doing so, the teacher designs a suitable environment for cognition, where the whole processes of learning develops. Elsewhere (Arzarello, 2005), I have called this environment *the cognitive space of action, production and communication* (APC-space).
The APC-Space is built up, developed and shared in the classroom. Its main components are:

- the body;
- the physical world;
- the cultural environment.

ICT are in relationship with all the three components in a complex way, which the analysis through criteria a) and b) above can focus properly. When students learn mathematics all these components (and possibly others, e.g. emotional ones) are active and interact. The APC-space is built up in the classroom as a dynamic single system, where the different components are integrated each other into a whole unity. The integration is a product of the interactions among pupils, the mediation of the teacher and the interactions with artefacts. The three letters A, P, C illustrate its dynamic features, namely the fact that three main components characterise learning mathematics: students' actions and interactions (with their mates, with the teacher, with themselves, with IC tools), their productions (e.g. answering a question, posing other questions, and so on) and communication aspects (e.g. when the discovered solution is communicated to a mate or to the teacher, using suitable representations).

In the presentation I'll illustrate how the notion of APC space can be used to face concretely the criteria sketched above and the two main subsequent Problems. To do that I'll discuss a teaching experiment that our research team in Turin (L. Bazzini, O. Robutti, D. Paola, F. Ferrara, C. Sabena) is developing since a couple of years, where functions and pre-calculus are approached in secondary school starting from grade 9 with a systematic use of different ICT environments.
I'll sketch both the a-priori analysis, which entails both the *embodied* and the *cultural* nature of function's concept, and some findings that we have drawn. I'll show that the APC-space model allows properly studying the so called *perceptuo-motor* features in the processes of knowing (Antinucci, 2001; Nemirovsky & Borba, 2003), which reveal crucial for learning in ITC environments. Namely, it allows to illustrate how action and perception determine the processes of learning and to describe them so that doing, touching, moving and seeing appear as their important ingredients. This allows to frame the criterion b) above according to the most recent results of research: a learning approach based on perceptuo-motor activities, requires suitable modalities of teaching, in which the students are actively involved in the construction of mathematical concepts. In this perspective, the artefacts that are introduced in the didactical practice, hence all ICT, are designed to support and mediate in an essential way the construction of the experiential base, which is necessary for learning. It embraces pupils' sensory-motor experiences, the embodied templates that they activate and the languages, signs, representations they use to interact with the environment (mates, teacher, artefacts and so on).

As to criterion a), one must look for the genesis, developments and changes of the function concept in the course of history, namely one must pursue its epistemological and historical roots (Tall et al., 2000). Some of these roots have also a cognitive and educational interest, as widely discussed in the literature (for a short summary see: Arzarello, in print; see also the general discussion in Furinghetti & Radford, 2002). Of particular interest are the phenomena of *change* and *motion*, which we find incorporated into many ICT devices (e.g. motion detectors connected to computers, see Nemirovsky, 2003). As widely discussed in Kaput et al. (2002) such devices are genuine new representational infrastructures, which can produce a positive cognitive resonance in pupils and support their learning.

The findings in our teaching experiment, with all their limits, confirm that this approach to ICT is an useful research tool to understand the ways in which technological artefacts can mediate/support the construction of the student's mathematical knowledge. In fact, the analysis of the cultural and cognitive ingredients of the technical tools used in the classroom allows to consider the value-added component provided by the technology not limited to its purely technical features. As well, the APC-space components allow to consider the emergence of the expected knowledge not as a result of a game definitely confined to the relationships between the subjects and the 'milieu' but in an environment where the 'game' consists in a semiotic mediation that involves the students, the ICT and the teacher, who rules and supports the evolution of the personal senses, which students attach to their actions with ICT, towards the scientific shared sense (see the discussion in Mariotti, 2002). The teacher's task consists in promoting the integration of the cultural and biological roots of the mathematical ideas within suitable representational infrastructures. This approach allows to nurture their cognitive resonance in students and produce what I call *learning in a natural setting* (the idea is from Tall, 2001).

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