“Dignam mathematicam docere” - Axiomatics meets axiology

Paola Ghiringhelli e Gemma Testin. Scuola Elementare “A.Scarpa” via Clericetti. Milano
Margherita Locatelli. Scuola Secondaria di primo grado “Aldo Moro”. Presezzo (BG)
Paolo Longoni, “Didactics Laboratory of Mathematics and Philosophy” – Bergamo
Gianstefano Riva, Instituto “Maironi da Ponte” – Presezzo (Bergamo) giansri@tin.it
Ernesto Rottoli, Instituto “Secco Suardo” – Bergamo ernerott@tin.it

We try to show, through some historical examples, how mathematics could be a text where to read dynamics of society.
Today crisis of universality joins mathematics, science and society, and produces a bewilderment in teaching mathematics.
Dignity of particular opens a new ethic horizon to thought mathematics, that becomes so the text of a hoped society. Paper folding shows didactic dignity of thinking by hands

Teaching is going across a phase of bewilderment that seems new and nearly unforeseen if compared with enthusiasm that accompanied it along most of last decades. We believe that this fact is due to the crisis of universality that has involved whole actual society and has reduced propulsive pressure of fundamental elements in didactic of mathematics: utility of taught mathematics, axiomatisation, technology. Just these elements, that during many years nourished enthusiasm, are now at the centre of an articulated debate. For example, about utility, while Freudenthal maintained that “we can no longer keep silent about teaching mathematics so as to be useful”, Maria Dedò sustains that useful mathematics in every day life is really little. About axiomatisation Kahane asserts that “when unity of mathematics seemed posed upon its foundations, it was possible to try to start from foundations to teach mathematics … Now teacher must be a bringer of a part of universal culture”.
These elements often recur in the sub-themes of this conference: sub-theme 1 speaks about to “provide students with useful, basic technical tools”; sub-theme 3 fronts “the impact of technology, and its influences on students’ skills and attitudes”; sub-theme 4 is centred in the change concerning “the importance of rigorous and systematic thinking”.
To show how the crisis of universality is intertwined with difficulties about these elements, we intend to start from our direct didactic experience of teaching, and from a historic-philosophical point of view that is not much frequented by debates in didactic of mathematics. We think that this approach should permit us to enlighten implicit elements of today’s discussion.

Crisis of universality interests many aspects of actual Western society and invests its criteria of truth, values and, in general, references and ideal tensions. The deep link between mathematics and Western society can be found just in the idea of universality: since the beginning, mathematical truth becomes necessary for the "universe" of considered objects, a prototype of rationality and a chance of discovery. By referring to the philosopher Lévinas, (*Totalité et Infini*) it is possible to maintain that a double movement is implicit in the tension to the universal: the opening one and the closing one. The opening happens when the becoming of world's things rises paths to open spaces: mathematical truth unveils itself as "in-finite" event, as a "necessary" course to a "focus", of which every historical time takes marks. But there is also a structuring movement towards totality, in which the dimension of closure prevails: mathematics set itself as enclosed system.

Besides, by freely inspiring to Ricoeur's phylosophy, it is possible to state that in some historical times, interconnection between mathematics and society is so deep that mathematics becomes a text to read some characteristics of society and to individuate the ways of its movement to the universal. The antyphairesis is an example: to follow this archaic procedure in comparing two magnitudes, permits to meet again not secondary aspects of original Greek philosophy. It is the text to read Pythagoreans, to listen the rhythm of their thinking and to feel the opening of their tension towards the universal; mathematics unveils itself as "in-finite" event.

Euclid's *Elements* are, instead, an example of the aspiration to the universal in which the movement of closing is prevailing. On the one side they contain and tell the stages of forming western rationality. On the other side, however, in its final draft, they constitute a form of closing in regard of debate and potentialities of Greek thinking in two preceding centuries.

Newtonian physics has become the prototype of modern science: through its mathematical language, it has strongly intertwined with modern society, and has shared aspirations and the idea of universality with that. So mathematics newly has become a text for modern society.

With the will of freedom that explodes with non Euclidean geometries, the idea of universality discloses itself as plurality of truth and centrality of freedom: "Essence of mathematical power lies in its freedom" [Toth]. Anxiety of freedom that goes across these geometries, finds consonance with ideals of Italian Risorgimento. During that one, Italian mathematics starts to flourish again; in that process, non Euclidean geometries found more attention than elsewhere (Battaglini, Beltrami) [Forti]. On the other side, plurality of truth advocated by non Euclidean geometries rises strong oppositions, and finds resistances, not only among mathematician but on the inside of society too, mirroring difficulties to accept that idea. Soon an opposed view of universality as closure matches together mathematics, science and society: the project of a total axiomatisation agrees with the

aspiration of unification of science and reverberates in totalitarianisms that have grown in the society.

Now we try to have a look to the present. The current time sees the defeat of totalitarianisms, the difficulties of science in fronting complex systems and the disillusion of mathematics as regards the axiomatisation. Mathematics, science and society live together a sense of bewilderment in face of the idea of universality. At the same time, technology, the main product of interaction among Western society, modern science and mathematics, is pushing to globalisation. So the idea of universality of Western society has to front the active presence of other cultures that makes relative reason, criteria of truth, values, rights.

The today's paradoxical risk is that, even if we live in a mathematised society [Kahane], mathematics should lose its function of text of society and should be confined to instrument for technology or slang for few super-specialists.

The bewilderment produced by this crisis leaves teachers without certain references and produces a teaching without link with universality; a teaching that becomes a training to technicism and mere memorisation.

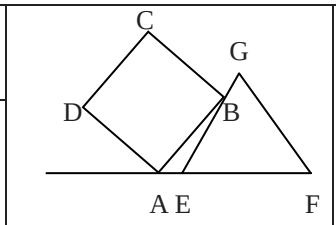
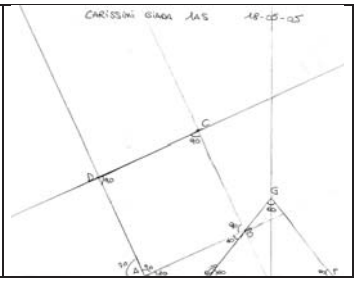
The answer could stay into recover the "flavour" of universality that is suggested in original meaning of term "axiom": *axios = dignum, id est worthy* ; teaching mathematics needs to invent dignity of what is taught. "A royal way" to mathematics doesn't exist; "master" styles and subjects of teaching don't exist. They must be reconsidered in function of moment, of background, of context: dignity is sometimes criterion of choice, but, above all, a challenge to make re-flourish taught subjects. Dignity of the particular could be the starting point to a new attention to the universal. It brings out those single moments and processes which are fundamentals of mathematics. So a new path of opening unveils. "*The whole stays in the fragment*".

In this new beginning that start from dignity of the particular, different perspectives open on questions concerning the role of ethnomathematics [D'Ambrosio], proto-mathematics [Chevallard]. exotic mathematics [Bishop], but of our mathematisation too.

We have two example through which show what we mean for dignity of particular, referring to a teacher. In one example, dignity lies in recover deep, basic meaning of quantity; in the other, dignity lies in to recover corporeity in teaching and learning of geometry, through the use of paper folding.

We now develop this second example. We have collected a series of activities of paper folding and we have experimented them in our classes, from primary grades to high school. Aims differ according ages: for younger children the aim is to facilitate logical capabilities and memorization by imaging; a generalized aim is to widen and to deepen students knowledge about geometric forms,

by creation, modification, investigation; for higher students the aims are to project the sheet of paper and to structure the succession of folds, to experiment and explore spatial reasoning by practical activity , to verify and check results, to communicate through practical demonstrations.
We have proposed the following example to a class of the first year of a high school, "Liceo delle Sciense Sociali. Its dignity rises from faculty to involve interest of students and force them to careful analysis of the text in term of elementary operations of paper folding.

ABCD = square EFG = equilateral triangle • *Construct the figure with paper folding* • *Esteem* ∠BAE • *Determine* ∠EBA		

In a first exploration we compare the results of our class in drawing this figure with the ones of a parallel, more quoted class. The slightly better result in our class suggest us the hypothesis that activity of paper folding could deepen the attention of students in the first two steps toward the resolution of problems geometric: interpretation and representation [Polya].
In conclusion, the need of dignity for what is taught, demands a different way to be a teacher. It is an opportunity of dignity for the teacher itself. The *axios* discloses as ethical dignity.
So mathematics is not only a site of formalisation but open itself to axiology, that is to an horizon of ethical values, chosen in function of human dignity. It could become a text of a hoped society.

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