Algebra for primary grades: construction of sense of quantity (second part)

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We now present the continuation of activities of algebra for primary grades, that started three years ago. While the first report at CIEAEM 55 concerned the second degree, now we report about the activities made in third and forth degrees.

At CIEAEM 55, held in Plock, we presented a work which dealt this question: is it possible to direct children’s thought towards algebra even since the first years of elementary school? We described there our activities with children of second degree [Bonetto].

In the following, we have continued this type of activities with children of third and forth degree. Here we report the carried out activities, the drawn observations and some proposals. We believe that our activities suggest some answers to the question of “the importance of providing primary school students with useful basic tools”, that is posed in sub theme 1.

There is a number of approaches for developing meaning for the objects and processes of algebra [Chick & Stacey]. The most widespread one is generalization approach: before children become familiar with arithmetic procedures; when they apply arithmetic rules and they learn quantitative relations, they should uncover the basic of structural properties and so they accumulate indications about some regularities; these last lead them in following abstraction of general principles and relations [Morris]. But experimental literature have stressed the fact “the generalization of numerical patterns and the symbolic formulation of relations between variables raise specific problems for novice students [Radford]. So questions arise about little effectiveness of the generalization approach. To answer this questions, different attitudes have been taken.

For example, Radford investigates the way students use signs and endow them with meaning in their very first encounter with the algebraic generalization of patterns. NCTM asserts that background of generalization must be broadened by a rich pattern of exercises that should allow to develop skills in constructing and representing regularities, in consciously generalizing, in actively exploring and in formulating conjectures.

A different attitude has been supported by Davydov. During sixties of twentieth century, he experimented in some primary school of Moscow, an approach to algebra by activities that are independent and parallel to arithmetic ones: algebraic activities in primary school have a different target respect to arithmetic ones. In fact Davydov observed that “Three components exist in each arithmetic calculation: numbers, arithmetic rules and relations. While children are executing an arithmetic procedure, their attention is principally turned to numbers and arithmetic rules, but not to relations”. The failure in perceiving relations entails often the lack of base for inductive generalisation. So Davydov, with his collaborators, planned and experienced a series of activities oriented to give younger children opportunities to cultivate, in a natural way, simple forms of reasoning that involve the relations of equality and order. [Davydov].

We have chosen to make use of the Davydov’s activities as starting point for our “research in teaching”. We have reorganized them by keeping in mind the recognition that the language of letters and the transformations are the key elements of elementary algebra. Therefore, first of all, activities have been set in order to turn the mind of children to the language of letters; real quantities have been considered only in the relation of whole and parts and so, usually, there has been no need to count, to measure, to weigh. Afterwards, to transform literal expressions, by using properties of operations, has become the prominent aim.

When we have adapted these activities to our classes, we have found ways of presentation partially different from the Davydov’s ones and entirely different times. 1) Times have been wholly different because, while Davydov’s activities were presented to children attending to classes that correspond to second degree, we have expanded them throughout the four years from second to five degree and we have tried to integrate them with other arithmetic activities. 2) Aims too have been modified and, where Davydov spoke of “understanding”, we have talked about “intention”, about “turning minds of children to …, in the most natural possible way”. Proposed activities should let in the mind of children a “trail”, instead of specific knowledge. Children would have recourse to this trail later on, when learning of algebra actually starts.

According to Davydov, also in our activities children have withdrawn from observations of real objects and they have expressed them by means of relations among letters, writing literal expressions. Then they have advanced evaluations about literal expressions, by using intuitive rules of logic. Later on they have been concerned with transformations of literal expression: they have started again from transformations among real objects and then gradually have reduced the role of these ones, while their familiarity with properties increases. At last, they have been involved in translation of problems in their resolutive formula.

Children have worked in group; the discussions among them and with teacher have contributed to grow their interest and curiosity and to engage them at most of their capabilities.

The structure we have proposed in our classes is the following:

- For second degree
    - To register, by means of relations among letters, the changes of physical quantities
    - To recognize the changes expressed by literal relations and to represent them by physical quantities
    - To describe and discuss the changes in physical quantities and in literal expressions
    - To record by strips of paper the changes in physical quantities and in literal expressions
    - To write the expression of "simple" additive problems

- For third degree
    - To reconstruct literal relations by strips of paper
    - To restore the equality by using strips of paper
    - To recognize inequalities among letters without using physical quantities and to restore equalities
    - To determine the added or taken away quantity in literal relations
    - To write the expressions of adequate problems

- For forth degree
    - To translate a problem in a literal expression and to represent it by strips of paper
    - To recognize relation among letters in equalities and inequalities;
    - To determine the added or taken away quantity in literal relations
    - To translate literal expressions in problematic situations
    - To determine added or taken away quantities in problematic situations.

Our activities have continuing until the end of school year.

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