History of Mathematics in the Curriculum and the Possibility of a New *Paideia*

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At *CIEAEM 51* in Chichester, Fried (2000) pointed out what he saw to be a theoretical dilemma in the attempt to combine history of mathematics and mathematics education. This was later sharpened and elaborated in Fried (2001). Very briefly, the argument can be framed as follows. Given the usual mathematics curriculum—and it is this, in part, that we shall want to challenge in the end—the attempt to combine history of mathematics and mathematics education entails a clash of commitments. Mathematics educators are committed to presenting *modern* mathematics; they are committed to students' understanding of the mathematical techniques and concepts used so powerfully in so many applications and fields of study; they are committed to students feeling at home with modern mathematics and recognizing it as a tool which they have the power to use. Historians of mathematics are interested in *shaking off* modern mathematics, "unthinking much that has become part and parcel of [their] mental equipment," as the historian W. K. C. Guthrie (1975, p.3) put it (speaking about the historian's task regarding the history of Greek thought); they are committed to seeing how the mathematics of the past diverges from mathematics as it is understood today, how diverse mathematical thinking can be; mathematics, for them, is never just mathematics, never a completely coherent body of knowledge that changes only by accumulation. Thus, the dilemma is that if one is true to the commitments of mathematics education, as they are traditionally conceived, one is forced to adopt what Butterfield (1951) called a *Whiggist* brand of history--history which sees history always in modern terms, history *which is not* history (in the sense of truly trying to understand the past on its own terms). On the other hand, if one is true to the commitments of the history of mathematics one is forced to spend time on mathematical and philosophical ideas which may not be relevant to the general mathematics curriculum.

While that argument may be correct, and the dilemma it lays out real, its critical character makes it appear also a dead end. This is ever a problem with criticism—by asserting what should not be, what is wrongheaded, what is ineffective it seems to send only a negative message. Yet, criticism contains, more often than not, a latent *positive* message as well. In the oral presentation that we are proposing for CIEAEM 57, we want to try and begin to uncover what that positive message might be. We want explore the possibility of a framework for mathematical education which sees a crucial role for the history of mathematics and which conceives mathematics as essentially a *humanistic* subject. In other words, we treat the dilemma described above as an invitation to reconsider the general orientation of mathematics education.

The points of departure for the paper are necessarily broad and include, besides that already mentioned, studies on mathematics in the light of culture (e.g. Bishop, 1988; Powell & Frankenstein,1997; Lave, 1988) and semiotic aspects of mathematics education (e.g., Radford, 2003; Fried, 2004). Another point of departure, and the one

we shall use as a focus for the questions we want to ask, comes from the history of mathematics itself, specifically, from a chapter in that history where mathematics was conceived *as* a component in cultural education. We refer to the idea of *paideia*, the classical educational ideal which began with the rhetorician Isocrates and crystallized in late antiquity.

At first sight, it may seem odd and Eurocentric to turn to an ideal from the classical world (especially one associated in some ways with conquest in Hellenistic times) for insight into how to meet the challenge of mathematics education in light of changes in society. To understand why this is not so, one must understand what *paideia* meant to the classical mind. For this, it is useful first to recall that in Roman times *paideia* came to be translated as *humanitas*. What it was to be a humanistic subject then was precisely to be related to *paideia*. Thus, Marrou (1982) points out that although the word *paideia* was derived from *pais*, child, in its eventual application it was directed towards the *whole human being* into which the child develops. To be a human being, for the classical mind, was to be whole and to engage in activities of which human beings are capable—"For Man included all this, and any kind of choosing meant self-mutilation" (Marrou, 1982, p.220). The wholeness that *paideia* embraced was both the wholeness of a culture and the wholeness of an individual human being. Mathematics being then (and now) part of both also had a place in both. This then is the first lesson of *paideia*, that mathematics education is education for *whole* human beings engaged in human activity. This implies even a stronger position than that mathematics *can* "...become a subject that can be shared with schoolmates, a 'social medium'"; it implies that we *must* teach our students that mathematics, rather than a subject for a special sort of person, is part of what makes every student a person. History of mathematics, in this light, is crucial since it both presents mathematics *as* the product of human activity and it shows the variety of ways in which that activity can be understood. In this way too, history of mathematics brings mathematics more in line with the project of 'mathematics for all' and with the aspiration towards broad 'mathematical literacy'.

The second lesson, going in the opposite direction, has to do with how *paideia* entered into and formed mathematical practice. In studying the late classical mathematical texts of Pappus and Proclus, Bernard (2003a ,2003b) has shown that *paideia*, especially as reflected in its rhetorical practices, gave those texts their particular shape. This is evident in the vocabulary used in those texts, words such as *problēma*, *hypothesis*, and *heuresis* being clear examples. But, more significantly, such texts were written by people trained to write rhetorical texts that inspire rhetorical practice. A text written with this background "...therefore functions as a kind of *trap* for its reader or its listener...Mathematical texts, that is, texts that are *mathemata* in the true sense, '*learning matters'*, also share in this particular form" (Bernard, 2003b, p.409). This is an important and subtle point. Although the *paideia* of classical times represented a particular culture it was not closed; it invited reflection on the tradition it represented and engaged the reader to move beyond it. In fact, this is built into the very meaning of 'tradition', whose root, *tradere*, means both 'to pass down' and 'to betray' (see Brann, 1979, p. 67). The idea of a mathematical tradition, then, entails having one foot in the past and one foot in the present; involvement in tradition, in this reading, means becoming part of a greater community whose continuity extends also over time. This can be related to Lave and Wenger's (1991) idea of 'communities of practice' and the theory of learning connected with it, namely,

'legitimate peripheral participation'. According to Lave and Wenger, learners are always engaged in the practices of a community, and, by participation in these practices, eventually become active and productive members of the center of the community; it is a view which conceives learning, thus, as "an evolving, continuously renewed set of relations" (p. 50) (see also Amit and Fried (2005)). In sum, the second lesson of *paideia*, is that a curriculum taking seriously mathematics in the context of a mathematical tradition, encourages learners, at the very moment they are learning modern techniques and ideas, to see those things alive and changeable and to see themselves as a part of an active ever self-modifying community.

In general, the historical study of mathematics in the context of classical *paideia* may lead one to think about mathematics education as deeply, indeed inextricably, entwined with cultural education. Mathematics education conceived in *this* way engages learners to reflect on the very question of their own culture and on themselves as *whole*, not fragmented, human beings. It forces one to confront the problem of tradition and the way an engagement in the past makes us fuller and active, not submissive, members of a greater and present community. In a sense, this approach sharpens rather than solves the problems set out in Fried (2000): paradoxical though it may seem, consideration of the old *paideia*—whose sense of tradition involved a *betrayal* of the past, as we mentioned above—gives historical legitimacy to the tendency toward anachronism in mathematics education. Yet, we believe, that the awareness of this precarious relationship to the past that comes with thinking about the *paideia,* as well as the issues raised in Fried (2000), may in fact be lead us back into a humanistic tradition and not away from it. This new *paideia*, in turn, may be an opportunity for students to see themselves, when studying mathematics, as participating and forging that new tradition, rather than engaging in an exclusive and specialist activity.

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