Mathematical Contest in Modeling and the Teaching of Mathematical Modeling

Qiyuan Jiang[1], Jinxing Xie[2], Qixiao Ye[3]

Abstract: China Undergraduates Mathematical Contest in Modeling (CUMCM) is a national annual team competition for undergraduates in China. The aim of the contest is to give students an opportunity for practicing the whole mathematical modeling process and to improve students' understanding of mathematics, especially mathematical modeling and mathematical experiments, and to enhance students' motivation for studying mathematics and to cultivate students' creativity and overall ability. In order to benefit more students and teachers a two-year project on penetrating ideas and methods of mathematical modeling through into the main mathematical courses initiated by the National Organizing Committee of the CUMCM in 2004 will be briefly presented. The Project's emphases are put on designing mathematical modeling modulus, which include whole mathematical modeling process from real world problems and can be effectively penetrated through into the existing courses teaching. A sample module "why a Coca Cola can takes such a shape?" is given.

1. China Undergraduate Mathematical Contest in Modeling

1.1 Aims, Scope and History. CUMCM is a national annual contest in China for undergraduates. The aim of the contest is to give students an opportunity for practicing the whole mathematical modeling process and to improve students' understanding of mathematics, especially mathematical modeling and mathematical experiments, and to enhance students' motivation for studying mathematics and to cultivate students' creativity and overall ability.

In the contest, up to three students (it doesn't matter they are from majors in mathematics or from non-mathematical fields) advised by at most one teacher form a team. The team's task is to develop in three days mathematical and computer models to solve a simplified real-word problem in engineering, management, etc. During the contest days, the students can use any references or materials they can find (including the materials on the INTERNET), but discussions with the advisor or out-of-team members are forbidden.

The Mathematical Contest in Modeling (MCM) was initiated and organized by the Consortium for Mathematics and its Applications (COMAP) in USA in 1985. Teams from universities in China have participated in the contest every year since 1989, and recently more than half of the teams are from China.

Recognizing the mathematical contest in modeling is beneficial to the students and helpful to the reform of mathematics education in universities, the China Society

---

[1] Department of Mathematical Science, Tsinghua University, Beijing 100084, P. R. China. Email: qjiang@math.tsinghua.edu.cn
[2] Department of Mathematical Science, Tsinghua University, Beijing 100084, P. R. China. Email: jxie@math.tsinghua.edu.cn
[3] Department of Mathematics, Beijing Institute of Technology, Beijing 100081, P. R. China. Email: yeqx@bit.edu.cn

for Industrial and Applied Mathematics (CSIAM) firstly organized CUMCM in 1992. CUMCM is co-organized by CSIAM and the Ministry of Education of China since 1994. The contest is very welcomed by the students, thus the number of participating students grows very fast from year to year. Currently, CUMCM becomes the most widespread out-of-class scientific activity for undergraduates in China. In 2004, there are 6881 teams from 724 institutions in China (including Hong Kong Special District) participated in the contest, in which more than 80% of the participants are not majoring in mathematics[4].

1.2 Organizing and Evaluation System. There is a CUMCM National Organizing Committee (NOC), which is setup jointly by the Ministry of Education of China and CSIAM. In most of the provinces or regions in China, there is a CUMCM Local Organizing Committee (LOC).

When the contest ends, the LOCs will evaluate the contest papers and rank the teams from their own local regions. Only the top 12% papers can be submitted to the NOC to compete the nation-level awards. The nation-level awards are classified into 1st and 2nd prizes. About 15 best papers resulted from the contest will be selected and published in the journal *Engineering Mathematics*, which is one of the official journals of CSIAM.

1.3. Training and Influence. Most students who register to the contest have some training in mathematical modeling from their mathematical modeling course or related courses. Some students prepare the contest just by teaching themselves with out-of-class materials related to mathematical modeling. They can also get some advices from their advisors before the contest.

Teachers are the key to the successful education. In order to train the university teachers so that they can become a good advisor for the students, in recent years, we have run several short-term training seminars on different scales. A national conference titled CCTMMA (China Conference on the Teaching of Mathematical Modeling and Applications) is also organized every two years to exchange the experience among the teachers. These activities have promoted the teaching reform on mathematical education, and have enhanced the teaching quality of courses related to the mathematics in most universities, especially by penetrating the mathematical modeling thoughts into the main courses such as calculus, algebra and probability theory, etc. The contest also boosts the publication of many innovative textbooks on mathematical modeling and mathematical experiments.

The contest is very welcomed by the students and the special experience in the contest is a real help to their innovative potential and cooperative spirit. They conclude the experience with the sentence "Once participated, benefit for life long". Through the training and preparation before the contest, the hard work in the three days contest, and the summarization and discussion after the contest, the students' creativity and overall ability are improved significantly. Indeed, most of the winners in CUMCM have done very well in their positions in both academic and industry areas. Therefore, the institutions and industries are getting to know more and more about CUMCM, and they are glad to accept the students who have the experience of

---

[4] More details can be found from http://www.comap.com and http://www.mcm.edu.cn .

the contest when they go to graduate schools or find jobs after their graduation.

1.4. Difficulties and Prospects. CUMCM also encounters some difficulties as more and more students come to participate in the contest. The most important task faced by the organizers of the contest is about how to improve the quality of the contest including quality of the contest problems[5], the equity and fairness of the contest etc.

In the following years, there are definitely more and more students in China will participate in and benefit from the contest. The students from out side China are also welcomed to participate in CUMCM. The organizer hopes to make CUMCM an international mathematical contest in modeling for university students all over the world.

2. Penetrating Ideas and Methods of Mathematical Modeling through into the Teaching of Calculus

2.1 Its importance. More and more professors, teachers and administrators from universities and Ministry of Education in China realize that mathematics teaching at the university level is so important that it will have great influence on students' careers. Especially, they realize that mathematical modeling techniques are important from the success of the China CUMCM. Various required or optional courses on mathematical modeling at different level are offered in more than 400 universities, but only a small part of students take these courses. We have around five million students each year entered into universities, most of them have to study calculus for one or two semesters. Many of them don't know that why they have to spend such a lot of time to study calculus and why it is important for their future careers, so their study are not active. In order to solve these kind problems, the National Organizing Committee of CUMCM initiated a two-year project in 2004 titled "penetrating ideas and methods of mathematical modeling through into the main mathematical courses". The project's emphases are put on designing feasible modulus on mathematical modeling (MM), which include the whole mathematical modeling process from real world problems and can be embedded or effectively penetrated through into the existing courses teaching.

2.2. A sample module — "why a Coca Cola can takes such a shape?" Almost all the calculus course includes a section on optimization problems as applications of differentiation. A typical example: A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can. Or if the volume of a can is known, what is the size (its radius and high) to minimize the surface. Teachers can use the module to teach the optimization problems making them more attractive.

First, we have a sub-section on what is mathematical model and what is mathematical modeling. A mathematical model is a (rough) description of a class of real world problems or phenomena expressed using mathematical symbolism. The process of building, solving, and validation of it is called mathematical modeling. Almost all the persons from ancient to modern who use mathematics to solve the real-world problems are

---

[5] All the contest problems in English during 1992-2004 can be downloaded from http://www.mcm.edu.cn.

using the ideas and methods of mathematical modeling. There are 7 key steps of mathematical modeling process.

2.3. The teaching of this module. Before you teach the problem "why a Coca Cola can takes such a shape" (may be before the ending of your last lecture), ask students to review the related text and do the typical example mentioned above, and take a Coca Cola can to measure its dimensions and other data (for instance, approximately the radius of its top cover is 3 cm, the radius of its middle part is 3.3 cm, and its height is 13cm). Then you can teach the MM process step by step together with your student in the classroom when you start your next lecture.

A simplified model: Simplification. Assume that the can is a right circular cylinder. Determine variables and parameters. The radius r and height h of the can are variables, the volume of the can V is a given parameter. The thickness of the other part is b cm and the thickness of top cover is αb. Math model. Denote surface area of the can by S, we have

$$S(r,h) = \pi b[(1+\alpha)r^2 + 2rh]$$
$$G(r,h) = \pi r^2 h - V$$

We need to find the r and h such that the minimum surface area under the constraint $G(r, h) = 0$, i.e. the math model is

$$\min_{r>0,\ h>0} S(r,h),\ s.t.\ G(r,h) = 0$$

Solving this math model. If $\alpha = 3$ (someone measures the thickness of the top cover and other part it is just so), then the ratio $r : h = 1 : 4$. The measurement roughly tells us that it is right!

The students will discuss whether this model and the result are correct or not, or if we need to modify the model, and so on.

2.3. Exercises. We design several exercises for students to practice the modeling process, especially, to get insight of the importance of mathematics and the necessity and convenient of using mathematical software, such as Mathematica, MATLAB, etc.

2.4. Assessment. (Teamwork) Ask each team to write an essay on an optimization problem about a bowl shape (the frustum of a right cone) container. It is also a mathematical experiment.

2.5. Materials for further reading. We also design a reading material on isoperimetric problems in order to increasing their interest in studying math and to strengthen their math knowledge, which includes the following two quotations. For instance, "The Legend of Princess Dido"(The Isoperimetric Problem).

References

[1]   *China Undergraduate Mathematical Contest in Modeling* (in Chinese), Edited by Li Daqian, 2nd Edition, Higher Education Press, 2001.

[2]   Jiang Q (1998), *Teaching of Mathematical Modeling in China*, in Mathematical Modeling: Teaching and Assessment in a Technology-Rich World, P. Galbraith *et al.* (eds.), Horwood Publishing, Chichester, pp.337-344.

[3]   Qixiao Ye, *Penetrating Ideas and Methods of Mathematical Modeling and Mathematical Experimentation into the Teaching of Calculus* (in Chinese), Engineering Mathematics, v.20(2003), no.8, 3-13.