The effects of different modes of representations in mathematical problem solving

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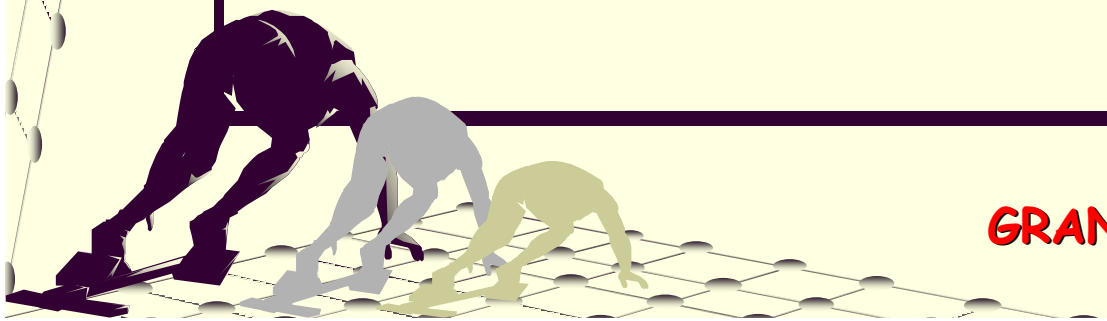
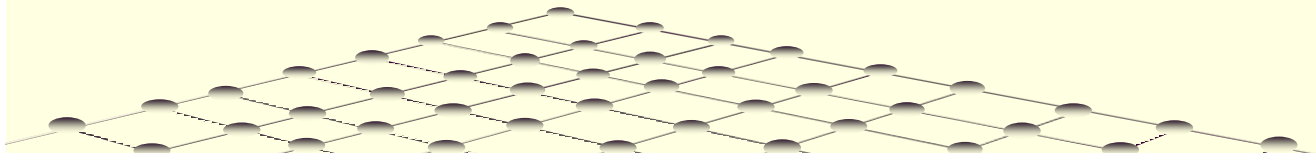


GRANDE FESTA DELLA MATEMATICA

Introduction

Un'immagine vale più di mille parole…Ma che tipo di immagine risulta più efficace nelle attività di problem solving?

- What is the role of representations in the understanding and learning of mathematics?
- Why do we use so many pictures in mathematics textbooks?
- Can a picture attain the function of a text?

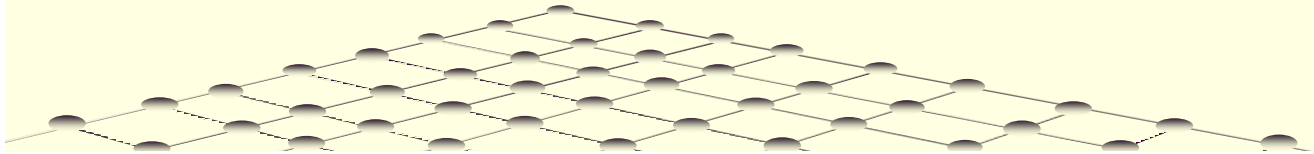


Introduction

The importance of the diversity of representations in the learning of mathematics

- It is at the core of mathematical understanding.
  - *A concept in mathematics is accessible only through its semiotic representations.*
  - *A representation cannot describe fully a mathematical concept.*
  - Each representation has different advantages and capabilities *(Duval, 2005).*

- The recognition and processing of a mathematical concept in a variety of representations and the conversion from one representation to another are considered as basic components of the understanding of the concept.

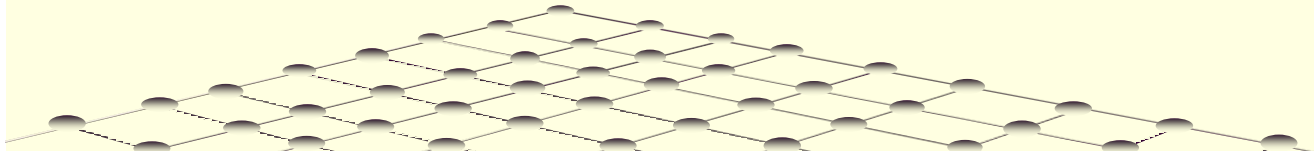


Introduction

The focus of this presentation

Number line

Pictures



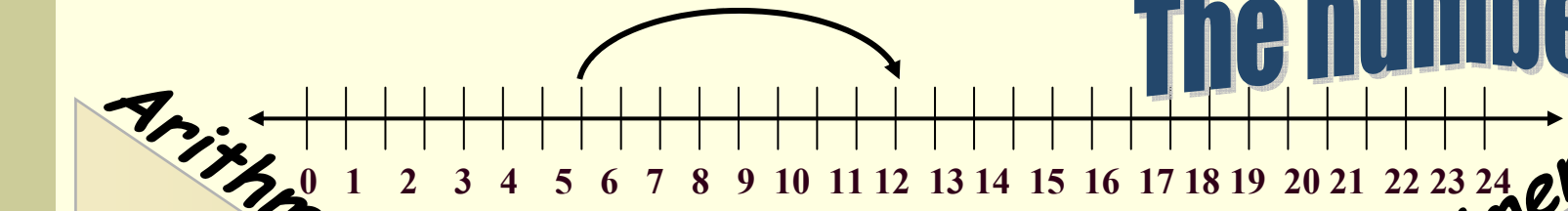
The number line

- In elementary mathematics a representation that plays an important role in the teaching of basic whole number operations, and generally in arithmetic is the number line (Klein, Beishuisen & Treffers, 1998).

- Despite the widespread use of the number line as an aid to whole number addition and subtraction, doubts about the appropriateness of using it have been raised (Hart, 1981).

- It constitutes a geometrical model, which involves a continuous interchange between a geometrical and an arithmetic representation → Dual nature of number line

The number line



Arithmetic dimension

Points on the line can be numbered

Geometric dimension

The numbers depicted on the line correspond to vectors

The simultaneous presence of these two conceptualizations may limit the effectiveness of number line and thus hinder the performance of students in arithmetical tasks (Gagatsis, Shiakalli, & Panaoura, 2003).

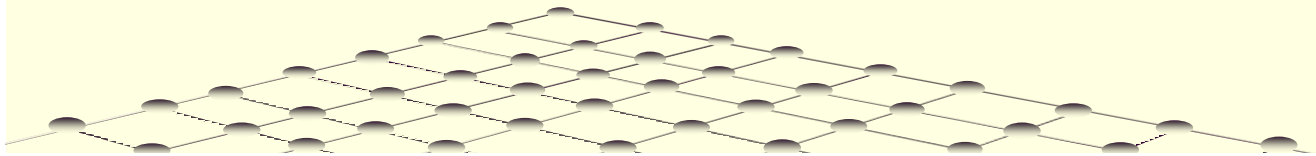
The number line

Test A

Write the appropriate number in the □

14+4=□ 13+□=16 19-4=□ 18-□=16

8+7=□ 9+□=15 12-5=□ 13-□=9



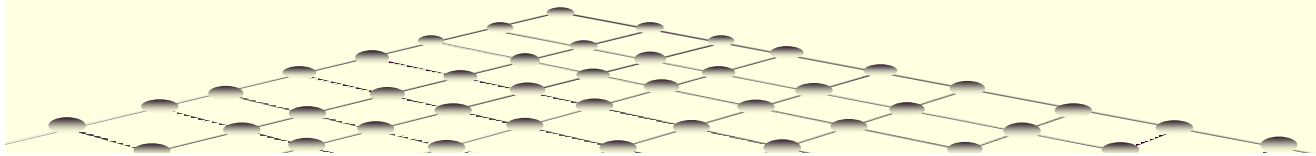
The number line

Test B

Put the appropriate number in the □
If you want use the number line model.

14+3= □ 13+□=16 19-4= □ 18-□=16

8+7= □ 9+□=15 12-5=□ 13-□=9

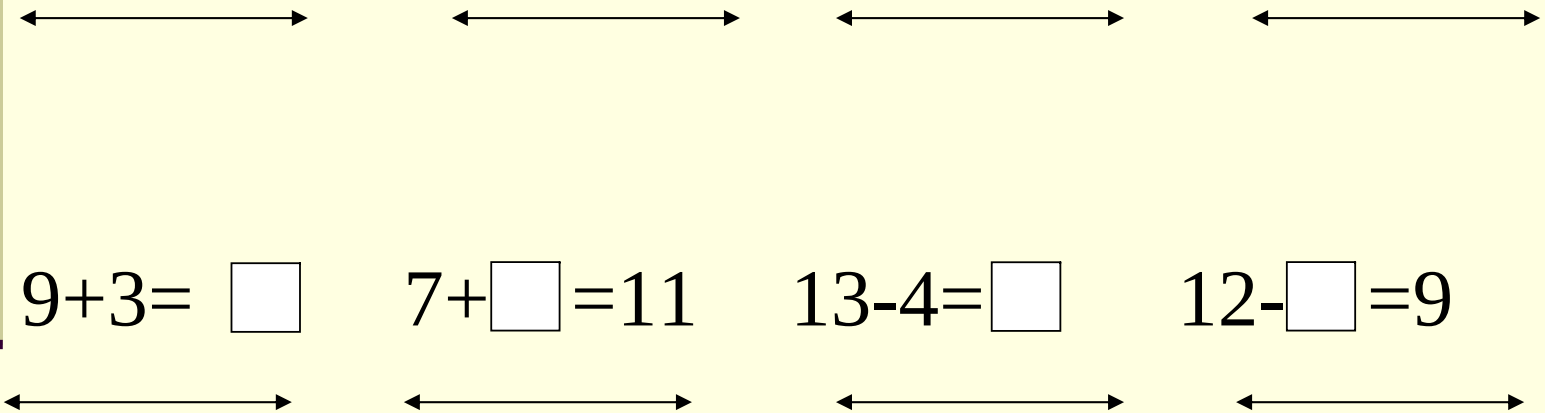


The number line

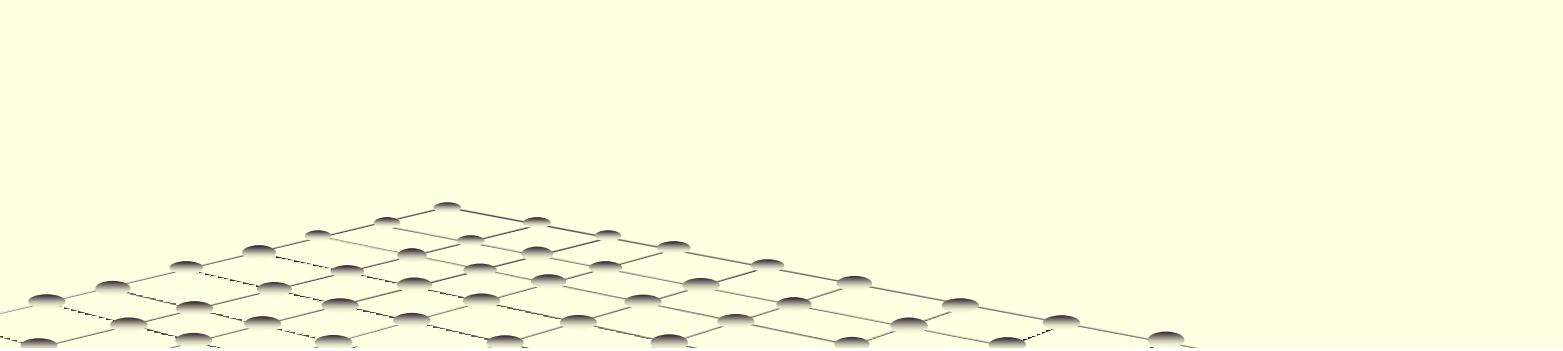
Test C

Show the mathematical proposition on the number line and find the result.

12+4= □ 16+□=19 18-3=□ 17-□=15



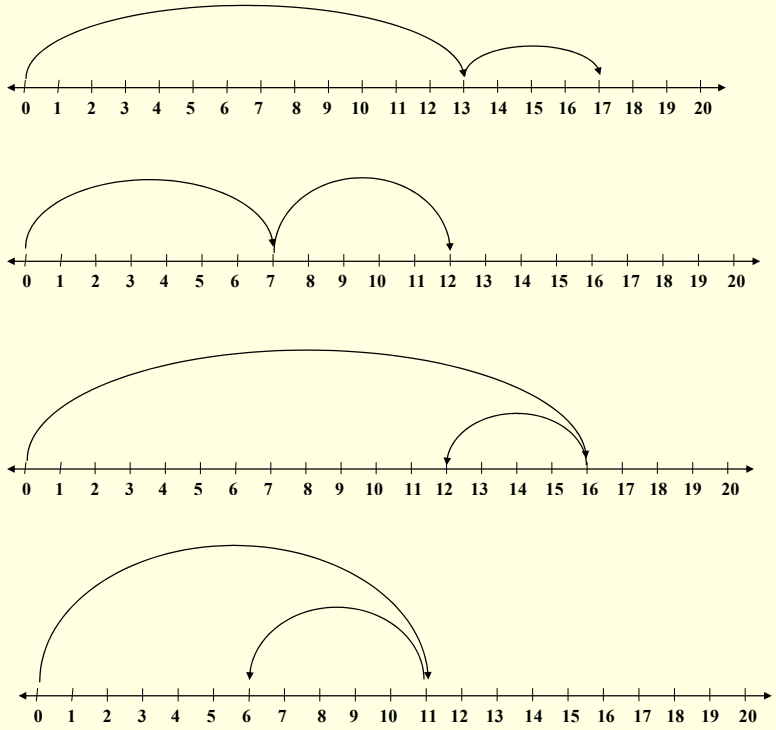
9+3= □ 7+□=11 13-4=□ 12-□=9



The number line

Test D

Write the addition or the subtraction sentence that matches each number line model and find the result.



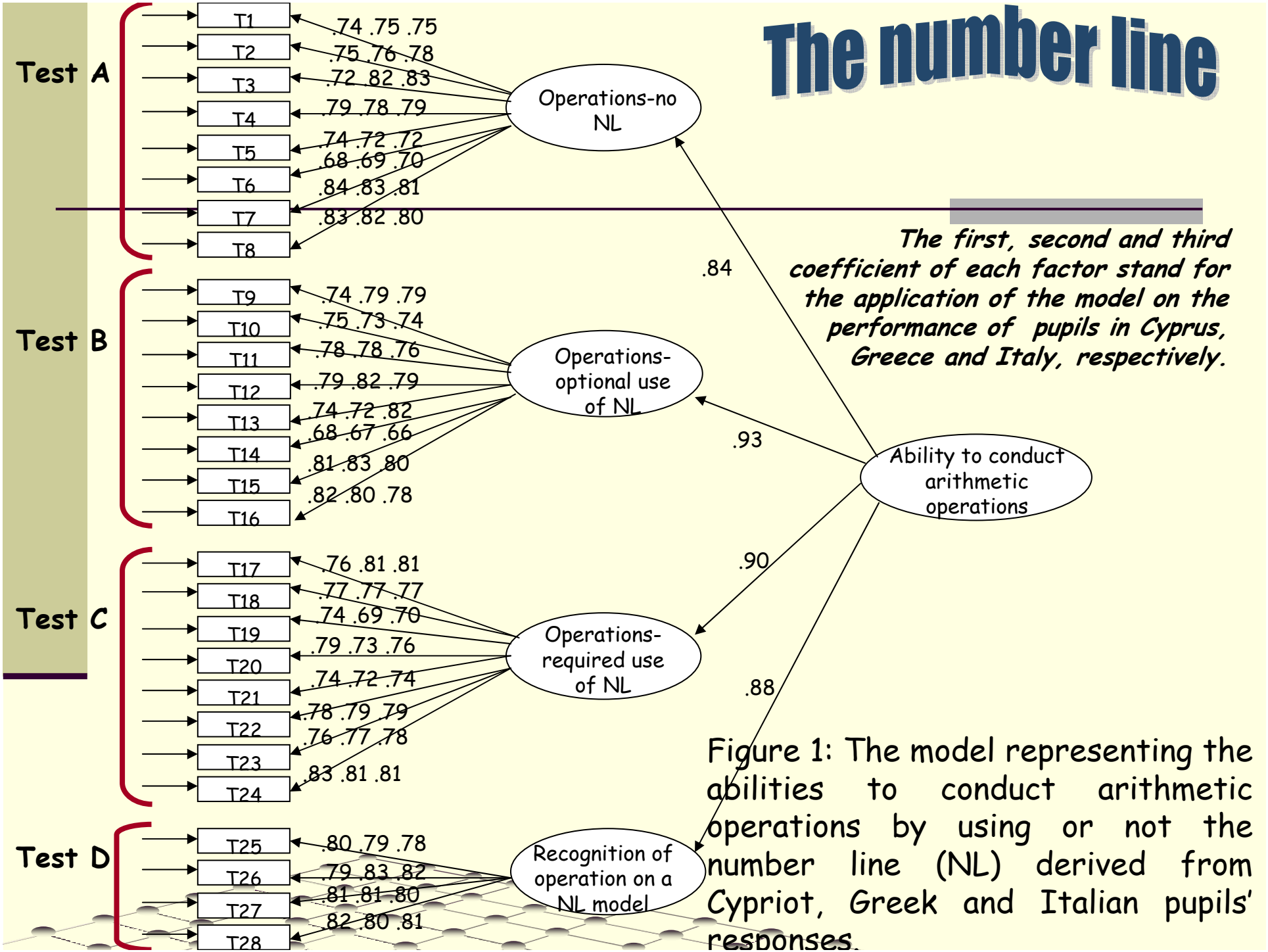
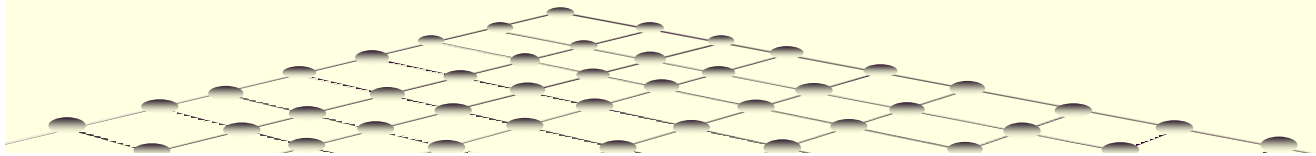


Figure 1: The model representing the abilities to conduct arithmetic operations by using or not the number line (NL) derived from Cypriot, Greek and Italian pupils' responses.

The number line

- The model raises a question of a great importance and interest for the mathematics educators of the three countries whether it is possible to teach or assess whole number operations in isolation from other skills and concepts such as those related to the use of number line diagrams.

- The model that was empirically validated in this study provides evidence to the **multidimensionality of pupils' skill in conducting arithmetic operations** and suggests that at least four different dimensions could be taken into account in the teaching and assessment of this skill.



Pictures

Four different functions of pictures in problem solving

Informational pictures

- provide information that is essential for the solution of the problem

Organizational pictures

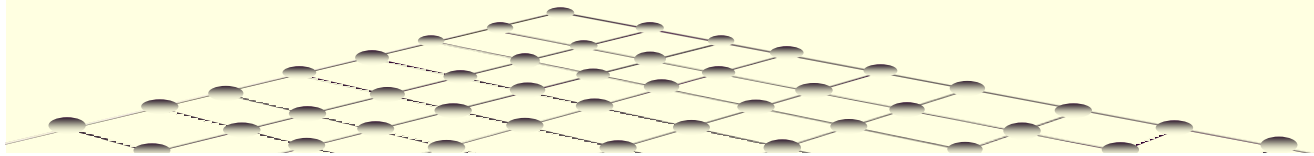
- provide directions for drawing or written work that support the solution process

Decorative pictures

- do not give any actual information concerning the solution of the problem

Representational pictures

- represent the whole or a part of the content of the problem



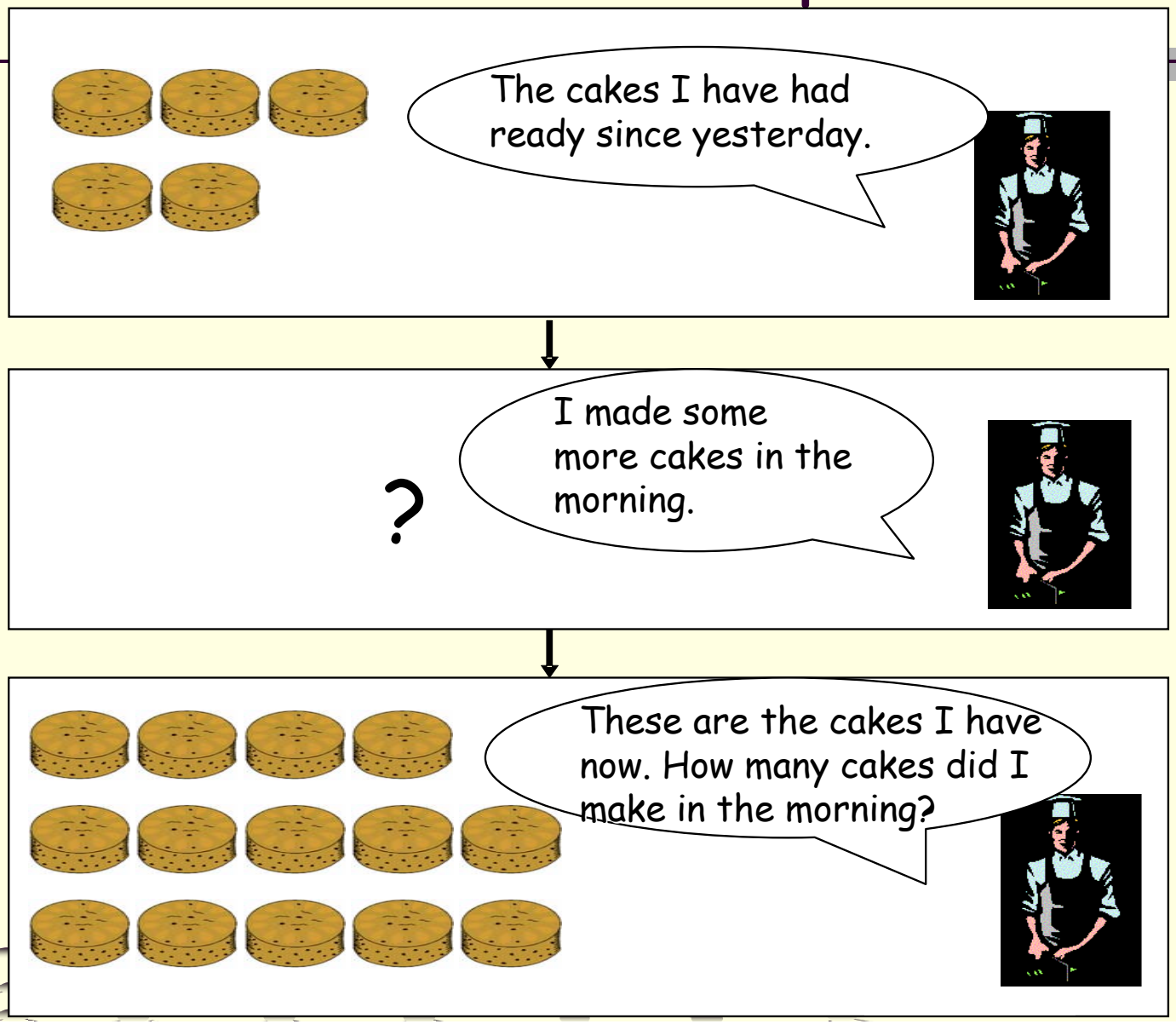
Pictures

Organizational picture

Five children shared 15 bars of chocolate. How many bars of chocolate did each get? ……. How many bars of chocolate remained? …….

Pictures

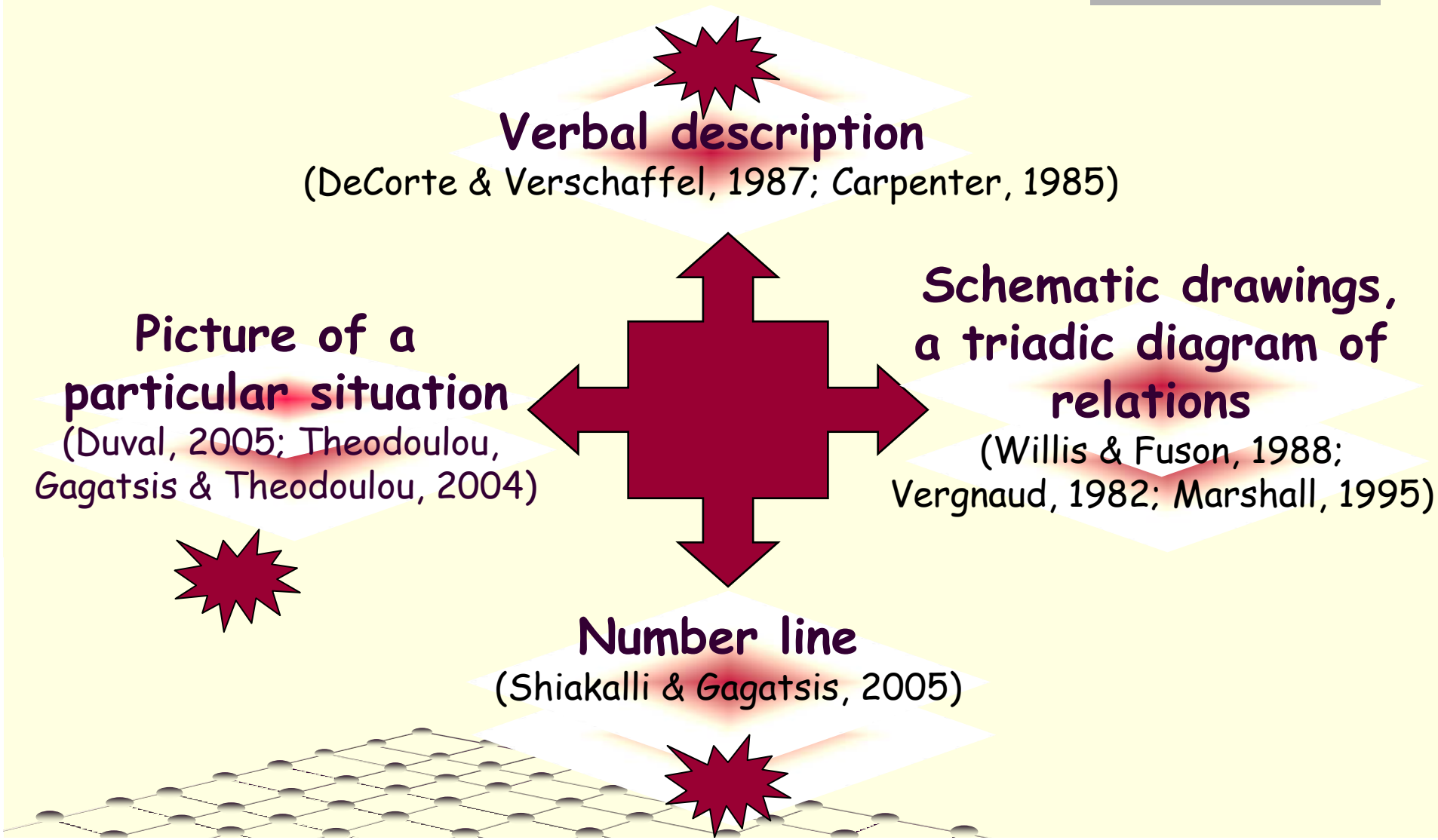
Join situation problem with the unknown in the transformation: The informational picture



(IJb)

Pictures

Representations used in additive problem solving



Pictures

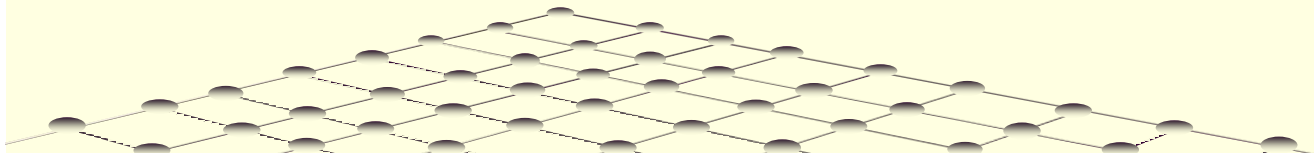
The effects of different modes of representation on additive problem solving

PhD Iliada Elia (2006)

Purpose

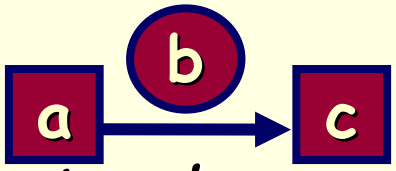
- To explore the effects of the **informational picture, the number line and the verbal description** (text) on the solution of one-step change problems of additive structures.

- To investigate the possible **interaction of the** various **representations with the mathematical structure** and more specifically with the placement of the unknown, on students' ability to provide a solution to additive change problems.



The test

Pictures



18 one-step change problems
(measure-transformation-measure)

Type of the relation

9 join situation (J) | *9 separate situation (S)*

The placement of the unknown

start.amount(a) transf.(b) fin.amount(c) start.amount(a) transf.(b) fin.amount(c)

V P L V P L V P L V P L V P L V P L

Representation

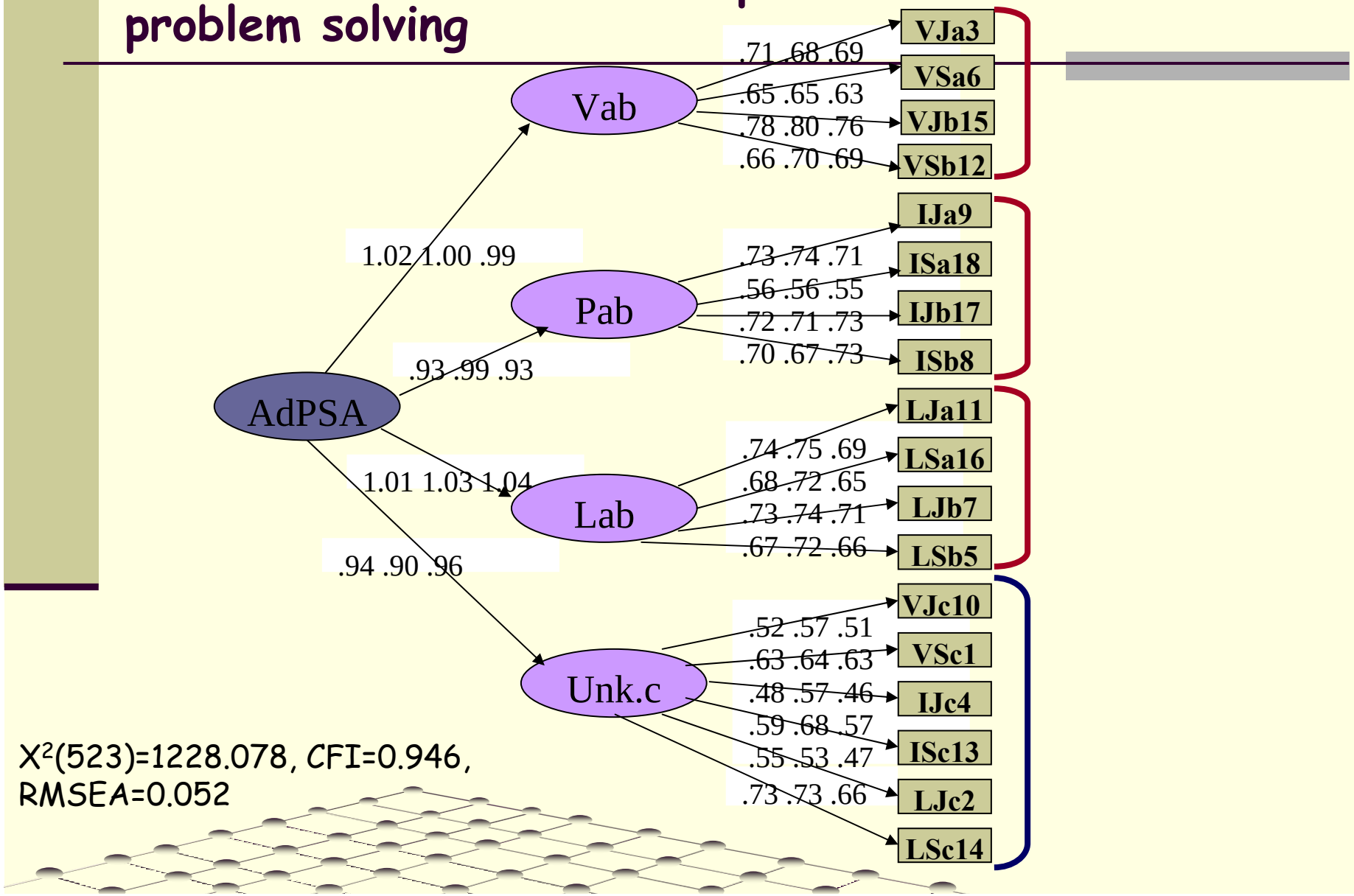
V= verbal, P= informational picture, L = number line

An example of the symbolization of the variables:

VJb= a verbal problem of a join situation having the unknown in the transformation

Pictures

Figure 2: The model for the role of the positions of the unknown and of the representations on additive problem solving



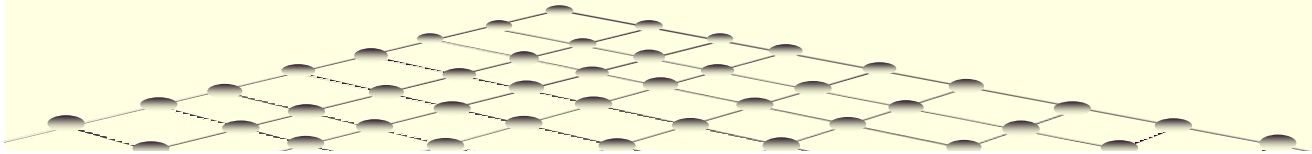
$X^2(523)=1228.078$, CFI=0.946, RMSEA=0.052

Pictures

Concluding remarks on the role of representations in problem solving

- The findings revealed that pupils **dealt flexibly and similarly with problems of a simple structure** regardless of the mode of representation. However, **when they confronted problems of a complex structure they activated distinct cognitive processes** in their solutions **with reference to the mode of representation.**

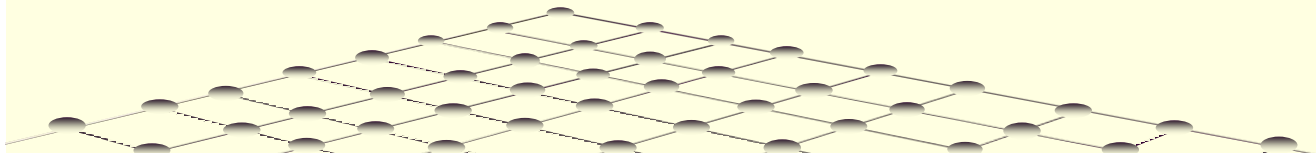
- Apart from the **semantics** of the problem, the **different modes of representation** do have an effect on additive problem solving.
- **There is an important interaction between the mathematical structure and the mode of representation in problem solving.**



Pictures

- **The connection among different representations in problem solving depended on the difficulty of the structure of the problem.** Pupils carried out the transition from one representation to another more flexibly in solving problems with the unknown in the final amount compared to solving more complex problems, i.e. with the unknown in the other positions.

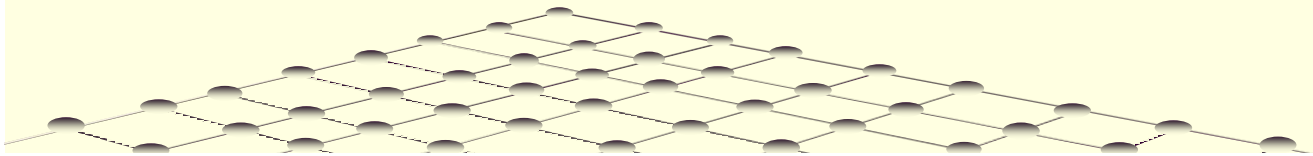
- The assertion that **the understanding and success in problem solving are closely linked with the ability to coordinate different representations of the same situation** (Duval, 2002) is empirically validated in the case of the additive change problems.



Pictures

The function of the informational picture in the second study

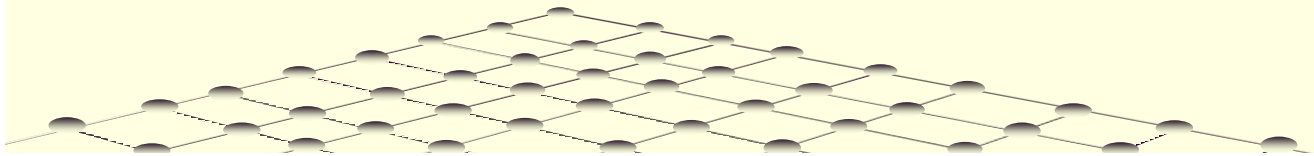
- **The function of informational pictures in problem solving was found to differ significantly from the use of the other forms of representation.**
- ✓ *Pupils dealt in a unique and less flexible way with these problems.*
- ✓ *The cognitive demands of the informational picture in the context of mathematical problems were different from the other forms of representations.*
  - It seemed to be incongruent with the mathematical situation of the problems.
  - It might require more complex mental processes relative to the other representations as the understanding and the solution of the corresponding problem involves the coordination of three different modes of representations combined in the informational picture, i.e. pictorial, verbal and diagrammatic.



Conclusion and implications

Both studies unraveled students' differentiated ways of thinking in arithmetic or problem solving tasks of different representations. These findings provide a strong case for the role of different modes of representation on problem solving, as well as, on the understanding of mathematical concepts such as arithmetic operations.

- **It is important for mathematics educators** to teach additive problem solving as well as arithmetic operations, by using **the semantic approach** integrated with **the perspective which refers to the diversity of representations** and transferring information from one representational mode to another.
- **Informational picture and number line need special attention** within the context of mathematical problem solving and the learning of arithmetic operations respectively, during instruction.



Epilogue

Given that

- a picture can be described through a sufficiently rich language
- not all verbal expressions can be represented pictorially

•(Un'immagine vale più di mille parole…)

•The well-known saying **"a picture is worth a thousand words"** refers to the economy of language and thought and not to the ability to represent.

•"Representationally" speaking: **"every word conveys a thousand pictures"** but **"a thousand pictures cannot represent some of the things we can represent using words"**.

Thank you for your attention