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THE DIALECTIC RELATIONSHIP BETWEEN RESEARCH AND PRACTICE IN MATHEMATICS TEACHER EDUCATION

ABSTRACT. This paper addresses issues linking research in student teacher learning with reflection on practice in mathematics teacher education. From a situated perspective on learning and practice, we explore our own practice as teacher educators while researching student teacher learning in our classrooms. We describe a study on student teacher learning, considering student teacher learning as a "process of becoming", and how the results of this research have affected our development as mathematics teacher educators and members of a community of inquiry. Our work shows how in the mathematics teacher education context the relationship between theory and practice becomes an element of both teacher educator and researcher development.

KEY WORDS: Situated perspective on learning and practice, use of tools of practice, relationship between research and practice in mathematics teacher education, community of inquiry

INTRODUCTION

A few years ago, Lerman (2001a) suggested the need to incorporate theories of teacher learning which draw on notions of developing individual teacher identities within a social context, including learning about teaching as a socio-cultural activity, into research on mathematics teacher education. For Lerman, "the classroom and seminar room are complex sites of political and social influences, socio-cultural interactions, and multiple positioning involving class, gender, ethnicity, teacher–student relations, and other discursive practices in which power and knowledge are situated" (Lerman, 2001a, p.44). He pointed out that individualistic accounts cannot explain all these forces.

Focusing on Lave's work (Lave, 1988; Lave & Wenger, 1991), Lerman indicated that "there are aspects of her theories that need work by researchers, such as that student teachers are learning about teaching from a teacher educator in a university setting, not a school setting. The teacher educator is not the master of the teaching practice, and some elaboration is required in terms of models of mastery being offered rather than the practices of the master'' (Lerman, 2001a, p. 46).

The need for teacher educators to construct research frameworks for analysing teacher learning from situated perspectives involves the creation of a research framework that analyses their own practice. The aspect of teacher educator practice considered here is the work with primary school[1] student teachers in teacher education programmes. Consequently, our understanding of student teacher learning must be made explicit. We assume that student teacher learning is a process during which knowledge and modes of reasoning similar to those of the experienced teacher should be acquired. Some features of this process are:

– it occurs through active participation in a context defined by ''authentic activities'' understood as ordinary cultural practices;
– learning is based on developing a way of participating in a community of practice;
– activity acquires full meaning from previous knowledge and beliefs, and by positioning the student teacher in that practice (his/her goals, needs, and so forth) (Llinares, 2002); and
– participation in the activity can increase and/or modify the meaning of conceptual tools used.

We also think that social practice is an integral and inseparable part of learning. With regard to social practice, Hanks' Foreword to Lave and Wenger (1991) considers legitimate peripheral participation to be a basic concept: ''this central concept [legitimate peripheral participation] denotes the particular mode of engagement of a learner who participated in the actual practice of an expert, but only to a limited degree and with limited responsibility for the ultimate product as a whole'' (p. 14).

Participation takes place in 'communities of practice' that portray a social group in which its members share a given activity (goals, purpose, ends, means, etc.). Although we are aware that student teachers do not initially belong to the 'community of practice' of mathematics teachers, we do acknowledge that teacher education programmes must provide the means for qualifying student teachers for becoming members of that community, and favouring student teacher participation in what are called 'communities of learning' (García, 2000, 2003).

In this article, we describe how we have used theoretical constructs taken from situated perspectives to characterize the dialectical relationship that is established between research into student teacher learning and mathematics teacher educator development. From our point of view, the situated perspectives provide referents, such as the notion of 'learning through the use of conceptual tools', with which we can examine the relationship between research on teacher learning and development of mathematics teacher educators. We assume that, in teacher education, there are two communities of practice, a community of future mathematics teachers (community of learning) and a community of mathematics teacher educators. In some cases, and specifically in our case, the community of practice of teacher educators could be considered a community of inquiry, in which there are different roles: mathematics teacher educators, researchers and researchers/ mathematics teacher educators. In such a community, a co-learning situation emerges (Jaworski, 2003a, b). Researchers, such as Jaworski (2003b), have started to develop theoretical frameworks for studying and analysing the relationships in communities of inquiry.

The first part of the article focuses on how we understand mathematics teacher education practice. In the following sections, we describe a study on student teacher learning, and discuss how the results of this research have affected our development as mathematics teacher educators, members of a community of inquiry. The article ends with some conclusions and implications for future research in the field.

A WAY OF CONSIDERING MATHEMATICS TEACHER EDUCATION PRACTICE

Efforts made in recent years to articulate mathematics teacher education programmes have come up against several problems, among them being a lack of tradition in the coordination of those programmes and teacher learning research. To grapple with this issue, we focus on mathematics teacher education practice from the point of view of a community of inquiry. We assume that reflective practice is a characteristic of this community (Tzur, 2001; Zaslavsky & Leikin, 2004). In our community of inquiry, student teacher learning is an object of reflection. We conceptualize our learning as members of that community, using the same theoretical constructs as for student teacher learning. Both in the community of learning and in the community of inquiry, learning is regarded as increased participation in the practice of a community.

For us, "becoming a primary teacher" may be understood as the process of pre-service primary school teachers being introduced into the community of practice of those teachers, and acquiring an understanding of the teaching of mathematics. Learning to teach is seen as the identification and use of conceptual and technical tools in solving professional tasks (García, Sánchez, Escudero & Llinares, 2003b). This implies:

- learning to carry out teaching tasks,
- learning to use and justify the tools involved in tasks like planning (task design, choice of textbook and curricular materials), assessment, and handling students' mathematical communication.

In this sense, student teacher learning could be understood as progressive participation in the community of practice through the use of conceptual tools, which permit student teachers to understand and undertake professional tasks. For us, the term 'tool' not only denotes a physical object, but is extended to concepts and reasoning, etc., which enable and influence interaction within a community. Such tools may be classified as either *technical or conceptual tools*. *Technical tools* are those tools used in the 'practice', such as teaching materials and software, techniques for managing discussion of procedures, answers to problems, and so on. *Conceptual tools* are understood as those concepts and theoretical constructs that have been generated from research in teaching and mathematics learning leading to understanding and handling the situations in which mathematics is taught and learned.

To articulate the ideas above, we organized our practice according to a 'teaching–learning trajectory' (see Figure 1).

These trajectories are our way of making operative the notion of a reproductive cycle (Lave & Wenger, 1991), in which knowledge is integrated into the activity. According to Lerman (2001b), we assume that practices in teaching–learning trajectories "should be seen, therefore, as discursive formations within which what counts as valid knowledge is produced and within which what constitutes successful participation is also produced" (p. 100).

In these trajectories, meanings related to the skills and tools necessary to teach mathematics may be shared, discussed and negotiated with the different groups of students that are formed in the classroom (Sánchez, 1997). We try to generate learning environments, the goal of

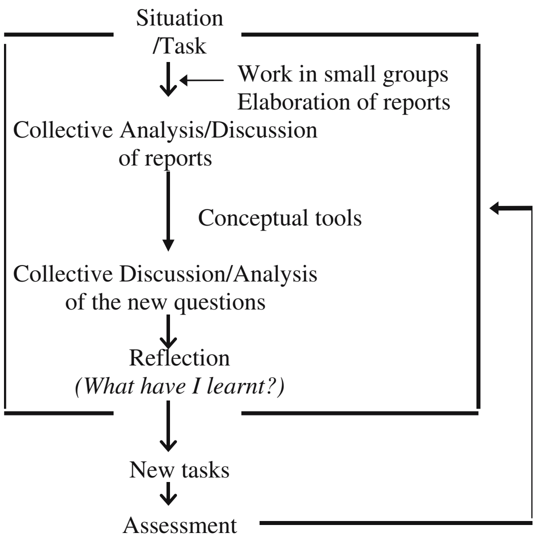

Figure 1. Teaching–learning trajectory (García, 2000. p. 63).

which is that student teachers progressively take part in professional teaching tasks, but without a teacher's responsibility (García, 2000, 2003). In addition, student teachers are encouraged to think of themselves as teachers, and share their comments and opinions with the group. The theoretical information providing student teachers with access to conceptual tools can be found in videos, articles from the literature on mathematics education, or information given by teacher educators (García & Sánchez, 2002).

An essential part of our classes is establishing a collaborative and cooperative atmosphere in which all the participants must contribute (Sánchez, 2003), making decisions grounded in their knowledge of teaching. In this sense, when the student teachers begin teaching practice, the regulating effect of that practice positions them in it. Following Llinares (2002), we believe that the regulating effect of teaching practice is a way of student teachers becoming teachers by developing their own identities as teachers. From this perspective, a teaching–learning trajectory is founded on the notion of "person-in-practice" (Lave & Wenger, 1991). According to Lerman (2001b), this theoretical construct includes the student teacher's previous experience, the ways in which his or her social relationships have been framed, how the teaching activities have been framed by teacher educators, the texts, and the histories and functions of didactic mathematics artefacts. We extend this construct to mathematics teacher educators.

Below, we make use of a study on student teacher learning to describe the processes by which our reflection is evidence of our own learning as teacher educators. In particular, the following question is addressed: How do the results of mathematics teacher educator reflections on student teacher learning in a community of inquiry contribute to teacher educators' own growth? In this sense, we have extended the ideas expressed by Zaslavsky and Leikin, "it follows that the community of mathematics educators (i.e. teachers and teachers educators) can be seen as learners who reflect continuously on their work and make sense of their histories, their practices, and other experiences" (Zaslavsky & Leikin, 2004, p. 6) from a community of mathematics educators to a community of inquiry.

A STUDY ON STUDENT TEACHER LEARNING

A problem that arises from our practice as mathematics teacher educators can be turned into an object of research within our community of inquiry. In particular, as teacher educators, student teacher learning is one of our main issues. From here, how student teachers learn – understood as student teacher use of the conceptual tools provided in teacher-learning trajectories—becomes a research problem within that community (Escudero, García, Llinares & Sánchez, 2002; García, Sánchez & Escudero, 2003a). In our study, this problem is expressed as the research question: How do student teachers use the conceptual tools to solve "professional teaching tasks" in the context of a trajectory? For us, the student teachers' use of conceptual tools is understood as "simultaneously setting in motion different tools, interaction and communication of the information coming from them, leading to reasonable decisions" (García et al., 2003b, p. 3).

From the different professional teaching tasks in our mathematics teacher education course that are the starting point of the various teaching–learning trajectories, we have chosen the planning process, and in particular, curricular analysis of the textbooks and teaching materials (a typical primary school teacher task). Specifically, this analysis focused on multiplicative structure problems (Greer, 1992). We prepared two abridged books for the design of this professional task. We chose two collections of textbooks, corresponding to all the primary courses, from different publishers that were very popular among primary teachers (labelled Publisher 1 (P1) and Publisher 2 (P2)). Although the texts chosen were not thoroughly analysed, some marked features were considered:

- Inclusion or not of certain characteristics in line with the traditional culture of primary school mathematics practice (subject revisions, recapitulations, etc.)
- Inclusion or not of supporting illustrations in the introduction of concepts and problems
- Integration of cross contents (other subjects or other mathematics topics).

The content of the textbooks was reviewed and all the pages relating to multiplicative-structure problems were selected. These pages were used to prepare the two abridged books, one from each publisher. Student teachers had to make a decision as to which book they would choose to use in their teaching. The first page of the abridged book described the professional task (a teacher must choose the textbook or published teaching material for his/her students) and posed several questions: What were the assessment criteria that you used to make your decisions? Do you agree with the content? Do you agree with the organization and presentation of that content? and so on. The purpose of the first page was to summarize the characteristics of a situation that is habitual in primary teaching practice and to stimulate thought about that situation with the questions proposed. The task was designed to allow the student teachers to situate themselves both personally and socially.

In addition to the task, as part of the teaching–learning trajectory, the student teachers were provided by the teacher educator with theoretical ideas, through articles, videos and other information, as conceptual tools. These tools included different multiplicative-structure problem typologies with different perspectives of analysis (Nesher, 1992; Vergnaud, 1991) of learning difficulties associated with these problems, and on the relationship between problem comprehension and the use of mathematical symbols. These conceptual tools thus gave student teachers a specific language, meanings, and connections that would allow them to think and speak as teachers.

Development of the study

The study included 130 primary school student teachers who were enrolled in two groups of our mathematics methods course. This four month course of four hours per week was part of the primary teacher education programme at the University of Seville. The specific trajectory that was object of our study took two weeks. In this paper, the data come from this trajectory. The student teachers were grouped into 23 small groups of 4–7 students. They were provided with the two "abridged books" referred to above and the conceptual tools, which were part of the trajectory. In addition, a list of five addition and five multiplication word problems of varying characteristics was also provided to each student. The student teachers were asked to identify the structure of the ten word problems that were presented in random order. They had to provide a justification of their answers.

With respect to the use of abridged textbook, although the task was initially introduced in the classroom, the different groups of student teachers carried out the work by meeting as many times as they considered necessary to discuss their decisions and negotiate their meanings. When the analysis was finished, each group had to write a final report including their decisions and arguments.

The discursive nature of their justifications (included in the individual answers) and the student teachers' group reports provided us with the data. From this data, we could see how student teachers use the theoretical constructs as tools in developing a professional task. On one hand, the individual answers identifying the set of problems were classified as to whether the student had identified the type of structure or not. When the identification was correct, we observed whether identification had been based on conceptual tools involved. On the other hand, the group reports were analysed by the following inductive process. First, units of analysis were identified and classified into the following sections:

- Criteria mentioned
- Elements considered basic
- Presence of the theoretical information in previous elements
- How difficulties (if any) are considered in the sequence set out in the introductions to the texts
- Relationships established (or not) between the types of problems/difficulties in that sequence
- Relationships of cross contents.

The individual answers showed that 99 of 130 students were able to identify the type of structure in all the sets of problems proposed. Of the students that identified the five multiplicative-structure problems correctly, 52 of 130 classified them properly according to the classification they has chosen (Nesher, 1992; Vergnaud, 1991), but only 50 students correctly analysed the elements involved in their classification. These results led us to delve into the origin of this low rate of identification. We found that a problem of multiplicative comparison, which accumulated the greatest number of mistakes, was the cause. The particularities of multiplicative-comparison problems have been reference by Greer (1992), with respect to children from different countries. Our findings have confirmed these particularities in Spanish-speaking student teachers.

The group reports also differed in their use of the conceptual tools for decision-making. Considering how student teachers made their choices, and used the theoretical constructs, we identified four different levels (see Table I).

On the first level, the student teachers are clearly situated at a personal stage, and base their decisions on previous experiences. They do not identify the conceptual tools as useful in carrying out this task. A detailed analysis of the reports by these groups allowed us to distinguish what characteristics of the abridged books, such as initial check of previous ideas, presence of drawings, final check, and criteria of what, how and what for in judging the contents, that is, aspects that they had found 'useful' in their student teaching experience, may have influenced their decisions. The comments quoted below illustrate these ideas:

'Our choice is supported by the following:

- Problem structure. In general, problems are illustrated by drawings that facilitate visual comprehension
- The final revision is rather detailed and constructive
- The content is developed at length and there is a brief summary at the beginning' (Group 2)

On a second level, student teachers were able to state that certain elements 'appeared' and detected when they 'did not appear', but they

TABLE I
Levels in the use of conceptual tools

Conceptual tools not identified	LEVEL 1
Conceptual tools identified, but not related to decisions	LEVEL 2
Conceptual tools provided are identified and applied	LEVEL 3
A. Difficulties and characteristics in the sequence are related to the difficulty of the specific mathematics content	
B. Difficulties and characteristics in the sequence are related to the conceptual level of pupils	
Conceptual tools are identified, applied and included in a more general framework	LEVEL 4

did not relate the presence/absence to anything else. The following response of a group of student teachers is representative of this level:

'In general, we think that some content is missing in P1. All types of problems appear except for the following: quotative division, Cartesian multiplication and multiple proportions. At first, this led us to think that the other publisher (P2) was better, since we found all the types of multiplicative structure problems. However, the problem presentation (the drawings) and organization (pages with too many problems) led us to choose the first Publisher (P1)' (Group 5).

In this level, we observed that some groups used information acquired from other sources, such as educational psychology. This can be considered positive, since it represents the presence of transverse knowledge, which has been transferred from other subject matter. Nevertheless, just as in the case of the different types of arithmetic problems, criteria for classification, pupils' strategies, and conceptual tools provided in our teaching–learning trajectory, their use does not go beyond their identification. Most of the groups (eleven) were found to be in this level.

On the third level, considering the basis of the choice of the student teachers, we might identify two subcategories, related to how the difficulty of arithmetic problems was considered. In one sublevel, student teachers related problem difficulties to semantic characteristics of the multiplicative structure problems.

'... we assessed the problems' quantity, quality and difficulty. Regarding quantity, we considered it to be positive that the textbook includes a greater number of problems. As for quality, we valued as positive that the textbook includes a wider variety of problems ... mapping rule, Cartesian multiplication, and multiplicative comparison. With respect to difficulties, we judged as positive the order of presentation by the difficulty of the different types of problems [following the Nesher's Semantic Analysis]...' (Group 21)

The student teachers in the other sublevel related the problem difficulties to the hypothetical conceptual development of the pupils, a broader idea that would include both the multiplication and addition structures. The following is representative of the student teachers' explanations:

'.. in Publisher P2 ... problems are introduced to make students transform addition into multiplication by using direct modelling, since they are urged to use counters...'(Group 12)

In the above-mentioned quotations, conceptual tools related to problem typology, learning difficulty and features facilitating the problem solving process are integrated in the decisions made in the professional task of analysing teaching artefacts (the problems) for teaching arithmetic problem solving. All student teacher groups that used these criteria chose the abridged book from P2.

Finally, those student teacher groups that identified the conceptual tools and used them to make a decision were placed on the fourth level. The way in which these groups reported their decisions might be considered a way of incorporating relationships between different conceptual tools in a more general framework.

'We did not choose Publisher 1 because:

- Some of the problems mentioned by the Nesher documents were not present ... the order of difficulty of the problems is not considered, given that the limited problems implying multiplicative comparison and Cartesian multiplication are treated superficially and not systematically. In other words, the Publisher considers them to be secondary problems and mere curiosities.
- The language used is complex and abstract.
- The exercises suggest an individual-oriented work methodology, obviating the advantages of students working in groups.
- Mental arithmetic is not sufficiently fostered.' (Group 9)

This group, which integrated problem typologies, learning difficulty, issues from language, how the textbooks integrated mental arithmetic in problem solving, etc., chose Publisher P2.

It is important to underline that the choice of abridged book made by the groups changed from P1 to P2 as level progressed. This may show that greater integration of conceptual tools in the "practice of analysing teaching resources" influences decision-making. 16 of the 23 groups were able to identify the type of multiplication problems in the abridged books and to some extent, considered the inclusion of different types of problems important. Nevertheless, only five of these groups established relationships among the different conceptual tools. We think that establishing relationships in the teachers' instructional practice is very important. Furthermore, it should be pointed out that the conceptual tool related to the different typologies and perspectives of analysis of multiplicative structure arithmetic problems were the ones that the student teachers identified best.

Our analysis of student teacher reports has shown the difficulty in establishing relationships among different conceptual tools when they are used in solving a task. Programme configuration through teaching–learning trajectories might enable student teachers to integrate concepts, ideas and ways of reasoning in the process of professional task solving as a way of becoming a teacher by peripheral participation in teaching practice (Lave & Wenger, 1991).

In the following, we try to show how the results of this research have affected our development as mathematics teacher educators and researchers, members of a community of inquiry.

FOCUSING ON OUR DEVELOPMENT AS MEMBERS OF A COMMUNITY OF INQUIRY

As teacher educators, the interpretation of the research on student teacher learning and its results involved the negotiation of meanings within our community of inquiry. Reflection on student teacher productions allowed us to add new characteristics to the design of teaching-learning trajectories. Analysing the written reports, we became more aware of the role played by the different aspects of the teaching–learning trajectories and the different uses of the conceptual tools. The task, as it was presented to the student teachers, required group work and the need to agree on their criteria through interaction. These aspects were important with regard to what learning was generated. The different positions adopted by the groups on the proposed task underlined its potential as an "authentic activity" in the community of learning, in the sense that it allowed student teachers to show their learning in different ways. Concerning the use of conceptual tools, the identification of different levels allowed us to examine the student teachers' different positions, needs and goals in depth. In our case, as teacher educators, the identified levels were tools that allowed us to assess student teacher learning.

The initial definition of learning (identification and use of tools) was extended to the characterization of differentiated uses of the tools (identified levels). This process enabled us to think about the use of these levels in the design (i.e., new tasks and other tools) and analysis of new trajectories, and in the assessment of student learning in these new situations. All of this led us to consider our development as mathematics teacher educators. As researchers, the identification and integration of different learning levels in our theoretical framework raised new research questions. These questions were related to better characterization of learning levels and their relationship with the development of different professional tasks, adding new aspects to the theoretical characterization of this learning. In this sense, we started to develop new research projects that have extended this characterization (García et al., 2003a; Sánchez, García & Escudero, 2004), contributing to our development as researchers. The combined development as teacher educators/ researchers is a characteristic of our community of inquiry.

DISCUSSION

Our study focused on two different levels, teacher education practice, and research on student teacher learning. For each, the notion of ‘‘the person-in-practice’’ was the unit of analysis describing the dialectical relationship between those levels, underlining the social aspects in them. This unit of analysis was made operative by considering the progressive integration of ‘‘conceptual tools’’ in solving professional tasks in each community of practice (community of inquiry and community of learning) as an evidence of learning.

The idea of person-in-practice was applied to:

– our own practice as teacher educators, and
– the discourse generated by student teachers grappling with professional tasks.

In the following discussion, the focus is on mathematics teacher educator development, assuming that as teacher educators, we learn through the analysis of student teacher practice in teaching–learning trajectories. In addition, we reflect on the student teacher learning as a growing use of conceptual tools in professional tasks. Finally, we show the relationship between research on student teacher learning and our development as members of a community of inquiry.

Development of mathematics teacher educators through their practice

Reflection about one's own practice as a teacher educator is a characteristic of our professional development that has been recognized by different authors (Doecke, 2004; Llinares, 2003; Tzur, 2001; Zaslavsky & Leikin, 2004). In our case, through the reflection on how our students use the theoretical ideas provided them from mathematics teacher education, considered as a scientific field, we have developed our own identity as teacher educators and have developed the way we participate in our community (Jaworski, 2003b; Wenger, 1998). As a consequence of such reflection, our practice as teacher educators started to be modified.

Our own learning was described in the early sections, when we showed how the information from our research about student teacher learning amplified the referents that we had used for articulating our practice. This provides evidence of our professional development as mathematics teacher educators through our engagement in our own practice.

The process followed was similar to that of the application by Zaslavsky and Leikin (2004) of the mathematics teaching and learning model proposed by Steinbring (1998). As teacher educators, we design teaching–learning trajectories in which student teachers can develop ways of participating in the community of learning, through their increasing use of conceptual tools in developing professional tasks. In our study, awareness of student teacher learning through these itineraries led to a new understanding about how they learn. This new understanding has led us to modify the trajectories, making them more appropriate for the student teacher. Those changes, based on our analysis of the relationship between teaching–learning trajectories and their effects on student teacher learning, may be seen as aspects of our own development.

Student teacher learning as a growing use of conceptual tools in professional tasks

As teacher educators, we see student teacher learning as the use and integration of conceptual tools during the performance of a professional task (textbook analysis). Other researchers (Goffree & Oonk, 2001) have also provided insight into knowledge construction from different perspectives (e.g., assimilation, adaptation, integration and theorizing). We also raised Goffree and Oonk's questions, "how can you observe the construction of practice knowledge? Since research does not extend to the fieldwork of student teachers, where practice knowledge in action could be observed?" (Goffree & Oonk, 2001, p. 138). We have to rely on student teachers' written reports and discussions, generated in the development of a trajectory. For us, the characteristic of 'authentic activity' of the proposed task and the activity that student teachers develop in the trajectory allow us to approach their practical knowledge in action.

This approach to how student teachers learn to teach considers their growing participation in a community of practice as a discursive practice. From this point of view, the negotiation of meanings associated with the conceptual tools that the students use, is related to how they participate in the community of learning (Llinares, 2002). The growing use of the conceptual tools in the tasks carried out by the group can then be considered a modification of student teacher practice, and a manifestation of the process of becoming a teacher.

Relationship between research on student teacher learning and our development as members of a community of inquiry

The characteristics of our community of inquiry lead to further reflection about the co-learning situation generated in that community. In the following, we use the four-dimension framework proposed by Jaworski (2003b) for showing the relationship between mathematics teacher educators (insiders), researchers (outsiders) and mathematics teacher educators/researchers (insider researchers). In this sense, we extend this framework to the context of mathematics teacher education, contributing to its characterization.

Knowledge and learning

Mathematics teacher educators' knowledge was apparent in the design (tasks/situation, conceptual and technical tools, etc.), implementation and assessment of the trajectories. The evidence of their professional development was shown in the decisions made with respect to the subsequent design of new trajectories (García et al., 2003a; Sánchez et al., 2004). The new designs were a result of the discussions generated by mathematics teacher educators and researchers in the community of inquiry. Teacher educator learning was observed in the assessment of what happened in the trajectories (student group briefs, development of the work groups).

The knowledge of researchers (outsiders) was made explicit through their knowledge of scientific production in the field related to both mathematics teacher education and student teacher learning. This knowledge made possible the identification of theoretical elements (in particular, situated learning—understood as the use of conceptual tools—and the characterisation of conceptual tools) within the community of inquiry, and their use in the design of the study on student teacher learning. Learning by researchers was developed through reflection in the community of inquiry on the results of the study. This enabled identification and characterization of student teacher learning levels (García et al., 2003b), which then become a new tool for the members of the community. The members considered this new tool from different perspectives, as a consequence of their different theoretical referents.

Inquiry and Reflection leading to development
Both teacher educators and researchers were inquiring. The teacher educators inquired into their use of the trajectories and their reflection contributed to enlargement and modification of some trajectory elements. As the study developed, the researchers inquired into student teachers' learning. Reflection on the results led to the inclusion of new theoretical elements in the conceptualization of this learning. This inclusion generated new research studying their validity and coherence. As insider researchers, reflection allowed us to improve the mathematics teacher education course.

Insider/outsider research(ers)
Insider researchers and outsiders conducted the study developed for the purpose of finding answers to a problem in mathematics teacher education practice, specifically, the characterisation of student teacher learning. The mathematics teacher educators learnt, through reflection with the other members of the community, about the process and its results, while the outsider learned from the study and from reflection on the learning of the insider researchers and insiders. This learning originates and produces results in the community of inquiry, raising new questions about student teacher learning and the professional development of teacher educators.

Individual/community
Individual mathematics teacher educators, researchers and mathematics teacher educators/researchers were the individual learners in the context of mathematics teacher education. A problem—students learning in a specific classroom—that emerged in our work as mathematics teacher educators relates to our community of inquiry. In this community, due to its inherent characteristics, it was considered a problem for research. This does not occur in other communities of inquiry in which mathematics teacher educators may be members. The researchers, as individuals, design the research project, which is developed within the community out of discussion among the members, who have their own individual referents with respect to the problem. Development of the project in a mathematics teacher education course allows individual mathematics teacher educators to develop his/her own professional work. In addition, the combined reflection in the community on the discourse generated by student teachers provided us with some results that have been assumed by the different members from the perspective of their individual identities. The relationship be-

tween individual and community established in this way contributes to showing its potential as a learning tool that favours co-learning in a community.

CONCLUDING REMARKS

In conclusion, we would like to underline several points. First is the necessary coherence that has to exist between the way student teacher learning and teacher educator learning are conceived. We have shown how the use of the "person-in-practice" as the unit of analysis, viewed through the successive incorporation of conceptual tools in the development of the practice, can provide new perspectives on the dialectic relationship between research on student teacher learning and practice in mathematics teacher education.

Consideration of student teacher learning as a process, viewed through the increasing use of the conceptual tools in the performance of professional activities at the university, provides the context in which the teacher educator can design new activities. The relationship between the analysis of the practice as mathematics teacher educators and what this practice favours (student teacher learning) is the centre of teacher educator development, what Tzur (2001) calls "orienting reflection on activity-effect relationships".

Reflection on the relationship between the activities designed (in our case, teaching–learning trajectories) and the nature of student teacher learning generated (e.g., the identification of different levels of usage of conceptual tools in the development of a professional task) becomes an element of teacher educator development. In this work, a co-learning community as described by Jaworski (2003b) has been characterized through a community of inquiry and three learning groups (mathematics teacher educators, researchers and mathematics teacher educators/researchers). Our study is related explicitly to research in the education of mathematics teachers, and concerns research on the crucial relationship between theory and practice. In these pages we have tried to show how the analysis and organization of mathematics teacher educator practice as a community of inquiry can generate a relationship between theory and practice in mathematics teacher education.

NOTE

[1] Primary school in Spain includes students aged 6–12.

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