

1 Building virtual learning communities and the learning of mathematics teacher student

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The reform in the structure and goals for Higher Education in Europe underscores the link between instructional design and learning. This reform places high demands on Teacher Training. Although the “routes” through which a student may become a mathematics teacher are different in many countries (Fandiño, 2003), in all of them it is begin to link teacher learning to design of learning environments. Nowadays, it is a challenge in Teacher Education to enable students carry out autonomous work and, at the same time, development of the identity of being a mathematics teacher belongs to a community of practice. One way of facing up to this challenge in teacher training is by using the media provided by new technologies of communication that increase the spaces and the time for interaction and communication professional on real case base studies between the student teachers.

In this paper I shall show a way to face up to these new challenges from a situated perspective of learning for student teachers. Firstly, I shall describe the characteristics of the design of learning trajectories in a video-based learning environment focusing on the exploration of mathematics teaching to help student teachers to develop and use knowledge about mathematics teaching during their initial training. Next, I shall show one way of reporting on the process of reification of the teacher student’s knowledge and beliefs (generation of a objectified discourse) considering the communication in a virtual debate. The identification of “conversational chains” in the virtual debate allows to describe the teachers’ manner of participating when they perform “worthwhile professional tasks”. The conversational chains determine possible moments for the negotiation of meanings and reification of knowledge and beliefs about mathematics specific pedagogical content knowledge. However, prior to this it is necessary to characterise:

- i) teacher’s learning and its implications for teacher training from a situated perspective, and
- ii) teaching of mathematics as a practice and its implications for the development and use of the teacher’s knowledge.

A situated perspective of mathematics teacher learning

Linking mathematics teacher training to theoretical perspectives on student teacher learning raises issues regarding the design of learning environments. Learning as a process of becoming a participant in a practice is not a direct adaptation from general perspectives. From a socio-cultural outlook, becoming a mathematics teacher means acquiring an understanding of the teaching of mathematics as a practice. That is to say, learning to use and justify the tools involved in professional tasks like planning (task design, choice of textbook and curricular materials), interpreting and analysing students’ mathematical thinking, assessment, and handling of students’ mathematical communications.

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From these perspectives, teacher learning might be understood as the progressive participation in a community of practice through the use of conceptual tools for understanding and handling the teaching task (learning as changes in participation in socially organized activity) (Lave & Wenger, 1991). Participation takes place in 'communities of practice' that portray a social group in which their members share a given activity (goals, purposes, ends, means). Teacher training programmes must therefore favour student teacher participation in so-called 'apprentice communities' (communities of learners) through social interaction – communication by tools - while solving professional tasks (Peressini et al, 2004; Shulman & Shulman, 2004).

Mathematics teaching as a practice: Developing and using knowledge about teaching

For the profession of being a mathematics teacher, the activity linked to it is that of "teaching mathematics". Posing what it means to learn how to teach mathematics means, from the perspective of "learning a practice", understanding the notion of practice as:

performing some tasks (system of activities) to achieve a purpose,
making use of some instruments, and
justifying their use (reflection and meta-cognition)

We might consider three "systems of activity" (knowing and know-how to do) that are relevant to teachers' instructional practices and that structure mathematics teaching as a practice: i) selecting and developing worthwhile mathematical tasks (e.g., organising mathematical contents to be taught); ii) initiating and guiding the mathematical discourse in the classroom, and iii) interpreting and analysing students' mathematical thinking. In each system of activity we may find elements of the knowledge base for teaching and the ways of using them. Examples include

organising the mathematical contents for teaching it: being familiar with the mathematical contents as a teaching-learning object, using the knowledge of mathematics for designing, selecting and analysing worthwhile mathematical tasks (for example, establishing ranges of cognitive demands), using the knowledge about the mathematical contents to design, analyse and select lessons and curricular resources

Student teachers come to teacher education differed in their subject matter knowledge for teaching, and in the use of this knowledge in selecting and analysing mathematical tasks. Student teachers differ both in the different aspect of concepts they emphasis and the use of representation repertoire to structure learning activities (Sánchez & Llinares, 2003). These studies underline the need to consider this system of activity of teaching mathematics in teacher training.

managing the mathematical contents and discourse in the classroom: being familiar with and identifying the phases and types of lessons of mathematics, being familiar with and identifying the characteristics that the mathematical interaction in the classroom (for example, the different socio-mathematical norms, the didactical contract, ...) identifying the constraints and mechanisms which maintain the interactive course of mathematical communication in the classroom (Steinbring, 1998); being familiar with and identifying characteristics of the management of debates as instruments for mathematical learning – asking questions that will allow prior conceptions to be compared with what is new and knowing how to underline the different contributions supporting the development of metacognition in the pupils and

proposing questions that are mathematically challenging, to encourage the progress of the pupils throughout the performance of the mathematical problems.

Teachers' management of mathematical content during the interaction with his/her students might determine the opportunities for mathematical learning (Llinares, 2000; Escudero & Sánchez, 2002). So, we should consider aspects of mathematical discourse and management of mathematical content during interactions as relevant to the teacher's instructional practice; but the development and use of this knowledge in this expertise domain in mathematics teaching is a challenge for teacher student.

analysing and interpreting the mathematical productions of the pupils: becoming familiar with the theories for the learning and construction of mathematical knowledge, and the characteristics of the learning of mathematical concepts and procedures, using the prior knowledge for interpreting and analysing the pupils' mathematical thinking (through audio, video record and written productions, in specific problems or in projects,...)

So teacher student must begin to take into account what is known about the ways in which students learn specific mathematical ideas, and its use in teachers' practice.

Considering jointly the situated perspective on (i) the practice of teaching mathematics, and (ii) the process of learning this practice, determines some conditions in the design of learning environments in the programmes for teacher training (initial training and professional development). In the following sections, I shall describe one way of understanding these implications.

Designing learning environments in teacher training

The setting-up of structures for social participation amongst the student teachers is not an easy task. In this context the means provided by ICT are helpful (Daniel, 1996; Horvath & Lehrer, 2000). We can adopt the notion of "learning trajectory" (Baroody et al, 2004) in Teacher Education as a combination of:

- a) goals for meaningful student teacher learning,
- b) authentic tasks from mathematics teaching as a practice, and
- c) aspects of a local learning theory with "structures of participation" that are characterised by the relationship between the individual and the social dimension.

For some years now, teacher educators have been studying the conditions that would allow the interactions between student teachers to be strengthened, in order to develop the social process for the construction of professional knowledge (García, 2000). Features of learning environments designed *ad hoc* are

- solve professional tasks ("authentic tasks") such as exploring the practical rationality of mathematics teaching, and
- generation of "participation structures" that are characterised by interaction and communication in small groups and writing reports where student teachers might reconcile alternative interpretations of analyses of teaching episodes. Here, the aim is to have an interplay between theoretical and

practical requirements. Integrate real and virtual debates as spaces to support interactions between mathematics student teachers is a decision made.

Individual and collective reflection is a precondition both for developing an identity as a teacher and for membership of a community of practice. Writing reports helps student teachers to reflect and the negotiation of meanings and allows the interplay between theoretical and practical requirements. In teacher training, the student teacher's reflection is particularly important (Flores, 1998).

To continue, I shall describe the type of work that is being carried out from these perspectives. Firstly, I will focus on the characteristics of the design of a learning environment in the initial training for mathematics teachers in secondary education, the target of which is the analysis of a mathematics lesson, and in which the use of videos and virtual debates through the web are integrated. Next, I will report on the implications of the situated perspectives on the analysis of the communication between teachers in virtual debates in a context of professional development.

Developing and using knowledge of mathematics teaching: A learning trajectory in a video-based learning environment

This section describes the characteristics of a interactive multimedia learning environment designed to allow student teachers to observe, analyse and discuss on a mathematics lessons. The aim was to favoured the construction of spaces for social interaction, using virtual debates on the web and incorporating videos for analysing a mathematics lesson (considered as an “authentic activity”).

The design of the learning environment using the analysis of a mathematics lesson as an “authentic activity” is backed by three basic ideas:

- i) analysing the teaching of the mathematics to understand the practice of teaching (Contreras & Blanco, 2002). This first idea involves the use of the recording of lessons as material for student teachers (Ball & Cohen, 1999; Lampert & Ball, 1998).
- ii) the construction of spaces for interaction as a medium for the social construction of knowledge. The integration of virtual debates allows the student teachers to interact with the material and with their classmates without needing to coincide in a given place or at a given time; and
- iii) the evolutionary nature of the process for constructing the knowledge needed for teaching. This idea involves developing a “learning trajectory” that allows the student teachers to explain their conceptions, negotiate new meanings and the possibility of slowly but surely integrating the use of conceptual tools in the analysis of situations for teaching mathematics. This characteristic is shown by the fact that the learning environment is structured through a set of work “sessions”. Each one of the sessions has a specific target and aims to favour the evolution from the student teachers' prior conceptions up to the development of components of professional knowledge. That is to say, understanding the teaching of mathematics as a practice (performing a task, making use of some tools and justifying their use)

The analysis of the mathematics teaching in the courses of methods in teacher education has become important since the recognition of “the teaching of the mathematics as a practice” showing the potential of the use of recordings of mathematics lessons as materials and resources for training. Videos in teacher training have demonstrated their potential as a medium that allows student teachers to gain access to real class situations so making it easier to analyse the mathematics teaching-learning process (Ball & Cohen, 1999; Canters, et al. 2002; Dolk et, 2002; Goffree & Oonk, 2001; Mousley & Sullivan, 1996). The use of fragments of classes recorded on video grants the student teachers a more and more progressive use of the conceptual tools that may allow them to go beyond the identification of the superficial characteristics of teaching.

Video-based learning environments with “virtual debates” are implemented on a web site called a “virtual campus” belonging to the University of Alicante which the teacher students can access using a password from any computer connected to Internet. The web site allows the teacher student to access the materials – watching videos and downloading documents in text format – at any time and from anywhere, thus providing the screening of the lessons and participating in the debates regardless of whether they are or not at the actual University Campus. This design allows the teacher student to

- explore a mathematics lesson as a whole or certain fragments and
- establish different virtual debates (social space of interaction) with their classmates asynchronously and with the support of materials (transcription of the lesson, the activities used by the teacher during the lesson, documents with theoretical information on the mathematical discourse in the classroom, ...).

The *learning trajectory* is constructed taking into account virtual “sessions of work”. In each one of these sessions the students have the chance to observe aspects of a mathematics lesson from different perspectives (ranging from their own initial conceptions to positions in which they use some elements of conceptual knowledge introduced in the training programme). This organisation of the learning environment through sessions aims to reflect the evolutionary nature of the process for constructing the knowledge needed for teaching (Goffree & Oonk, 2001). Moreover, the research focusing on teacher students’ learning have shown that the construction of the knowledge needed for teaching mathematics is a process in which the conceptual tools are progressively included in the analysis and reflection. This process for constructing the knowledge needed for teaching mathematics begins with the possibility making the student teachers’ initial conceptions public and debatable, regarding the nature of mathematics, the learning and the teaching of mathematics and the role of the teacher. The progressive use of the conceptual tools in the activities for analysing and interpreting the teaching-learning situations, and progressive modification in the way of participating in the spaces set up for social interaction are manifestations of the knowledge construction process (LLinares, 2002-a).

The work sessions are made up by

- a script of questions that allows the student teachers’ activity to be organised and guided,
- some materials (documents in doc format, videos, links to web pages, ...), and
- a virtual debate.

The teacher students’ work in the different “sessions” allows them to go deeper into the analysis of the lesson progressively and incorporate the conceptual tools gradually in the

debates. Hence, the order of the different sessions allows the student teachers' work to be guided.

For example, a learning environment, for students on the Degree Course in Mathematics who were studying on a course in Teaching Mathematics (Didactics of Mathematics), is meant to analyse a mathematics lesson for pupils aged 13-14 (introduction to symmetry) and it was organised through three "virtual sessions" that formed a "learning itinerary" (Figure 1):

Session 1: Analysis of a mathematics lesson 1: Introduction

Targets: To elicit the student teachers' conceptions from the fact that they grant meaning to the mathematics class: the characteristics of the mathematical problems proposed by the teacher, how the teacher manages the mathematical contents, what the characteristics of the interaction are between the teacher and the students (what is the role of the mathematical interaction and communication in the learning process generated)

Session 2: Analysis of a mathematics lesson 2: In-depth study

Target. To introduce conceptual tools such as the idea of a didactic variable and the role of the socio-mathematical norms that govern the interaction between the teacher and the pupils in the process for constructing the mathematical meaning.

Session 3: Analysis of a mathematics lesson 3: Synthesis

Targets. To use the conceptual instruments for analysing a fragment of the lesson. To compare the contents of the different interventions in the debates held in the previous sessions to identify aspects of what they have managed to learn regarding the teaching of mathematics.

In each one of the sessions, the student teachers have access to a recording of the whole lesson, as well as segments of the lesson, which allows them to focus their attention on specific aspects of what happened in the classroom without having to watch the whole lesson. The use of segments of the lesson together with the whole recording allows the teacher students to be able to contextualise specific aspects of the teaching in the overall context of the lesson. This resource is meant to overcome the limitations deriving from just showing the student teachers sporadic sections from the lessons without any reference to the overall context that they are dealing with, as well as allowing them to concentrate on specific aspects within an overall context when the analysis requires it.

Figure 2 shows a computer screenshot from session 2 "*analysis of the teaching of mathematics. In-depth study*" stating the targets, the methodology and the list of documents. When the student teacher starts a session the computer screen is split into three windows. One of them describes the targets for that session and the work method (screening of a video, reading of documents, participation in a virtual debate). Another window shows the list of the materials that can be used (transcription of the lesson, activities that the teacher provides for the pupils, access to the videos and documents with theoretical information). The text documents can be downloaded in pdf. format or displayed in a separate window. For example, this lets the student teachers watch the video and at the same time they can have the document with the transcription of the lesson open in another window. In the third window, the student teachers can watch and listen to the videos.

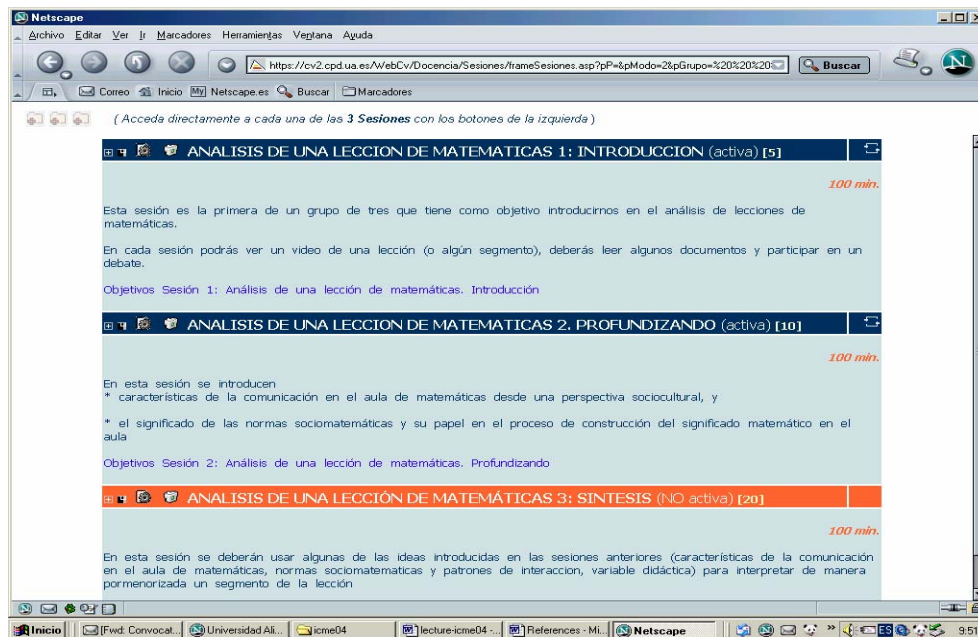


Figure 1. Screenshot with the structure of the “training itinerary” for the module “Analysis of a mathematics lesson”

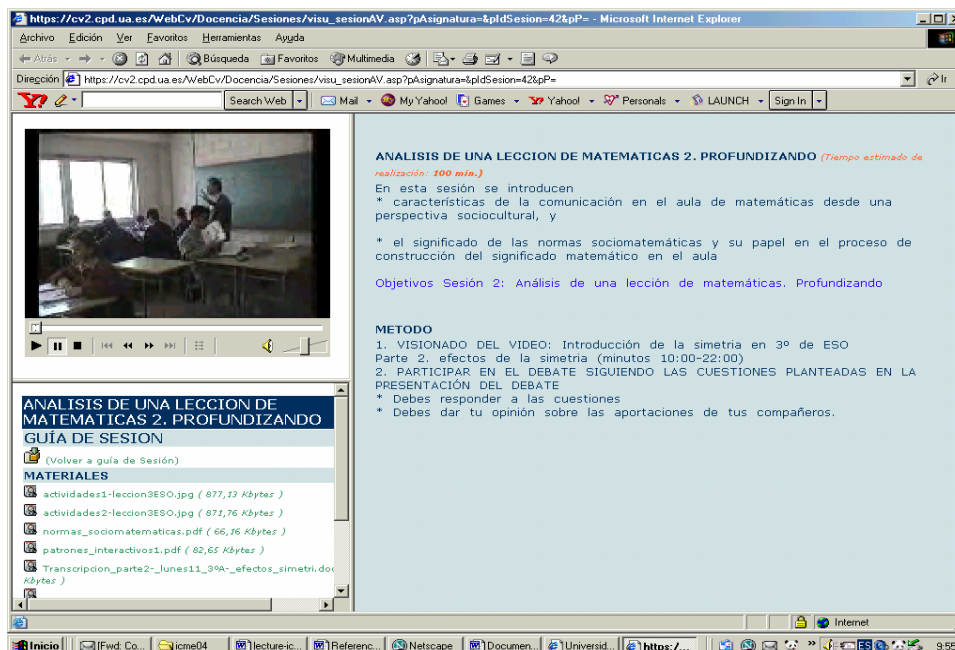


Figure 2. Session 2: Analysis of a mathematics lessons: to study in depth

The recordings of the lesson are accompanied by the transcriptions of the spoken interactions between the pupils and the teacher during the lesson that allow the student teachers to focus their attention on the different contributions from the pupils and the teacher during the process of mathematical communication. The video recordings and the different materials in text format are the resources in these learning environments so that they may be used in a number of virtual debates that form the space in which the interaction may be generated for the social construction of the knowledge (see Figure 3).

The virtual debates are designed to provide the student teachers with the possibility of explaining their initial conceptions regarding what constitutes the focus of the analysis at any time and of setting a first interaction with their classmates in which they may show discrepancies in the way of granting meaning to the different sections of the lesson. The debates are meant to help to transform the initial “superficial” approaches into more professional approaches by means of the introduction of the conceptual tools (Llinares, 2002-b).

For example, the questions that guide the debate for the second session “*Analysis of a mathematics lesson 2: In-depth*” are:

- a) *regarding the characteristics of the problems proposed:*
What range of skills do the tasks proposed for the pupils seem to develop?
What aspects of the concept of symmetry seem to be emphasised?
What do you base your answer on?
- b) *regarding the management of the mathematical contents by the teacher:*
At what times does the teacher support his intervention on the management of a didactic variable? What do you base your answer on?
- c) *regarding the communication in the classroom and the process for generating mathematical meaning:*
When do you think that the teacher is using a communication of a univocal or dialogue type?
What role do you think that these forms of communication play in the process whereby the pupils grant meaning to the idea of symmetry?

The use of case studies allows “students to learn how to reason about the situation, learn how to generalize their thinking, developed for one case to form the basis for thinking about many cases. The situation becomes a tool to think with” (Dolk et al, 2002; 163). Video-based sessions are conceived as cases studies and function as “mediating tools” in the construction of meaning to conceptual tool about teaching. The incorporation of the virtual debates also allows the student teachers to relate with their classmates without having to meet at the same time or in the same place. Furthermore, the fact that they have to write down their reflections (texts of the participation in the debates) allows the reflection and the analysis to be incorporated since they do not have to provide an immediate response as usually happens in the sessions with physical presence.

The possibility that the student teachers may

- i) “observe” the lessons or specific fragments,
- ii) analyse the different aspects, and

- iii) discuss the different interpretations through a virtual space that does not demand that they have to be together physically or even connected at the same time

introduce new characteristics in the design of the opportunities for learning to teach and in the development of the identify of being a mathematics teacher when feeling that he/she is a member of a “virtual community of practice” (sharing targets, media and resources).

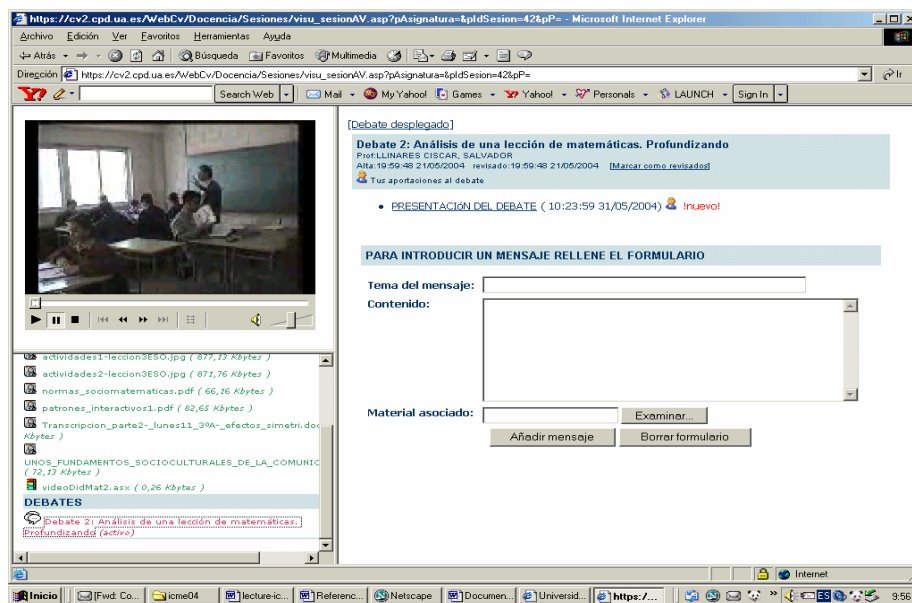


Figure 3. Appearance of the window for the session in video format that includes the possibility of watching a video, participating in a debate and a list of additional documents that may be consulted at any time

Virtual debate allows to teacher student generate un discourse in which the theoretical tools may be used to analysis the case. In this moment, the possibility of teacher students may generate an objectified discourse help to determine the professional learning. The following example describes the way in which we start to analyse the teachers' participation in the virtual debates and to what extent virtual debates may be considered to be spaces in which a social construction of knowledge is generated. From situated perspectives the unit of analysis for understanding how the understanding of a practice is generated by the individuals is the individual-in-practice (Lerman, 2000). Hence the interaction (communication in the virtual debate) between teacher students and the way in which they participate in a virtual debate when performing “professional tasks” may show us the characteristics of the understanding of the practice of teaching mathematics that have been generated. This entails viewing professional learning as an initiation to “certain discourse”. We look at two factors that may dictate what may be learned if a person is to become a skill full participant of a given discourse (Sfard, 2001) mediating tools, and meta-discursive rules that regulate the communicative effort.

Reification and conversational Chains: Interaction in virtual debates and building of communities of learners.

It is also necessary to point out the difficulty of generating productive interactions and establishing communities of learning as a means for enabling mathematics teachers students to develop social practices as teachers. Wenger (1998) argues that through the negotiation of meaning people gain experience about the world and can situate the meaning in a process that involves participation and reification. The reification process produces “objects” (focus of the attention around which the negotiation of meaning is organized). For example, in the research performed by Rey and colleagues (Rey, et al. 2004) in a programme of professional development for mathematics teachers (training for psycho-pedagogues) the interaction in the virtual debates was analysed to identify how the teachers negotiated the meanings of the conceptual tools whilst they solved “professional problems”. In this learning environment the “professional worthwhile task” for the teachers was to interpret and analyse students’ mathematical thinking on whole-number operations and understanding base-ten, place-value skills and number sense and operations (by a case study). The focus was on the type of interactions and the conversational chains generated as an aspect of the reification process of knowledge and beliefs (as a example of Sfard’s objectified discourse; Sfard 2001).

The teacher students participated in a learning environment that integrated the work “real” (the teachers met each other to perform the tasks) and the participation in virtual debates that allowed them to carry on exchanging information (interpretations and negotiation of meanings) without physically coinciding in a given place or at a given time. From situated perspectives of learning, the evolution in the way in which the teacher participates in the environments (now understood as communities of learners) and the generated discourse are indications of the learning achieved. From this viewpoint, in order to analyse the learning it is necessary to characterise

- how the groups of teacher students participate in the debates, and
- the progressive use by the teacher of the “conceptual tools” in solving authentic activities.

The graphic description of the interactions that allows us to see the evolution and the changes are shown graphically using “graphs of interactions”. On the graph each line represents an interaction, the source of the line indicates who makes the contribution and the end represents to which it is addressed (Figure 5). To describe the way in which the teachers participate in the debates we identify “conversational chains” and use the notion of “modes of participation” to elicit the modes of communication.

The “modes” in which the teacher students participate in the debates (the form of the participation and meta-discursive rules) allows information to be obtained regarding the process for negotiating the meanings. Bearing in mind that the participations in the virtual debates are performed using written texts, the “mode of participation” indicates “the attitude of the speaker with respect to the linguistic statement”. That is to say, the attitude of the teacher students (understood in this case as a cognitive individual when functioning by means of shared meanings) in response to the contribution from another teacher students. Sfard (2001) considers meta-discursive rules are the molders, enablers and navigators of communicational activities that regulate the communicative efforts. Rey et al

(2004) use the following categories for describing the “mode of participation in the virtual debate”

Response (RS): initial contribution referring to the task itself. Answer to the questions asked by the moderator throughout the debate.

Questions for reflection (PR): questions that lead to greater reflection on a subject at a given time throughout the debate: review of the contribution, extension of the information... It includes both the questions that arise throughout the discourse of a group and the questions that a participating group launches directly.

Questions for clarification (PA): contributions that demand the clarification of a concept – idea used by a participant.

Responses for clarification (RA): are the answers given by the groups in response to the questions for clarification or the questions for reflection asked by any of the participants.

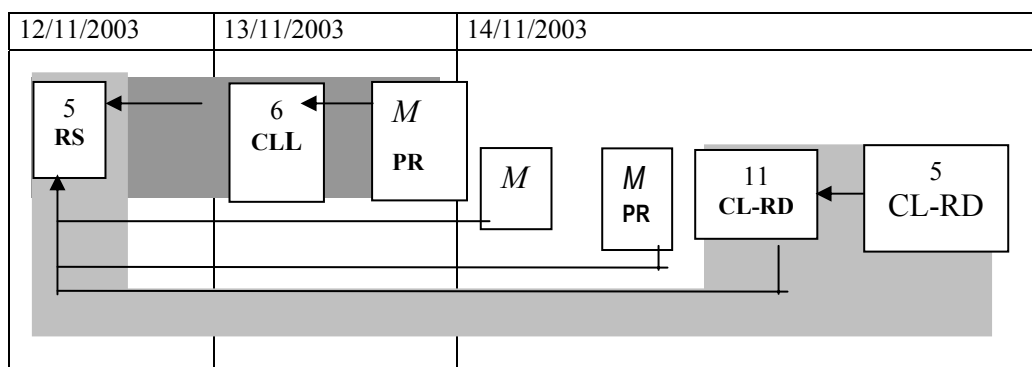
Disagreement (D): shows disagreement with the contribution that is sent to it.

Refutation (RF): shows disagreement with the contribution that is sent to it, accompanying it with arguments that back this disagreement.

Endorsement(RD): shows agreement with and support for a given contribution.

Clarification (CL): Extends and/or refines any previous contribution, whether it is its own or from another participant, by means of the use of new information, describing their own experiences or contributing to the group opinion.

On the other hand , a conversational chain” is a sequence of interactions between teacher students with a specific focus. The conversational chains may be described using graphs showing the order of participation and the type of interaction. Conversational chains allow identify what the teacher students have focused and what is being learnt. The following graph (Figure 7) shows an example of conversational chains in a virtual debate in which the participants were three groups of teachers (named 5, 6, 11; an a moderator M).



*Figure 5. Graphic description of the conversational chain generated in a virtual debate.
RS= initial contribution; CL= Extends an/or refine the previous contribution; PR=
question ask ; RD= shows agreement*

In this example, two conversational chains are identified: group 5 and group 6 (extend information) and the moderator (who asks a question for reflection) participate in the first of them. So, the response to the task issued by group 5 generated questions for reflection (PR) by the moderator and contributions for clarification (CL) from groups 6 and 11. The second one arises between group 5 and group 11 (highlighted in grey). Faced with the response from group 5 to the task. Group 11 shows its agreement with the contribution (endorsement) (CL) and offers new information about the situation (RD). Group 5 confirms its contribution by stating its agreement to it and provides information again (CL) .

The use of the constructs “modes of participating” and “conversational chains” allows to characterize the forms of participation and interaction generated in the debates. The growth of mutual understanding and coordination between teacher students is the focus. So, the aspects of the “person-in-action” are identified as a unit of analysis in which the teacher educator and researcher can look at “what is being talked about”. The contents of the conversational chains and the modes of participation allow the negotiation of the meanings to be identified, the way in which the conceptual tools are used for resolving the task, etc..., i.e., the reification of the knowledge and beliefs that are generated in the community of practice.

In the analysis of the communication in the virtual debate, the participating teacher students concentrate on accumulative interactions. At certain interactive points, a change towards exploratory interactions take place, so that they give explanations, justifications or ask for clarification depending on the contributions of their colleagues, i.e. high level skills are being brought to bear.

Once we have considered the aspects of participation in interaction, we feel that it is important to highlight the mediating tools that teacher student use as the means of communication. In this case, the conceptual tools from mathematics pedagogical content knowledge are the shapers of the content of interaction and whose meanings are being negotiated and developed by the participants in the interaction. These give meaning to the discourse carried out by those interacting during the activity, showing their evolution throughout the process. During the interactions the participants show their mathematical and didactical meaning with regard to the activity, and while this takes place, the participants negotiate and modify these meaning, even if the general meaning that emerge from activity can be distinct from the significant negotiated by participants.

On the other hand , the interactive moments are linked to true learning communities, which have synthesised the group of students considered in this research. Community identification was characterized by the assumption of collective objectives by means of carrying out activities in the virtual learning environment, and by statements, negotiation, comprehension and exploitation of knowledge involved in these activities. Furthermore, we can safely say that the three characteristics of a practice that determine the establishment of a learning community as stated by Wenger (1998) occurred. The first of these is a mutual commitment recorded in the interactions and interactive moments that characterized the virtual debate’s conversational networks. The second is the collective enterprise of making contributions and responses to improve the carrying out the activities. The third is the shared list due to the availability and access to all conversational registers

by all participants carrying out the activities. We therefore conclude that the interactions are a means for constructing and negotiating meanings of conceptual tools from mathematics pedagogical content knowledge.

Some final observations

Using the theoretical constructs taken from situated perspectives for characterising the teacher's learning and the teaching of mathematics as a practice creates opportunities for thinking about how learning arises. Integrating virtual debates in video-based learning environments allows us to consider the progressive development of student teachers' knowledge for teaching from

- observing
- sharing and discussing their first interpretation,
- negotiating the meaning of conceptual tools and their usefulness for analysing and interpreting a real situation,
- writing reports and reviewing them with reflection, and
- answering questions such as "What have we learnt?"

On the other hand, at the present time we are now starting to understand how virtual debates can be used and how they influence the process of becoming a teacher (reification process of knowledge and initiation of a objectified discourse), forms of participating and development of the identity). In this sense, virtual debate as a space where student teachers/teachers can share their observations and analysis is a means by which the theory-practice dichotomy at University can be reduced. But it is necessary that teacher students learn new communicative skills in a new interaction space. Now more research is required into how learning to teach arises in these kinds of environments. More experimentation and research into this line will allow us to get to know

- the potential of situated perspectives for learning and practice when they are applied to the learning of the mathematics teacher, and
- the new forms of learning that are generated when including the media for communicating and the interrelationship between the people that are provided by the new technologies (web, internet, , ...).

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