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Acalculia and Dyscalculia

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Even though it is generally recognized that calculation ability represents a most important type of cognition, there is a significant paucity in the study of acalculia. In this paper the historical evolution of calculation abilities in humankind and the appearance of numerical concepts in child development are reviewed. Developmental calculation disturbances (developmental dyscalculia) are analyzed. It is proposed that calculation ability represents a multifactor skill, including verbal, spatial, memory, body knowledge, and executive function abilities. A general distinction between primary and secondary acalculias is presented, and different types of acquired calculation disturbances are analyzed. The association between acalculia and aphasia, apraxia and dementia is further considered, and special mention to the so-called Gerstmann syndrome is made. A model for the neuropsychological assessment of numerical abilities is proposed, and some general guidelines for the rehabilitation of calculation disturbances are presented.

KEY WORDS: acalculia; dyscalculia; numerical knowledge; calculation ability.

INTRODUCTION

Calculation ability represents an extremely complex cognitive process. It has been understood to represent a multifactor skill, including verbal, spatial, memory, and executive function abilities (Ardila et al., 1998). Calculation ability is quite frequently impaired in cases of focal brain pathology (Ardila and Rosselli, 1992; Grafman, 1988; Harvey et al., 1993; Hécaen et al., 1961; Rosselli and Ardila, 1989) and dementia (Deloche et al., 1995; Grafman et al., 1989; Parlatto et al., 1992; Rosselli and Ardila 1998). The loss of the ability to perform calculation tasks resulting from a cerebral pathology is known as *acalculia* or *acquired dyscalculia*. *Acalculia* has been defined as an acquired disturbance in computational ability (Loring, 1999). The developmental defect in the acquisition of numerical abilities, on the other hand, is usually referred to as *developmental dyscalculia* (DD) or *dyscalculia*.

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Acalculia is frequently mentioned in neurological and neuropsychological clinical reports, but research directed specifically to the analysis of acalculia is rather limited. During 1990–99, there were 83 Medline and 56 PsychInfo entries for "acalculia/dyscalculia" as key word. The word *acalculia* or *dyscalculias* was included in the journal paper or general publication titles 31 and 13 times respectively according to these two databases during this 10-year period (Table 1).

The same applies to testing for calculation abilities. Tests for calculation abilities are always included in the psychological or neuropsychological evaluation of cognition. Calculation abilities are included when testing for general intelligence (e.g., WAIS-III; Wechsler, 1997) and most neuropsychological assessment procedures worldwide include the assessment of calculation abilities. Moreover, they are frequently included in the Mini-Mental State Examination (MMSE) (Folstein et al., 1975) and other brief neuropsychological assessment procedures (e.g., Neuropsi; Ostrosky et al., 1997). But, a specific standardized test battery with norms for acalculia is hardly found (Deloche et al., 1994).

Thus, acalculia is in a somewhat peculiar position amid the cognitive disturbances encountered in cases of brain pathology. Although there is a general consensus that calculation ability represents an extremely important

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Table 1. Number of Entries of Some Neuropsychological Syndromes Found in Medline and Psychinfo During 1990–99

	Medline		PsychInfo	
	As key word	In the title	As key word	In the title
Dementia	18,180	5,580	4,269	1,978
Amnesia	2,528	713	965	318
Aphasia	2,344	632	972	417
Alexia/dyslexia	1,278	409	601	317
Apraxia	665	240	222	91
Agnosia	505	104	146	61
Agraphia/dysgraphia	266	84	123	39
Acalculia/dyscalculia	88	31	54	13

type of cognition, and calculation abilities are tested in virtually any psychological and neuropsychological assessment procedure, research on acalculia is quite limited.

Historical Note

Henschen (1925) proposed the term *acalculia* (*akalkulia*). He then defined *acalculia* as the impairment in computational skills resulting from brain injury. Nonetheless, before Henschen some mention of calculation disturbances associated with brain damage is found in the literature. In most cases, those calculation disturbances were interpreted as sequelae of the language impairment (aphasia).

Lewandowsky and Stadelmann published the first detailed report of a patient suffering from calculation disturbances in 1908. Their patient presented with focal brain damage associated with right homonymous hemianopia, and had significant difficulties in written and mental calculations. The authors further reported that the patient had impairments in reading arithmetical signs, even though the patient could carry out arithmetical operations. This paper represents a landmark in the development of the concept of acalculia, because it considers that calculation disorders are different and dissociated from language disturbances.

Henschen (1925) reviewed 305 cases in the literature that reported calculation disturbances associated with brain damage, and 67 of his own patients. He identified those patients presenting calculation disturbances without evident language impairments, and proposed an anatomical substrate for arithmetical operations, different from but close to the anatomical substrate for language and musical ability. The third frontal convolution was suggested to represent the center for the pronunciation of numbers. The angular gyrus and the fissure interparietalis were proposed as the brain areas participating in number reading, and the angular gyrus was regarded as the brain structure responsible for writing numbers.

Berger (1926) introduced the distinction between primary and secondary acalculia. Primary or “pure” acalculia corresponds to the loss of numerical concepts and to the inability to understand or execute basic arithmetical operations. Secondary acalculia refers to the defect in calculation derived from other cognitive deficits (e.g., memory, language, etc.). This distinction became particularly influential, and then was increasing recognition that calculation disturbances could be associated with and dependent upon other cognitive defects, such as aphasia, alexia, and agraphia. The polemic, however, has revolved around the possible existence of a primary acalculia, because some authors have questioned the existence of acalculia as an independent cognitive deficit (Collington et al., 1977; Goldstein, 1948).

Gerstmann (1940) proposed that primary acalculia is associated with agraphia, right–left disorientation, and digital agnosia, conforming a single brain syndrome that since then has been known as “Gerstmann syndrome.” Neuroimaging methods have shown a correlation between the Gerstmann syndrome and left posterior parietal injuries (Mazzoni et al., 1990).

Lindquist (1936) distinguished different types of acalculia associated with lesions in different brain areas. He conjectured that calculation disturbances are not homogeneous, and in consequence, acalculia subtypes may be distinguished. As a result, several classifications of acalculias have been presented (e.g., Ardila and Rosselli, 1990; Grafman, 1988; Grafman et al., 1982; Hécaen et al., 1961; Lindquist, 1936; Luria, 1973), and different patterns of errors have been described in patients with right and left hemispheric injuries (Levin et al., 1993; Rosselli and Ardila, 1989).

Boller and Grafman (1983) claim that calculation abilities can be impaired as a result of different types of defects. They believe that calculation skills can be altered as a result of (1) inability to appreciate the significance of the names of numbers, (2) visual–spatial defects that interfere with the spatial organization of the numbers and the mechanical aspects of the operations, (3) inability to remember mathematical facts and to appropriately use them, and (4) defects in mathematical thought and in the comprehension of the underlying operations. Interestingly, occasionally brain damage may result in a relatively restricted disorder in performing arithmetical operations, for instance, a limited alexia for arithmetical signs (Ferro and Botelho, 1980), or a specific deficit for arithmetical procedures (Semenza, 1988), without additional calculation disturbances.

Calculation ability implies the use of numerical concepts. The concept of numbers can be associated with the presence of at least four factors: (1) immediate representation of quantity, implicit in the number; (2) understanding the numerical position within the system of other numerical symbols (i.e., its position in the series of digits and its place in class); (3) understanding the relationships between a number and other numbers; and (d) understanding the relationships between numerical symbols and their verbal representations (Tsvetkova, 1996).

McCloskey et al. (1985, 1986, 1991a,b) and McCloskey and Caramazza (1987) proposed a cognitive model regarding the processing of numbers and the relationship of arithmetical operations. This model includes a distinction between the processing system of numbers (an understanding mechanism and the production of numbers), in addition to the numerical calculation system that includes the necessary processing components to accomplish mathematical operations. In the event of brain injury, these components can be disassociated (Dagenbach and McCloskey, 1992; Pesenti et al., 1994). The principles (multiplication tables), rules ($N \times 0 = 0$), and procedures (multiplication proceeds from right to left) form part of the numerical calculation system. Errors of calculation observed in patients with brain injury and in normal participants can result from inappropriate recall of principles, inadequate use of rules, and/or errors in procedures. Cognitive modeling has helped to establish similarities between acquired acalculias and DDs (Temple, 1991).

Clark and Campbell (1991) have presented a "specific integrated theory" of calculation abilities. This theory presumes that visuospatial, verbal, and other modality-specific number codes are associatively connected as an encoding complex and that different facets of number processing generally involve common rather than independent, processes. This point of view emphasizes the participation of multiple components in calculation. It seems evident that normal calculation ability requires verbal, visuospatial, and other fundamental cognitive skills. Calculation impairments, as a matter of fact, can be observed in cases of a wide diversity of brain disturbances: Left or right hemisphere, frontal, parietal, temporal, and occipital. Moreover, subcortical lesions can also result in some calculation defects (Dehaene and Cohen, 1997). Almost any type of brain pathology may produce difficulties on calculation tests, even though the specific pattern of difficulties can be different (Rosselli and Ardila, 1989).

Calculation Ability: One or Several Abilities? The Question of Modularity

Calculating has been identified as a concept and goal formation cognitive skill (Mandell et al., 1994). Calculation ability under normal circumstances requires not only the comprehension of numerical concepts, but also that of conceptual abilities and other cognitive skills. In the neuropsychological domain, however, very little research indeed has been carried out to explore the relationship between mathematical test performance and performance on other cognitive tests. Ardila et al. (1998) administered a comprehensive neuropsychological test battery to a 300-participant sample, aged 17–25 years. All were right-handed male university students. The battery included some basic psychological and neuropsychological tests directed to assess language, calculation abilities, spatial cognition, praxis abilities, memory, perceptual abilities, and executive functions. The following tests were included: (1) Auditory Recognition (recognition of songs and the Seashore Rhythm). (2) Verbal Fluency (phonologic and semantic). (3) Nonverbal Fluency. (4) Serial Verbal Learning (5) Finger Tapping Test (FTT) (left hand and right hand). (6) The Rey–Osterrieth Complex Figure (ROCF) (copy and immediate recall). (7) Ratcliff's Mental Rotation Test. (8) Arithmetical Abilities. Two different tests were used: (a) Mental Arithmetical Operations (two additions, two subtractions, two multiplications, and two divisions) and (b) arithmetical problems. Sixteen arithmetical problems were orally presented. The participants were allowed to use pencil and paper if so desired. (9). Localization of cities on a map. (10) Orthography Test. (11) Perceptual Recognition (similarities between two figures, differences between two figures and hidden figures). (12) Reading Speed. (13) Wechsler Adult Intelligence Scale (WAIS). (14) Wechsler Memory Scale (WMS). And (15). Wisconsin Card Sorting Test (WSCT). Forty-one different scores were calculated.

Four different tests were considered in the analysis of the calculation abilities (Mental arithmetical operations, Arithmetical problems, WAIS Arithmetic, and WAIS Digits subtests). Numerical ability tests turned out to present a notably complex correlation system. Mental arithmetic significantly correlated with 25 (out of 41) test scores, and Arithmetical problems with 17 test scores. WAIS Arithmetic subtest correlated with 15 test scores, and WAIS Digits correlated with 7 test scores. Some correlations were quite understandable (e.g., different mathematical test scores are highly intercorrelated), whereas other correlations were rather unexpected (e.g., arithmetical ability tests highly correlate with the Orthography test). Main correlations were observed with verbal memory, visuospatial, visuoperceptual, language, and visuoconstructive ability tests (Table 2). It may therefore be assumed that arithmetical ability is associated with and depends upon some verbal, visuoperceptual, visuospatial, and memory abilities. Consequently, it is not surprising that there is such a wide variety of calculation disturbances observed in brain pathology. Calculation ability disturbances are even associated with body knowledge disturbances (autotopagnosia, finger agnosia) (Gerstmann, 1940).

Table 2. Correlations over.15 ($p < .01$) Between Different Tests[a]

Test	Correlates with	r
Mental arithmetic	Arithmetical problems	.49
	Orthography test	.37
	WAIS: Arithmetic	.35
	WMS: Associative Learning	.29
	Ratcliffs test	.25
	WMS: Logical Memory	.26
	Perceptual speed: Similarities	.25
	WAIS: Information	.24
	WAIS: Similarities	.23
	WAIS: Comprehension	.23
	WAIS: Digits	.23
	Phonologic Verbal Fluency	.22
	WAIS: Vocabulary	.21
	Localization of cities on a map	.20
	WAIS: Picture completion	.20
	WAIS: Picture arrangement	.20
	Perceptual speed: differences	.19
	Seashore Rhythm Test	.18
	Perceptual speed: Hidden	.18
	WAIS: Digit–symbol	.18
	WAIS: Block design	.17
	WAIS: Object assembly	.16
	Reading speed	.16
	WMS: Orientation	.16
	WMS: Mental Control	.15
Solving arithmetical problems	Mental arithmetic	.49
	WAIS: Arithmetic	.46
	Orthography test	.32
	WAIS: Block design	.31
	WAIS: Information	.28
	Ratcliffs test	.26
	WAIS: Digit–symbol	.26
	WAIS: Comprehension	.24
	WMS: Mental Control	.24
	WMS: Logical Memory	.23
	WAIS: Similarities	.22
	WAIS: Picture Arrangement	.21
	WAIS: Digits	.20
	WAIS: Vocabulary	.19
	Perceptual speed: Similarities	.18
	Perceptual speed: differences	.18
	WAIS: Picture completion	.18
WAIS: Arithmetic	Arithmetical problems	.46
	WAIS: Information	.40
	Mental Arithmetic	.35
	WAIS: Comprehension	.32
	WAIS: Vocabulary	.31
	WAIS: Picture completion	.26
	WAIS: Digits	.22
	WAIS: Block design	.21
	Phonologic Verbal Fluency	.21
	Localization of cities on a map	.21
	WAIS: Similarities	.18
	Semantic Verbal Fluency	.17
	Orthography test	.17
	Perceptual speed: Hidden	.16
?>	WAIS: Picture arrangement	.15
WAIS: Digits	Mental arithmetic	.23
	Phonologic Verbal Fluency	.23
	WAIS: Arithmetic	.22
	Arithmetical problems	.20
	Orthography test	.17
	FTT: Left hand	.16
	WAIS: Similarities	.16

[a]Mental Arithmetic, Arithmetical Problems, WAIS: Arithmetic, and Digits Subtests (adapted from Ardila et al., 1998a).

A factor analysis with varimax rotation disclosed five factors accounting for 63.6% of the total variance. Interestingly, none of these factors was a "calculation" or "numerical" factor. The first factor, accounting for over one fourth of the variance, was clearly verbal ability. It was best tested with the Similarities, Information, Vocabulary, and Comprehension subtests from the WAIS. This result reinforces the assumption that Arithmetic and Digits subtests are not purely verbal subtests. Factor II was a perceptual or nonverbal factor, whereas factor III was simply a WCST factor, nonsignificantly correlated with other test scores. Minor correlations were observed with the WMS Information and Orientation subtests. Factor IV was a type of fine motor factor or (motor and verbal) fluency factor. Factor V was a memory, especially verbal memory, factor (Table 3).

"General intelligence" (Full Scale IQ) as understood in the WAIS was best associated with the mathematical ability tests and the orthography test (Table 4). These correlations were particularly high and significant. The association with some perceptual tests was somehow lower, yet strongly significant. All of these tests (mathematical, orthographic, and perceptual tests) presented a very complex intercorrelation system with a wide variety of other psychological and neuropsychological tests. Consequently, they may be measuring a diversity of cognitive abilities. Still, mathematical abilities represented the best predictors of the "general intelligence" according to the psychometric concept of intelligence. Interestingly, no

Table 3. Correlations Between the Different Calculation Tests and the Five Factors (Adapted from Ardila et al., 1998a)

	Factors				
Tests	I Verbal	II Perceptual	III WCST	IV Fluency	V Memory
Mental arithmetic	.40	.35	.10	.03	.19
Arithmetical problems	.37	.40	−.03	.07	.23
WAIS: Arithmetic	.44	.28	−.05	.15	.19
WAIS: Digits	.30	.02	.17	.18	.08

Table 4. Correlations over.15 ($p < .01$) Between Verbal IQ, Performance IQ, and Full Scale IQ with Different Test Scores (Adapted from Ardila et al., 1998a)

	Correlates with	*r*
Full Scale IQ	*Arithmetical problems*	.40
	Orthography test	.36
	Mental arithmetic	.35
	Perceptual speed: Similarities	.32
	Seashore Rhythm Test	.28
	Perceptual speed: Hidden	.27
	Phonologic verbal fluency	.25
	ROCF: Memory	.24
	Localization of cities	.24
	Reading speed	.20
	Semantic verbal fluency	.19
	Ratcliffs test	.16
Verbal IQ	WAIS: Vocabulary	.81
	WAIS: Comprehension	.68
	WAIS: Information	.67
	WAIS: Similarities	.59
	WAIS: *Arithmetic*	.50
	Arithmetical problems	.32
	Phonologic verbal fluency	.32
	Mental arithmetic	.31
	Orthography test	.31
	WAIS: *Digits*	.28
	Localization of cities	.25
	Semantic verbal fluency	.18
Performance IQ	*Arithmetical problems*	.35
	Perceptual speed: Similarities	.33
	Seashore Rhythm Test	.28
	Mental arithmetic	.28
	Perceptual speed: Hidden	.28
	ROCF: Memory	.24
	Orthography test	.24
	Reading speed	.20
	Ratcliffs test	.17

single WAIS verbal or performance subtests, according to current results, could be considered a good predictor of Full Scale IQ.

In conclusion, calculation abilities are associated with a diversity of other cognitive abilities, including verbal, perceptual, spatial, memory, and executive function abilities. In this regard, calculation ability represents a multifactor ability. To a significant extent, it reflects the ability to manipulate acquired knowledge (Mandell et al., 1994). The ability to use numerical information is highly correlated with general intellectual level. It is not surprising that calculation disturbances are heterogeneous, and can be observed in cases of brain pathology of quite different locations. Virtually, any cognitive defect may result in impairments in calculation abilities. By the same token, mental calculation impairments have been demonstrated to represent an important factor in predicting cognitive deficits in cases of dementia (Roudier et al., 1991). Noteworthy, using functional magnetic resonance imaging (fMRI) and positron emission tomography (PET) while performing arithmetical operations, a complex pattern of activity has been demonstrated. Most active areas include the left prefrontal areas and the posterior superior temporal gyrus (Burbaud et al., 1995; Sakurai et al., 1996).

HISTORICAL DEVELOPMENT OF CALCULATION ABILITIES

Calculation abilities have followed a long process, since initial quantification of events and elements, to modern algebra, geometry, and physics. Some rudimentary numerical concepts are observed in animals and no question prehistorical man used some quantification. However, the ability to represent quantities, the development of a numerical system, and the use of arithmetical operations, is found only in old civilizations.

Numerical Concepts in Animals

The origin of mathematical concepts can be traced to subhuman species. Throughout recent history different reports have argued that animals (horses, rats, dogs, chimpanzees, dolphins, and even birds) can use numerical concepts and perform arithmetical operations.

There is general agreement that some rudimentary numerical concepts are observed in animals. These basic numerical skills can be considered as the real origin of the calculation abilities found in contemporary man. For instance, pigeons can be trained to pick a specific number of times on a board, and rats can be trained to press a lever a certain amount of times to obtain food (Calpadi and Miller, 1988; Koehler, 1951; Mechner, 1958). It could be conjectured that pigeons and rats can count, at least up to a certain quantity: They can recognize how many times a motor act—to pick on a board or to press a lever—has been repeated. If this behavior can or cannot be really interpreted as counting is nonetheless questionable. But it is observed, at least, after a long and painstaking training. Interestingly, these animal responses (to pick or to press the lever) are not accurate but just approximate. In other words, when the rat is required to press the lever seven times, the rat presses it *about* seven times (i.e., five, six, seven, eight times) As Dehaene (1997) emphasizes, for an animal $5 + 5$ does not make 10, but only about 10. According to him, such fuzziness in the internal representation of numbers prevents the emergence of exact numerical arithmetical knowledge in animals. Using highly controlled and sophisticated designs, it has been pointed out that chimpanzees can even use and add simple numerical fractions (e.g., $1/2 + 1/4 = 3/4$) (Woodruff and Premack, 1981). These observations support the assumption that some quantity concepts can be found in different animals.

Counting (or rather, approximately counting) motor responses is just a motor act as it is walking or running. "Counting" lever pressing is not so different to estimate the effort (e.g., number of steps or general motor activity) required going from one point to another. Counting could be linked to some motor and propioceptive information.

Not only chimpanzees but also rats and many other animals can distinguish numerosity (i.e., global quantification): they prefer the bowl containing the larger number of nutritive elements (chocolates, pellets, or whatever) when selecting between two bowls containing different amounts (Davis and Perusse, 1988). It may be conjectured that global quantification (numerosity perception) and counting (at least the approximate counting of motor responses) represent kind of basic calculation abilities found at the animal level. Rats prefer the bowel containing 10 pellets to the bowel containing 20 pellets; however, they do not prefer the bowel containing 20 pellets to the bowel containing 21 pellets. Obviously, numerosity perception is related to size and shape of the visual image projected to the retina. It can be assumed that 20 pellets in a bowel result in a larger and more complex retinal image than do 10 pellets. But the visual image corresponding to 20 pellets is difficult to distinguish from the visual image corresponding to 21 pellets.

Calculation Abilities in Prehistorical Man

Chimpanzees are capable of various forms of numerical competence including some correspondence constructions for low quantities (Davis and Perusse, 1988; Premack, 1976). Most likely, these numerical abilities also existed in prehistorical man. *Homo sapiens* antecessors may have been capable to use correspondence constructions in some social activities such as food sharing. It has been proposed that *Homo habilis* (ancestor of *Homo erectus*, living about 2.5 million years ago) required to use correspondence constructions when butchering large animal carcasses (Parker and Gibson, 1979). Distributing pieces of a divided whole (e.g., a pray) into equal parts required the ability to construct one-to-one correspondences. Probably, Paleolithic man was able to match the number of objects in different groups. And eventually, the number of objects in a collection with the number of items in some external cue system, for example, fingers or pebbles (incidentally, *calculus* means pebbles).

The immediate recognition of certain small quantities is found not only in animals but also in small children. Animals and children can readily distinguish one, two, or three objects (Fuson, 1988; Wynn, 1990, 1992). Beyond this point, however, errors are observed. Oneness, twoness, and threeness seemingly are basic perceptual qualities that our brain can distinguish and process without counting. It can be conjectured that when the prehistoric humans began to speak, they might have been able to name only the numbers 1, 2, and perhaps 3, corresponding to specific perceptions. To name them was probably no more difficult than to name any other sensory attribute (Dehaene, 1997). Noteworthy, all the world languages can count up to three, even though three may represent "many," "several," or "a lot" (Hurford, 1987). "One" is obviously the unit, the individual (the speaker may also be "one"). Two conveys the meaning of "another" (e.g., in English and also in Spanish, "second" is related with the verb "to second" and the adjective "secondary"). "Three" may be a residual form from "a lot," "beyond the others," "many" (e.g., "troppo" that in Italian means "too much" is seemingly related with the word "three" -*tre*-). In the original European language, spoken perhaps some 15,000–20,000 years ago, apparently the only numbers were "one," "one and another" (two), and "a lot," "several," "many" (three) (Dehaene, 1997). In some languages, two different plurals are found: a plural for small quantities (usually 2, sometimes 3 and 4) and a second plural for large quantities.

Interestingly, in different world languages the word *one* has *no*t any apparent relationship with the word *first*, and the word *two* is not related either with the word *second*. *Three* may sometimes but not always holds some relationship with *third*. Beyond three, ordinals are clearly associated with cardinal numbers (Table 5). The conclusion is obvious: for small quantities, cardinals and ordinals must have a different origin. For larger quantities, ordinal numbers are derived from cardinals. As a matter of fact, one/first and two/second correspond to different conceptual categories.

It may be speculated that for the prehistorical man the first person and the second person in a line (or the first animal and the second animal during hunting or whatever) do not seem to be related with numbers 1 and 2. For small children "first" has the meaning of "initial" (e.g., "I go first") whereas "second" is related to "later" or "after" ("you go second"). They have a temporal and also spatial meaning, but not an evident cardinal meaning. The associations between "one" and "first," and between "two" and "second" seem a relatively advanced process in the development of numerical concepts. That is, the numerical meaning of "first" and "second" seems to appear after its temporal and spatial meaning. The association between

Table 5. Cardinal and Ordinal Numbers in Different Languages

English	Spanish	Russian	Greek	Persian	Arabic	Hindi	Aymara[a]	Ibo[b]
One	Uno	Odin	Ena	Yek	Wahid	Ek	Maya	Nbu
First	Primero	Pervie	Proto	Aval	Awal	Pahla	Nairankiri	Onye-nbu
Two	Dos	Dva	Dio	Dou	Ethnaim	Do	Phaya	Ibua
Second	Segundo	Vtoroi	Deftero	Douvoum	Thani	Dusra	Payairi	Onye-ibua
Three	Tres	Tri	Tria	Seh	Thalatha	Tin	Kimsa	Ito
Third	Tercero	Treti	Trito	Sevoum	Thalith	Tisra	Kimsairi	Nke-ito
Four	Cuatro	Cheterie	Tesera	Chahaar	Arrbaa	Char	Pusi	Ano
Fourth	Cuarto	Chetviorti	Tetarto	Chaharoum	Rabiek	Chautha	Pusiiri	Nke-ano
Five	Cinco	Piat	Pente	Pang	Khamsa	Panch	Pheschka	Ise
Fifth	Quinto	Piati	Pemto	Panjoum	Khamis	Panchvan	Pheskairi	Nke-ise
Six	Seis	Shest	Exi	Shash	Sitta	Chha	Sojjta	Isi
Sixth	Sexto	Shestoi	Ekto	Shashoom	Sadis	Chhatha	Sojjtairi	Nke-isi
Seven	Siete	Siem	Epta	Haft	Sabaa	Sat	Pakallko	Isaa
Seventh	Septimo	Sidmoi	Evthomo	Haftoom	Sabieh	Satvan	Pakallkoiri	Nke-isaa
Eight	Ocho	Vosiem	Octo	Hasht	Thamania	Ath	Kimsakallko	Asato
Eighth	Octavo	Vosmoi	Ogdoo	Hashtoom	Thamin	Athvan	Kimsakallkiri	Nke-asato
Nine	Nueve	Dievit	Enea	Nouh	Tisaa	Nau	Llatunca	Itonu
Ninth	Noveno	Diviati	Enato	Houhum	Tasih	Nauvan	Llatuncairi	Mke-Itonu
Ten	Diez	Diesit	Deka	Dah	Ashra	Das	Tunca	Iri
Tenth	Decimo	Disiati	Dekato	Dahoom	Asher	Dasvan	Tuncairi	Mke-iri

[a] Ameridian language spoken in Bolivia.
[b] Ibo: Eastern Nigeria.

ordinals and cardinals becomes evident only for larger quantities (more than three) and seems to represent a later acquisition in human evolution and complexization of numerical concepts. In many contemporary languages (e.g., Uitoto language, spoken in South America) there are not ordinal numbers. For “the first” Uitoto language uses “the beginning”; to express “second” the word *another* is used.

Arithmetical abilities are clearly related with counting. Counting, not simply recording the approximate amount of motor responses required to obtain a reinforcement, but to say a series of number words that correspond to a collection of objects, is relatively recent in human history. Counting is also relatively late in child development. In human history as well as in child development (Hitch et al., 1987) counting using number words begins with sequencing the fingers (i.e., using a correspondence construction). The name of the finger and the corresponding number can be represented using the very same word (that means, the very same word is used for naming the thumb and the number 1; the very same word is used to name the index finger and the number 2, etc.). The fingers [and toes; as a matter of fact, many languages (e.g., Spanish), use a single word (*dedo*) to name the fingers and toes] are usually sequenced in a particular order. This strategy represents a basic procedure found in different ancient and contemporary, cultures around the world (Cauty, 1984; Levy-Bruhl, 1947). Interestingly, it has been demonstrated that children with low arithmetical skills also present a finger misrepresentation on Draw-a-Person Test (Pontius, 1985, 1989). This observation has been confirmed in different cultural groups.

Taking a typical example as an illustration, the Colombian Sikuani Amazonian jungle Indians count in the following way: the person (a child when learning to count or an adult when counting) places his/her *left* hand in supination to point the number 1, the right index points to the left little finger, which is then bent (Queixalos, 1985, 1989). The order followed in counting is always from the little finger to the index. To point to the number 5, the hand is turned and the fingers opened; for 6, both thumbs are joined, the left fingers are closed, and the right opened; they are opened one after the other for 7, 8, 9, and 10. Between 11 and 20, the head points to the feet and the sequence is reinitiated. The lexicon used is as follows:

1. *kae* (the unit, one)
2. *aniha-behe* (a pair, both)
3. *akueyabi*
4. *penayanatsi* (accompanied; i.e., the fingers together)
5. *kae-kabe* (one hand)

Numbers from 6 to 9 are formed with “one hand and (a certain number) of fingers.” Ten becomes “two hands.”

6. *kae-kabe kae-kabesito-nua* (one hand and one finger)

7. *kae-kabe aniha-kabesito-behe* (one hand and a pair of finger)
10. *aniha-kabe-behe* (two hands)

"Two hands" is maintained between 10 and 20. Toes (*taxawusito*) are added between 11 and 14, and "one foot" (*kae-taxu*) is used in 15. Twenty is "two hands together with two feet."

11. *aniha-kabe-behe kae-taxuwusito* (two hands and one toe)
12. *aniha-kabe-behe aniha-tuxuwusito-behe* (two hands and two toes)
15. *aniha-kabe-behe kae-taxu-behe* (two hands and one foot)
16. *aniha-kae-behe kae-taxu-behe kae-taxuwusito* (two hands, one foot, and one toe)
20. *aniha-kabe-behe aniha-taxu-behe* (two hands and two feet)

Fingers are named according to their order in counting (as mentioned previously, counting begins always with the little finger of the left hand). Sikuani language possesses number words only up to three (*kae*, *aniha-behe*, and *akueyabi*). Four (*penayanatsi* = accompanied, together) represents a correspondence construction. Strictly speaking, Sikuani language counts only up to three. From 4 to 20, they use a correspondence construction, not really counting; and for higher quantities, they recur to a global quantification.

Sometimes not only the fingers (and toes) but also other body segments may be used in counting: the wrist, the shoulders, the knees, and so on (Cauty, 1984; Levy-Bruhl, 1947). But sequencing the fingers (and toes) represents the most universal procedure in counting. Some languages (e.g., some Mayan dialects and Greenland Eskimo) use the same word to denote the number 20 (i.e., "all the fingers and all the toes") and "a person."

In different Amerindian languages, for higher than 10 or 20 figures, most often "many" is used (global quantification principle) (Cauty, 1984). Or, they can recur to other people's hands (correspondence construction) (e.g., 35 might be something like "my two hands, my two feet, my father's two hands, my father's one foot"). As mentioned, "20" sometimes becomes something like "one person," a sort of higher order numeral. It is interesting to note that in some contemporary languages (like English and Spanish) "one" means the unit, but it is also used as a sort of indefinite personal pronoun. In English and Spanish we can also use "one" as synonymous of "myself." Twenty is found to be the base number in the Maya's numerical system (Cauty, 1984; Swadesh, 1967). In many contemporary languages, a 10 and/or 20 base can be evident.

"Digit" (from *digitus*, Latin) in English or Spanish (*dégito*) means not only number but also finger. The correspondence construction between numbers and fingers is evident. Latin number notation was originally Etruscan (Turner, 1984), and referred (as everywhere) to the fingers. One, two, and three were written simply making vertical strokes. In four the Latin system recurs to a simplification. Originally, four was written IIII, but later on it became IV. Five (V) represented the whole hand with the arm bent (i.e., all the fingers of the hand), and 10 (X) the two arms crossed.

From a neuropsychological perspective, the strong relationship existing between numerical knowledge, finger gnosis, and even lateral (right–left) knowledge becomes understandable. Finger agnosia (and probably right–left discrimination disturbances) could be interpreted as a restricted form of autotopagnosia (Ardila, 1993). It is not surprising to find that a decimal (or vigecial) system has been most often developed. Simultaneously or very close in time, decimal systems appeared in different countries (Sumer, Egypt, India, and Creta). Different symbols were used to represent 1, 10, 100, and 1,000 (Childe, 1936).

There is, however, an interesting and intriguing exception: Sumerian and later Babylonians (about 2,000 BC) developed not only a decimal system but also a sexagesimal system: a symbol represented 60 or any 60-multiple, and other different symbol represented the number 10 and any 10-multiple. Thus, for example, the number 173 was then represented: 2×60 (the symbol for 60 repeated twice) $+ 5 \times 10$ (the symbol for 10 repeated five times) $+$ 3 (a symbol for units repeated three times). A base of 60 has remained for some contemporary time measures (e.g., hours, minutes, seconds). Twelve is also frequently maintained as a "second-order" unit (e.g., a dozen). Evidently, 60 results from "five times 12." Five obviously is "one hand," and the question becomes where 12 comes from. Which are the two additional units? It might be speculated that 12 means the 10 fingers plus the 2 feet or even the 2 elbows or the 2 shoulders or the 2 knees (individuality of components is easier to appreciate in the hands than in the feet). But this is only speculation, although feasible according to our knowledge about counting procedures used in different cultural groups (Levy-Bruhl, 1947).

It is interesting to note that the Maya Indians developed a similar system, but having 20 as a base (León-Portilla, 1986). They used different symbols to represent 20, 400 (20×20), and 8,000 ($20 \times 20 \times 20$) (Cauty, 1984).

So, reviewing the history of numerical concepts, it is found that world languages developed a base 10 (10 fingers) or 20 (10 fingers plus 10 toes) or even 5 (5 fingers) to group quantities. In some contemporary languages, for example in French, a residual 20-base can be found (e.g., in French 80 can be "four twenties"). In many contemporary languages, different words are used between 1 and 10. Between 10 and 20 the numerical systems usually become irregular, unpredictable, and idiosyncratic. From 20 ahead, new numbers are formed simply with the words "twenty plus one," "twenty plus two," and so on. Some contemporary languages still use a 5-base in counting. For instance, in the Amerindian language Tanimuca in South America, they count up to five. Between 5 and 10, numbers are "five one," "five two," and so on.

Further Developments of Arithmetical Abilities

Writing numbers appeared earlier in history than writing language. Some cultures (e.g., Incas) developed a number-representing system, but not a language-representing system (Swadesh, 1967). As mentioned, "calculus" means pebble. Pebbles, or marks, or knots, or any other element were used as a correspondence construction to record the number of elements (people, cows, fishes, houses, etc.). In summer the first number writing system has been found (about 3,000 BC) (Childe, 1936): Instead of using pebbles, fingers, or knots it was simpler just to make a mark (a stroke or a point) on the floor, or on a tree branch or a on board if you wanted to keep the record. In Egypt, India, and later in Crete, a similar system was developed: units were represented by a conventional symbol (usually a stroke) repeated several times to mean a digit between one and nine; a different symbol was used for 10 and 10-multiples.

Positional digit value is clearly disclosed in Babylonians, and about 1,000 BC the zero was introduced. Positional value and zero are also disclosed in Maya Indians (León-Portilla, 1986). Egyptians and Babylonians commonly used fractions. Small fractions (1/2, 1/3, and 1/4) are relatively simple numerical concepts, and even chimpanzees can be trained to use small fractions (Woodruff and Premack, 1981).

As mentioned previously, recognition of individual marks or elements up to 3 is easy: It represents an immediate perception readily recognizable. Beyond 3, the number of marks (strokes or dots) has to be counted and errors can be observed. Furthermore, it is rather time-consuming and cumbersome to be all the time counting marks. Noteworthy, the different digit notational systems always represent one, two, and three with strokes (or points, or any specific mark). It means, the numbers 1, 2, and 3 are written making one, two, or three strokes. But beyond that figure, digit writing may recur to other strategies. In our Arabic digit notation system "1" is a vertical line; whereas 2 and 3 were originally horizontal lines that became tied together by being handwritten. This observation may be related with the inborn ability to perceptually recognize up to three elements. Beyond three, errors become progressively more likely. Perceptually distinguishing 8 and 9 is not so easy as distinguishing between 2 and 3 strokes. The introduction of a different representation for quantities over 3 was a useful and practical simplification.

Not only the numerical system but also the measure units were developed departing from the body dimensions (fingers, hands, arm, steps, etc.). This tendency to use the human body not only to count but also as measure units is currently still observed in some contemporary measure units (e.g., foot).

In neuropsychology, some common brain activity for finger knowledge and calculation abilities can be supposed. Finger agnosia and acalculia appear as two simultaneous signs of a single clinical syndrome (Gerstmann, 1940), usually known as "Gerstmann syndrome" or "angular gyrus syndrome." For prehistorical man, finger agnosia and acalculia could have represented just the same defect.

Adding, subtracting, multiplying, and dividing were possible in the Egyptian system, but of course, following procedures quite different than those procedures we currently use. They based multiplication and division in the "duplication" and "halving" method (Childe, 1936). Interestingly, this very same procedure (duplicating and halving quantities) is also observed in illiterate people when performing arithmetical operations. So, in the Egyptian system to multiply 12 by 18, the following procedure was followed:

1	18
2	36
*4	72
*8	144
Total	216

(The number 18 is duplicated one or several times, and the amounts corresponding to 12 ($4 + 8$ in this example) are selected and summed up as follows: $72 + 144 = 216$. To divide, the inverse procedure was used. So, to divide 19 by 8 would be as follows:

1	8
*2	16
2	4
*4	2
*8	1

That is, $2 + 4 + 8$ ($2 + 1/4 + 1/8$), that is 2.375.

In brief, arithmetical abilities and number representation have been around only for some 5,000–6,000 years. Most likely, during the Stone Age only simple counting up to 3 was present and of course, "bigger" and "smaller" (magnitude judgment) concepts. Global quantification already was existing at prehuman levels. Correspondence constructions allowed increasing the amount of numbers. The most immediate correspondence construction is done with the fingers. Finger knowledge and counting represent in a certain extent the same cognitive ability, as it is still evident in some contemporary languages, as Sikuani language.

Counting, finger gnosis, and even lateral spatial knowledge may present a common historical origin. Seemingly, calculation abilities were derived from finger sequencing. Number representation and arithmetical operations are observed only for some 5,000–6,000 years. Currently, calculation abilities are rapidly evolving because of the introduction of modern technology.

Right–left discrimination (as well as the use of other spatial concepts) most likely was present in prehistorical man. Requirements of spatial abilities may have been very high, even higher, than in contemporary man (Ardila, 1993; Ardila and Ostrosky, 1984; Hours, 1982). Right–left discrimination and finger gnosis are strongly interdependent and even they can be interpreted as components of the autotopagnosia syndrome. It seems, in consequence, that there is a rationale for finding a common brain activity for finger gnosis, calculation, and right–left discrimination (and, in general, spatial knowledge mediated by language).

DEVELOPMENT OF CALCULATION ABILITIES IN CHILDREN

During child development, different stages in the acquisition of numerical knowledge are observed (Klein and Starkey, 1987). They include global quantification, recognition of small quantities, numeration, correspondence construction, counting, and arithmetics (Table 6). As mentioned, some fundamental numerical concepts can be observed at the animal level and it is not surprising to find them in small children. The initial levels of numerical knowledge are found in preschool children. The development of complex numerical concepts requires long school training. Complex arithmetical concepts depend upon a painstaking learning process, and they are not usually found in illiterate people. The different stages in the acquisition of numerical concepts are associates with the language, perceptual, and general cognitive development. Variability is normally observed, and some children can be faster in the acquisition of numerical abilities. The different stages appear in a sequential way, and the understanding of more complex concepts requires the acquisition of more basic levels. A percentage of otherwise normal children can fail in using numerical concepts normally expected at their age. The term *developmental dyscalculia* has been used to refer to this group of underperforming children.

Table 6. Different Levels of Numerical Knowledge (Adapted from Klein and Starkey, 1987)

Global quantification	What collection is bigger and smaller
Recognition small quantities	Differentiate one, two, and three elements
Enumeration	Sequencing the elements in a collection
Correspondence construction	To compare collections
Counting	A unique number name is paired with each object
One–one principle	Each object in a collection is to be paired with one and only one number name
Stable order principle	Each name is assigned to a permanent ordinal position in the list
Cardinal principle	The final number name used in a counting sequence refers to the cardinal value of the sequence
Arithmetics	Number permutability (e.g., adding, subtracting)

Numerical Abilities in Preschool Children

Global quantification or *numerosity perception* represents the most elementary quantification process. Global quantification supposes the discrimination between collections containing different number of objects (Davis et al., 1985). Global quantification simply means what set of elements is bigger and which one is smaller. Global quantification is observed at the animal level: Many animals can select the larger collection of elements when they have to choose. However, the ability to distinguish which collection is larger depends upon the number of elements in the collections. To distinguish 3 and 4 elements may be easy (4 is 25% larger than 3). To distinguish 10 and 11 elements is obviously harder (11 is only 10% larger than 10). By the same token, to distinguish 10 from 20 elements is easy (the double), but to distinguish 100 from 110 is hard (one tenth). It means, what is important is the ratio existing between the two collections of elements (so-called psychophysics Weber's fraction).

Global quantification is expressed in the language with words such as "many," "a lot," and similar terms.

For small quantities, the words *several, a few*, and similar quantity adverbs are included in the language. Quantity adverbs used in everyday speech represent global quantifiers. Quantity adverbs appear early in language history and also in child language development than in the numerical system. As mentioned previously, all known world languages use global quantification and possess words to refer to "many," "a lot." All languages oppose small quantities (one, two, a few) to "many," "a lot." As a matter of fact, "many," "much," and similar global quantifiers represent early words in language development.

Global quantification, however, does not represent yet a truly numerical process, because it does not suppose a one-to-one correspondence. *Enumeration* (sequencing the elements in a collection of elements; this process supposes the individualization of each element) represents the most elementary type truly of numerical knowledge (Klein and Starkey, 1987). Enumeration requires to distinguish the individual elements in the collection ('this, this, and this," etc.). In child language development, the most elementary distinction is between "this" and "other." "This" and "other" are also early words in child language development.

Correspondence construction constitutes a type of enumeration used to represent the number of objects in a collection and to compare collections. The amount of elements in a collection is matched with the amount of elements in an external aid (fingers, pebbles, knots, strokes, marks, dots, etc.). It implies, in consequence, a one-to-one correspondence: Each one of the elements in the collection corresponds to one finger or pebble, knot, stroke, mark, dot, or whatever. An external device can be used for making the correspondence construction. The most immediate devices are the fingers. During enumeration usually the fingers are used to point the objects.

Counting represents a sophisticated form of enumeration: a unique number name is paired with each object in a collection, and the final number name that is used stands for the cardinal value of that collection. The initial object corresponds to "one," the following to "two," and so on. Some times, the very same finger name is used as number name (i.e., the very same word is used for one and thumb, two and index finger, etc.). The collection has the amount of objects that corresponds to the last pointed object (cardinal principle). Arithmetics represents an advanced numerical system, which comprises number permutability (e.g., adding, subtracting).

Human infants are able to recognize numerosity for small quantities (usually up to 3–6 items) (Antell and Keating, 1983), but the ability to construct correspondences emerges only during the child's 2nd year (Langer, 1986). During the 2nd year the child also begins to use some number names, and usually develops the ability to correct counting up to 3. The child thus acquires the knowledge of two basic principles in counting: (1) one-to-one principle (i.e., each object in a collection is to be paired with one and only one number name) and (2) the stable order principle (each name is assigned to a permanent ordinal position in the list; the sequence of numbers is always the very same: one, two, three, etc.). At this point, however, the child does not exhibit yet a cardinal principle; that is, the final number name used in a counting sequence refers to the cardinal value of the sequence. If a collection is counted "one, two, three," it means that in that collection there are three objects (Klein and Starkey, 1987). Cardinal principle will be observed in 3-year-old children (Gelman and Meck, 1983).

At this point, the child can count small quantities, usually below 10. During this period the child is also learning how the numerical system works and memorizing the number words. Most often, the numerical system contains three different segments: (1) From 1 to 10 different words are used. (2) From 10 to 20 counting becomes idiosyncratic and quite frequently irregular. In English "11" has not any apparent relationship with "1"; "12" has an evident relation with "2" but it is a unique word number; from 13 to 19 the ending "teen" is used. In Spanish, from 11 (*once*) to 15 (*quince*) the ending *ce* is used. From 15 to 19 the word numbers are formed as "ten and six," "ten and seven" and so on, 20 (*veinte*) has not any apparent relation with 2 (*dos*). And (3) from 20 ahead the numerical system becomes regular. Word numbers are formed as "twenty and one," "twenty and two," and so on. Learning the whole numerical system usually is completed at school.

Computational strategies (e.g., adding; if a new item is included in a collection, the collection will become larger and the next cardinal number name will be given to that collection) are found in 3- to 5-year-old children, initially only for small quantities.

Development of Numerical Abilities at School

Adding and subtracting numerical quantities and the use of computational principles are observed during in first–second-grade children, but they only become able to manipulate the principles of multiplying and dividing after a long and painstaking training period, usually during third–fifth school grade.

Understanding that subtracting is the inverse operation of adding is usually acquired about 5–6 years. At this age the child begins to use three different procedures for performing additions and subtractions: (1) counting using the fingers, (2) counting aloud not using the fingers, and (3) memorizing additions and subtractions for small quantities (one plus one is two, two plus two is four, two minus one is one, etc.). The last strategy becomes progressively stronger when advancing age and schooling. Nonetheless, children continue using the finger for adding and subtracting larger quantities. From the age of 10 until about 13 years, counting using the fingers progressively disappears, but counting aloud, and performing arithmetical operations aloud, remains. Automatic memory not only for additions and subtractions but also for multiplications (multiplication tables) and divisions becomes progressively more important (Grafman, 1988; Siegler, 1987). As a matter of fact, adding and subtracting one digit quantities (e.g., $7 + 5 = 12$; $4 + 5 = 9$; $8 - 5 = 3$; etc.) represents a type of numerical rote learning, similar to the multiplication tables. Interesting to note, the performance of arithmetical operations aloud may remain during adulthood, even in highly educated people.

It should be emphasized that there is a significant variability in the specific strategies used by different children at the same age. Furthermore, the very same child can recur to different strategies when solving different arithmetical problems. In some situation, for instance, the child can recur to the fingers, whereas in a different operation, he may not require using the fingers. Or, the child can be able to use some multiplication tables whereas failing with others.

About the age 8–9 usually the children learn to multiply. This requires the memorization of the multiplication tables. The errors most frequently found when learning the multiplication tables are those answers that could be correct for other number within the same series (e.g., $4 \times 5 = 16$). These errors may be the result of some interference. They can be observed in children at any age, and even they are sometimes found in normal adults (Graham, 1987).

Development of abstract reasoning and increase in working memory span contribute to the use of mathematical algorithms (i.e., the set of rules used for solving arithmetical problems following a minimal number of steps). The development of algorithms begins when learning the basic arithmetical operations. Progressively, they become more automatic, representing basic strategies for solving arithmetical problems. Development of abstract thinking allows the use of magnitudes applied to different systems (use of the numerical system in measuring time, temperature, etc.) and the understanding of quantities expressed in a symbolic way.

DEVELOPMENTAL DYSCALCULIA

The term *developmental dyscalculia* refers to a cognitive disorder of childhood, impairing the normal acquisition of arithmetical skills (American Psychiatric Association, 1987). Frequently, dyscalculia has been used as a general term encompassing all aspects of arithmetical difficulty (Shalev et al., 1988). The term *DD* has been changed to Mathematics Disorder in the *DSM-IV* (American Psychiatric Association, 1994). However in the neuropsychology literature the term *DD* remains. It is estimated that approximately 6% of school-age children in the United States suffer from this disorder (Grafman, 1988; Gross-Tsur et al., 1996). The prevalence of the disorder is difficult to establish because it is frequently found in combination with other developmental disorders (American Psychiatric Association, 1994). Gross-Tsur et al. (1996) observed that 17% of the children with DD were diagnosed with dyslexia and 25% had ADHD-like symptoms. It is estimated that 1% of the school-age children have DD alone (American Psychiatric Association, 1994). DD is a disorder frequently encountered in children with epilepsy (Seidenberg et al., 1986) and in girls with sex chromosome abnormalities such as Turner's syndrome (Gross-Tsur et al., 1996).

Gender distribution has been controversial. Sometimes it is assumed that it is more frequent in boys, whereas some authors consider that DD tends to affect both sexes equally. Equal ratios between the sexes for arithmetical difficulties have been reported by Lewis et al. (1994) and Gross-Tsur et al. (1996). In Gross-Tsur et al., the ratio of girls to boys was 11:10.

The origin of DD has not been established. Some researchers considered DD to be a genetically determined brain-based disorder (Rourke, 1989). However some have hypothesized that the child's environment and social context are causal for mathematical disorders (Fergusson et al., 1990). In acalculia (acquired dyscalculia), contrary to DD, the mathematical disorder is consequence of a well-known cerebral lesion. The cognitive processes dysfunction that underlie the two defects could, however, be the same ones, because the types of errors that people present with either of the two-dyscalculia types, are similar.

Neuropsychological Characteristics

Children with DD can fail in a whole array of numerical tasks including performing arithmetical operation, solving arithmetical problems, and using numerical reasoning. According to Strang and Rourke (1985), the errors found in children with dyscalculia can be classified

Table 7. Errors Most Frequently Found in Children with DD (Adapted from Strang and Rourke, 1985)

Error	Characteristics
Spatial	Difficulties in placing numbers in columns, following appropriate directionality of the procedure, v. gr., to subtract the substrand from the minuend
Visual	Difficulties in reading arithmetic signs, forgetting the points of the units of thousand, etc.
Procedural	Omission or addition of a step of the arithmetical procedure, and application of a learned rule for a procedure to a different one, v. gr., 75 + 8 = 163, an operation in which the multiplication rule is applied in the sum
Graphomotor	Difficulty in forming the appropriate numbers.
Judgment	Errors that imply impossible results, such as one in which the result of subtracting is bigger than the numbers being subtracted
Memory	Problems in the recall of multiplication tables or arithmetical procedures
Perseveration	Difficulty in changing from one task to another one, repetition of the same number

into seven categories: (1) errors in spatial organization of quantities, (2) errors in visual attention, (3) arithmetical procedural errors, (4) graphic motor errors when writing quantities, (5) numerical judgment and reasoning errors, (6) memory errors for quantities, and (7) preserveration in solving arithmetical operations and numerical problems. Table 7 describes the characteristics of the most frequent types of errors found in children with DD.

Kosc (1970) described six types of difficulties observed in DD: (1) problems in verbal organization of numbers and mathematical procedures, (2) difficulties in the management of mathematical symbols or objects, (3) errors in reading numbers, (4) errors in writing numbers, (5) difficulties in the understanding of mathematical ideas, and (6) in the carrying over when performing arithmetical operations. Students with dyscalculia may present a cluster of problems in their ability to perform mathematical tasks.

It is unclear what arithmetical function is impaired in children with DD. Shalev et al. (1988) studied the arithmetical errors found in 11 children with dyscalculia and 10 matched control children. No differences were observed in number comprehension (matching numbers to quantities, appreciation of relative quantity, numerical rules, and serial order) and number production (counting, reading, and writing numbers) scores. But significant differences emerged in the performance of fact retrieval, addition, subtraction, multiplication, and division scores. The group with DD had difficulty in fact retrieval but could show that they knew how to calculate by using finger counting and other appropriate strategies. Cohen (1971) has proposed that short-memory difficulties explain the incompetence in arithmetics of children with DD. In fact, the inability to carry and recall number tables may be the result of memory deficits (Shalev et al., 1988). Davis et al. (1992) suggest a sequential processing deficit as the underlining deficit of DD.

In addition to memory deficits, children with DD present attentional difficulties. The association between attentional problems and dyscalculia has been well documented. Badian (1983) described attentional–sequential problems in 42% of the children with DD. More recently, Shalev et al. (1995a,b) demonstrated the presence of attention deficit disorder symptoms in 32% of the dyscalculia sample studied. Equally children with attention deficit syndrome with or without hyperactivity make mathematical errors secondary to impulsiveness and inattention (Sokol et al., 1994).

Rosenberger (1989) found that visual–perceptual and attention disorders were evident in children who had specific difficulties in mathematics. Strang and Rourke (1985) not only corroborated the presence of significant difficulties in visual–perceptual organization in children with dyscalculia, but also described difficulties in other neuropsychological tasks. These children manifested difficulties in the tactile analysis of objects, particularly with the left hand, as well as impairments in the interpretation of facial and emotional expressions (Rourke, 1987). Children with dyscalculia also present an inadequate prosody in verbal language (Rourke, 1988), and difficulties in the interpretation of nonverbal events (Loveland et al., 1990). These neuropsychological findings have suggested the presence of a functional immaturity of the right hemisphere as a structural fact underlying dyscalculia.

Hernadek and Rourke (1994) described the disorder of nonverbal learning associated with a dysfunction of the right hemisphere and characterized by visuospatial difficulties, visuomotor coordination and reasoning problems, defects in concept formation and in mathematical skills. This disorder has also been known as developmental right-hemisphere syndrome (Gross-Tsur et al., 1995; Weintraub and Mesulam, 1983). The children with nonverbal learning disorder also present defects in the recognition of faces and of emotional expressions and poor adaptation to novel social situations. In accordance with Hernadek and Rourke, children with dyscalculia associated with the disorder of nonverbal learning can be distinguished clearly from children with dyscalculia associated with reading disorder. In the later group, problems are manifested in the performance of language recognition tasks whereas performance in tactile and visual perception tasks is well preserved. Gross-Tsur et al. (1995) describe the clinical characteristics of 20 children with the nonverbal learning disability disorder. Dyscalculia was the most frequent encountered scholastic problem.

Although an evident association exists between dyscalculia and dyslexia, different underlying cognitive explanations have been suggested for the two disorders. Rosenberger (1989) found that those children with difficulties in mathematics present more evident visuospatial and attentional dysfunctions. These children obtained lower scores in visuomotor tests, for example, in the Bender Visuomotor Gestalt Test, and in the digit–symbol subtest of Wechsler Intelligence Scale, than did a group of children with specific difficulties in reading. The authors postulate that visuomotor and visuospatial execution and organization defects can distinguish children with dyscalculia from children with dyslexia. Not all researchers, however, have supported this distinction between children with DD and children with dyslexia. Rasanen and Ahonen (1995) have suggested functional communality between dyslexia and dyscalculia. They found that reading accuracy and reading speed correlate with the number of errors in arithmetical operations, particularly in multiplication. The authors concluded that difficulties in visuoverbal representations might explain both reading and mathematical disorders.

Some authors consider that DD does not appear like an isolated manifestation of cerebral dysfunction, but as part of a syndrome called Gerstmann syndrome. This syndrome is composed of the tetrad of dyscalculia, digital agnosia, dysgraphia, and left–right disorientation. This syndrome is found in skillful adults as a result of lesions in the left parietal lobe, but has been described in children with specific learning disabilities, and has been given the name of developmental Gerstmann syndrome (DGS). Children with DGS present intact language skills, and reading is usually at the normal grade level (PeBenito et al., 1988). However, Croxen and Lytton (1971) found that more children who are slow readers had difficulties with finger recognition and right–left discrimination.

Gerstmann syndrome may occur in children with brain damage or in children who are apparently normal. Gerstmann syndrome in brain damage children is usually associated with multiple symptoms, such as hyperactivity, short attention span, lower intellectual performance, and poor reading. This group accounts for most of the reported Gerstmann syndrome in children (Kinsbourne and Warrington, 1963; PeBenito et al., 1988; Pirozzolo and Payner, 1978). The nature of the neurological disorder in these cases is usually diffuse and bilateral. DGS, on the other hand, seems to be relatively common among individuals with fragile X syndrome (Grigsby et al., 1987). Few cases of DGS in otherwise normal children, however, have been described (Benson and Geschwind, 1970; PeBenito, 1987), and the syndrome may represent a delayed cerebral maturation (PeBenito et al., 1988).

Besides dyslexic children, those children with emotional problems can also fail easily in tasks that demand high attention levels such as in the case of mathematics. Problems in the emotional domain are recurrently mentioned in the context of arithmetical disabilities (Shalev et al., 1995a,b). Withdrawal and social problems have been noted in children with DD (Rourke, 1989).

In summary, children with DD can fail in a whole array of numerical and arithmetical tasks. Errors found in these children are of spatial organization, visual attention, procedural, motor, judgment, reasoning, and memory. DD is commonly associated with reading dysfunctions, attention disorders, and emotional difficulties. The involvement of one or both cerebral hemispheres in dyscalculia is still controversial.

Subtypes

DD is not a uniformed disorder. Children with DD can manifest an array of different numerical errors, and there are variations in terms of the type of dyscalculia and the severity of the disorder (Grafman, 1988). Different subtypes of DD have been proposed.

Kosc (1970) describes six dyscalculia subtypes characterized by difficulties in (1) the verbalization of terms and mathematical relationships, (2) the handling of symbols/mathematical objects, (3) the reading of numbers, (4) the writing of numbers, (5) the comprehension of mathematical ideas, and (6) the ability for "carry" in arithmetical operations.

Badian (1983) finds in children with DD a high frequency of spatial numerical difficulties, developmental anarithmetia (primary difficulties in calculating), and attentional–sequential defects, but very few cases of dyslexia and dysgraphia for numbers. Geary (1993) intended to classify dyscalculia in three groups based in three types of errors: (1) visuospatial, (2) semantic memory, and (3) procedural. Two types of developmental dysacalculias have been recognized by Rourke (1993): (1) dyscalculia associated with language problems (dyslexia), defects in the understanding of instructions and verbal problems, and reduction in the capacity of verbal memory; and (2) dyscalculia associated to spatial–temporal difficulties, with sequence problems and reversion of numbers (Spreen et al., 1995). The dichotomy between fundamental visuospatial dyscalculia indicative of right hemispheric dysfunction and another fundamental dyslexic dyscalculia, suggestive of left hemispheric dysfunction, has not been corroborated by other authors (e.g., Shalev et al., 1995a,b; Sokol et al., 1994). It is frequent to find that those children with difficulties in reading and writing also present defects in learning arithmetics. For the child with learning difficulties, the achievement of a mathematical problem becomes a task more difficult than reading, because the change of a single digit alters the result of the operation completely. Additionally, the achievement of any mathematical problem, even the simplest, demands the pursuit of certain systematic steps that can be a high-level difficult task for a child with dyslexia.

The prevalence of some right hemisphere dysfunction in DD has been recently reanalyzed. Shalev et al. (1995a,b) studied a group of children with DD and neuropsychological profiles suggestive of right or left hemispheric dysfunction. They analyzed the types of errors in each group, and correlated the scores of hemispheric lateralization with the results in tests of mathematics. Contrary to that proposed by Rourke et al., the children with supposedly dysfunction of the left hemisphere presented a significantly larger number of visuospatial errors and more notorious difficulty for the achievement of arithmetical operations that the group with right neuropsychological dysfunction. The authors did not find differences in the profile of errors of each group, neither a correlation between the hemispheric dysfunction and the type of errors in mathematics tests. Despite the fact that Shalev et al.'s data indicates that arithmetical impairments are more severe in cases of DD associated with left hemisphere dysfunction, their conclusion is that there is a participation of both cerebral hemispheres the arithmetical processes (Shalve et al., 1995b).

PROCESSES INVOLVED IN ARITHMETICAL OPERATIONS

Numerical handling represents a language that involves a system of symbols. This system of symbols implicated in calculations can be divided into two groups: (1) a logographic system including Arabic numbers from 0 to 9, and (2) a phonographic system that provides the verbal name to numbers; for example, "one," "nine." The performance of a given arithmetical operation begins with the numerical recognition, which depends on a verbal process and perceptual recognition: number–symbol or symbol–number. Each number provides two types of information. On the one hand the base group to which the number belongs (units, dozens, hundreds), and on the other, the ordinal position of the number inside the base. Thus, the number 5 belongs to the units and occupies the fifth place inside them. The successful performance of an arithmetical operation demands visuospatial discrimination ability to organize the numbers in columns, to arrange the appropriate spaces among numbers, and to begin the operation from right to left. The working memory, or operative memory, associated with sustained attention, obviously plays a central role in the performance of any arithmetical operation. The algorithmic plan of action that unchains the numerical symbol in a particular arithmetical operation is evoked, or recovered from previous learnings whose engrams are found in long-term memory (Boller and Grafman, 1985). The stages of the necessary cognitive process for the achievement of an arithmetical operation can be observed in a simple example: $34 + 26 = 60$.

When this operation is presented to a person, in the first place they have to perceive the spatial organization of the quantities, the relationship amongst themselves, understand the meaning of the "plus" (+) symbol, recognize the numerical symbols, and to know the steps that should be followed to carry out adding appropriately. The sum of the numbers 6 and 4 is automated, and the only answer that needs attention is the number 10. If the operation is not automated, the individual can use the controlled process of count. The number 10 should be maintained in memory whereas the zero is placed in the right column, and the unit 1 is taken and conserved in the short-term memory (mentally), or written on the following column. The same process is continued for the following column. The information that is stored in the long-term memory seems to correspond to two types: (1) syntactic information, that is, the knowledge of the rules of the numerical procedures, and (2) semantic information (i.e., the comprehension of the meaning of the procedures implied in the solution of particular problems). When the problems are solved using automatic codes, semantic reasoning is not required.

In performing an arithmetical operation, two different cognitive systems should be distinguished: the processor of numbers and the operating system of calculation. Additionally, during the processing of numbers it is necessary to distinguish between the comprehension and the production of numbers, and in each one of these subprocesses, in turn, it is necessary to consider the presence of a double code (verbal and numerical) and of a double analysis (lexical and syntactic). To process quantities in Arabic numbers, for example, "823," or numbers in verbal codes, "eight hundred twenty-three," demands a lexical analysis (comprehension and production of the individual elements) and a syntactic analysis (processing of the relationships among the elements).

McCloskey et al. (1985) and McCloskey and Caramazza (1987) focused on a detailed analysis of the calculation system. They proposed a cognitive model of the numerical processing on the basis of three basic

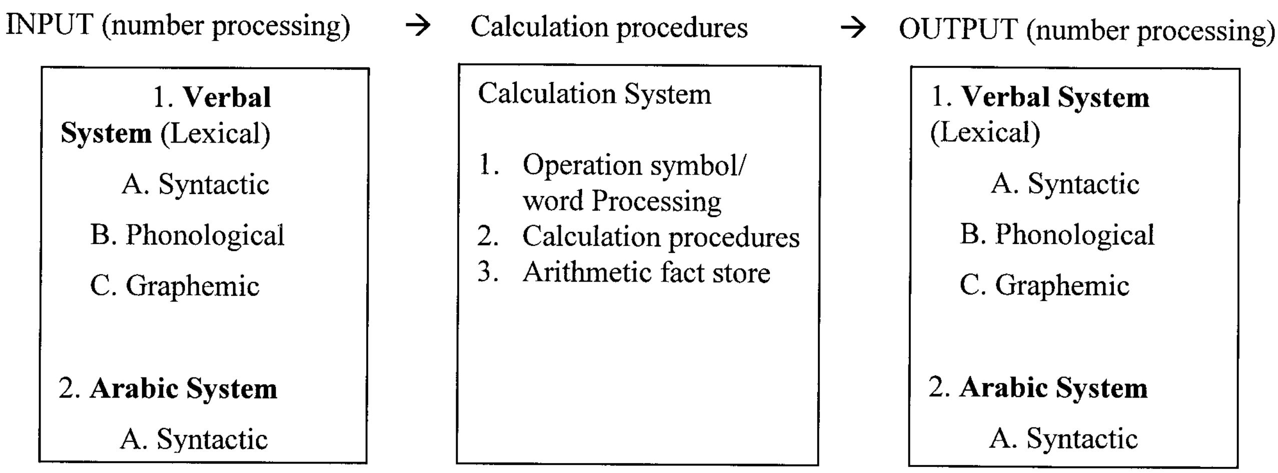

Fig. 1. Schematic representation of the number comprehension, number production, and calculation system (adapted fromCaramazza and McCloskey, 1987; Levin et al., 1993).

abilities: understanding of a number, production of a number, and processing of the mathematical procedures. The category of understanding of the number (input) includes the understanding of quantities, of the symbolic character of those quantities (lexical processing), and of the order of the digits (syntactic processing). Inside the numerical production (output) count is found, and the reading and the writing of numbers. The two subsystems, number comprehension and number production, include the verbal system (written or spoken words, e.g., "thirty") and the Arabic system (element of a number, e.g., 30). These subsystems are illustrated in Fig. 1. The verbal system is the lexical processing mechanism. Three are the mechanisms included inside the calculation system. The first one is the recognition of the arithmetical symbols, the second and the third ones are the understanding, memorization, and execution of the arithmetical facts (i.e., table facts, such as 2×3), and arithmetical procedures (i.e., when performing an addition, start at the right most column, sum the right most digits, write down the ones, carry the tens, etc.) (Hittmair-Delazer et al., 1995; Sokol et al., 1994). Arithmetical facts are retrieved from a semantic network system independently from calculation procedures. The calculation procedures are the sequence of steps necessary to perform multidigit operations. Although both components are learned, their functional independence has been demonstrated by several authors (McCloskey et al., 1985, 1991a,b; Sokol et al., 1994; Sokol et al., 1991; Warrington, 1982). Hittmair-Delazer et al. (1995) have pointed out that conceptual knowledge plays a crucial role in arithmetical processing, frequently neglected in the neuropsychology models of calculation. Hittmair et al. refer to conceptual knowledge as the understanding and use of arithmetical principles (i.e., principle of communality). They demonstrated, in a case study design, that conceptual knowledge can be dissociated from arithmetical fact and procedures. They found impaired fact retrieval with intact processing of arithmetical principles and problems. This selective vulnerability of arithmetical facts had been demonstrated before by Warrington (1982) and McCloskey et al. (1985). Arithmetical facts represent a separate subset of the semantic memory and may also dissociate from other mathematical and numerical knowledge (Hittmair-Delazer et al., 1995). Dissociations, according to the specific arithmetical operations, have also been reported. For example, patients with severe addition difficulties and selective preservation of subtraction have been reported (Dagenbach and McCloskey, 1992; McNeil and Warrington, 1994).

Using a cognitive neuropsychology perspective, McCloskey et al. (1985, 1986) have described different forms of acquired acalculia. They distinguished between disorders of number processing (reading, writing, production, comprehension, or repetition of numbers) and disorders of calculation (number facts, knowledge of procedures).

Caramazza and McCloskey's model has been applied to the acquisition of mathematical abilities in preschool children and to the study of DD (Sokol et al., 1994). An specific impairment in number processing has been described in a child with DD (Temple, 1989). This child had a selective impairment in the lexical processes of reading Arabic numbers with intact syntactic processes. Temple (1991) also reports two cases of DD in which an accurate number processing skill had been developed. However, one of the cases showed a selective difficulty with the procedures of calculation, and the other one showed a selective disorder in mastery of those arithmetical facts that comprised the multiplication tables. Sokol et al. (1994) studied the range of functional dissociations predicted by McCloskey et al.'s cognitive model in 20 students with DD. Their results supported the usefulness of the model to understand DD.

The modules delineated by McCloskey et al. (1985, 1986) may be also separable during the normal development of calculation abilities. Children are considered to have considerable conceptual knowledge before they acquire automaticity in number processing and calculation processes (Hittmair-Delazer et al., 1995). For example, children know counting principles before they count correctly (Gelman and Meck, 1983), and they can judge arithmetical transformations correctly before they can perform them (Starkey, 1992).

Cerebral Mechanisms

The appropriate solution of a numerical problem demands verbal, spatial, and conceptual abilities that very probably require of the active participation of numerous cerebral structures. It has been suggested, however, that the central neural mechanisms implied in the recognition of numbers seem to be different to those that participate in the solution of arithmetical problems as such mechanisms can be altered differentially in cases of cerebral focal damage. A patient can present difficulties in the recognition of numbers with appropriate conservation of the ability to carry out arithmetical operations. Additionally, the handling of numbers can be dissociated alternating independently from the numerical production and conserving their understanding (Benson and Denckla, 1969). McCloskey and Caramazza (1987) describe, as a consequence of localized cerebral damage, a dissociation between the capacity to understand and the capacity to produce numbers. Evidence of selective alterations exists for the processing of Arabic numbers and the processing of lexical numbers (Ardila and Rosselli, 1990; Deloche and Seron, 1987; Rosselli and Ardila, 1989). Similar dissociations have been described between the lexical and syntactic processing of verbal numbers and that of Arabic numbers (Caramazza and McCloskey, 1987). An example of a lexical mathematics error would be to represent two hundred twenty-one as "215," whereas the representation of five thousand six hundred as "50006" would be an error of mathematics syntax.

Moreover, Ferro and Botelho (1980) and Caramazza and McCloskey (1987) have observed patients with a defect exclusively in the analysis of mathematics signs. Warrington (1982) describes dissociations between the ability to develop simple arithmetical operations and arithmetical problems. Hittmair-Delazer et al. (1995) describe a patient that presented great difficulty in carrying out arithmetical operations despite having the appropriate knowledge of arithmetical principles. These dissociations among diverse mathematical components suggest a functional independence of each one of them.

The functional independence of the diverse mathematical components has been corroborated for the analysis of the errors of numerical count in patients with cerebral damage. Lesions in diverse parts of the cerebral cortex originate different errors (Rosselli and Ardila, 1989). Some patients can present difficulties in "carrying" quantities, others in the placement of the appropriate numbers, and in other patients the difficulty to execute arithmetical operations one can observe in the combination of the two procedures.

The studies of the alterations of number count in patient with cerebral damage have demonstrated that lesions in either of the two cerebral hemispheres can produce acalculia, although of different characteristics. The lesions in the areas of language in the left cerebral hemisphere produce alterations in the comprehension and in the production of numbers, and therefore in carrying out the arithmetical operations. On the contrary, the lesions in the right cerebral hemisphere cause alterations in the spatial organization quantities and in the comprehension and achievement of abstract problems (Ardila and Rosselli, 1990; Rosselli and Ardila, 1989). The experimental studies with dichotic audition and the tachistoscopic presentation of visual information support the participation of the two cerebral hemispheres in carrying out arithmetical problems (Grafman, 1988; Holender and Peereman, 1987).

CALCULATION ABILITIES IN NORMAL POPULATIONS

Few neuropsychological studies have approached the question of calculation abilities in the general population. Intuitive observation points to a significant dispersion of arithmetical abilities in normal people. Usually, however, it is assumed that any normal person should be able to tell one-digit multiplication tables, to use the four basic arithmetical operations, to solve simple arithmetical problems, to memorize seven digits after a single presentation, and to use diverse numerical information in the everyday life. Nonetheless, normative studies are scarce.

Normative Studies

Deloche et al. (1994) developed a standardized testing battery for the evaluation of brain-damaged adults in the area of calculation and number processing. With the purpose of obtaining some norms, the battery was administered to 180 participants stratified by education (up to 9 years of formal education, 10 or 11 years, and more than 11 years), age (20–39 years, 40–59 years, and 60–69 years) and gender. This battery, named as EC301, includes three notational systems for numbers: Arabic digits, written verbal forms, and spoken verbal forms.

Analysis of error rates indicated the effect of some demographic factors, principally, education (in counting, transcoding, written verbal numbers, magnitude comparisons, and arithmetical operations subtests); incidentally, gender (in digit numbers, and total mental calculations scores). No age effect is mentioned in the age range included in this study (20–69 years).

In the normative study, 88% of the participants presented at least one error in the EC301 test battery. The easiest subtest turned out to be "reading and writing arithmetical signs"; this subtest was failed by only 1% of the total participant sample. The hardest subtest was "written multiplication"; in this subtest 36% of the partcipants presented at least one error. So, errors were common in normal participants, but level of difficulty was variable.

The educational effect deserves some comments. The lowest educational group was "equal or below 9 years," but observing the mean educational level and the standard deviation, seemingly, all the participants in this group had 8–9 years of education (the mean education for this groups was 8.40, and the standard deviation was 0.50). So, as a matter of fact, the whole sample had at least 8 years of education. Nonetheless, a significant educational effect was observed in several tasks. It can be conjectured that if they had included participants with an even lower educational level, the education effect would be stronger and may have appeared in a larger amount of calculation subtests.

The gender effect was demonstrated only in Task 6 (Mental calculation) in the Arabic digit condition and in the total score. Males performed better than females. Surprisingly, no age effect was found in any of the subtests. It can be assumed that calculation abilities, at least for simple tasks, remain relatively stable up to the 1960s. A decline in calculation abilities would be evident only after the age of 70 years.

Ardila et al. (1998) analyzed the calculation abilities in a normal population sample composed exclusively of young people with a high level of education. A comprehensive neuropsychological test battery was assembled and individually administered to a 300-participant sample, aged 17–25 years. All of them were right-handed male university students. The battery included some basic psychological and neuropsychological tests directed to assess not only calculation abilities, but also language, memory, perceptual abilities, concept formation, and praxis abilities. Two arithmetical tests were used: (1) mental arithmetical operations (two additions, two subtractions, two multiplications, and two divisions). Maximum possible score was 8 points (1 point for each correct answer). And (2) arithmetical problems. Sixteen arithmetical problems were orally presented. The participants were allowed to use pencil and paper if so wanted. Maximum possible score was 16 points (1 point for each correct answer). In addition, two WAIS scores were analyzed: Arithmetic and Digits subtests. Thus, in total four calculation ability tests were considered. Means, standard deviations, and ranges are presented in Table 8. Noteworthy, a very significant dispersion was observed in the scores. Some university students were unable to solve mentally even a single addition. Other participants had a virtually perfect performance.

Table 8. Performance of 300 Normal University Students in Some Calculation Tests (Adapted from Ardila et al., 1998a)

Test	*M*	*SD*	Range
WAIS: Arithmetic (scaled score)	11.8	7.7	3–8
Digits (scaled score)	11.6	2.2	5–17
Mental arithmetics (maximum score 8)	5.3	1.9	0–8
Arithmetical problems (maximum score 16)	9.5	3.2	1–16

Ostrosky et al. (1997) selected 800 normal population participants in five different states of the Mexican Republic. The obtained sample included 665 participants (83.12%) from urban areas, and 135 participants (16.88%) from rural areas. Ages ranged from 16 to 85 years (mean age = 47.77; *SD* = 20.14). Education ranged from 0 to 24 years (mean education = 6.8; *SD* = 6.1). Fifty-two percent of the sample was women. Ninety-five percent of the sample was right-handed. Four age groups were formed: (1) 16–30 years, (2) 31–50 years, (3) 51–65 years, and (4) 66–85 years. In addition, each age group was divided into four different educational levels: (1) illiterates (0 years of education), (2) 1–4 years of education; (3) 5–9 years of education, and (4) 10–24 years of formal education. The NEUROPSI neuropsychological test battery (Ostrosky et al., 1997) was individually administered. It includes three items related with calculation abilities: (1) Digits backward, up to six digits (maximum score = 6 points), (2) serial 3 subtraction from 20 to 5 (maximum score = 5), and (3) Calculation Abilities subtest. In this subtest, three very simple arithmetical problems to

Table 9. Means and Standard Deviations Found in the Different NEUROPSI Neuropsychological Tests (n 800) (Adapted from Ostrosky et al., 1997)

Test	16–30 years	31–50 years	51–65 years	66–85 years	Maximum score
Illiterates					
Digits backward	2.2 (1.1)	2.8 (1.1)	2.9 (1.0)	2.7 (0.9)	6
20 minus 3	2.2 (1.6)	3.8 (1.5)	3.1 (1.8)	2.9 (1.8)	5
Calculation abilities	1.0 (1.1)	1.4 (1.1)	1.6 (1.1)	0.9 (1.1)	3
One to four years of education					
Digits backward	2.6 (1.0)	2.7 (0.7)	3.0 (1.0)	2.8 (0.8)	6
20 minus 3	3.5 (1.6)	3.6 (1.4)	4.3 (1.3)	4.4 (0.9)	5
Calculation abilities	1.3 (1.1)	1.5 (1.1)	1.6 (1.1)	2.0 (0.9)	3
Five to nine years of education					
Digits backward	3.4 (0.7)	3.4 (1.2)	3.6 (0.8)	3.4 (0.8)	6
20 minus 3	4.3 (1.3)	4.6 (0.6)	4.4 (0.9)	4.6 (0.2)	5
Calculation abilities	2.3 (0.8)	2.4 (0.6)	2.5 (0.6)	2.3 (0.9)	3
Ten to 24 years of education					
Digits backward	4.3 (0.9)	4.4 (0.9)	4.0 (0.9)	3.9 (1.0)	6
20 minus 3	4.7 (0.8)	4.7 (0.7)	4.9 (0.4)	4.8 (0.6)	5
Calculation abilities	2.6 (0.6)	2.6 (0.7)	2.7 (0.6)	2.5 (0.8)	3

be mentally solved are presented (“How much is 13 + 15”; “John had 12 pesos, received 9 and spent 4. How much does he have”; and “How many oranges are there in two and half dozens”) (maximum score = 3). Normative results are presented in Table 9. It is observed that in general scores increase with educational level and decrease with age. It is interesting to note that the highest scores, particularly in the illiterate group, are obtained not in the youngest group (16–30 years) but in the second age group (31–50 years). Despite representing a very easy calculation test, even some people with relatively high education failed some points. This observation emphasizes the significant dispersion in calculation abilities found in normal populations.

Educational effect presented a very robust effect (Table 10). In the highest educational group, scores in the three subtests are about the double those in the illiterate group. Age effect, however, was notoriously weaker and was observed only in the second subtest (20 minus 3). Most important, even though people up to 85-year-old were included in this study, the age effect in this subtest cannot be interpreted as a score decrease associated with age. In all the educational groups, performance in the oldest participants was higher than that in the youngest participants. The age effect simply means that the performance in this subtest was associated with the participant’s age. Scores tended to increase up to the 50s, and further remained stable or presented a very mild decrease.

Table 10. *F* Values for Education and Age Variables, and Interactions Between Education and Age[a]

Test	Education (E)	Age (A)	E × A
Attention: digits backward	108.00***	1.85	2.09
20 minus 3	63.46***	6.27***	3.37***
Calculation abilities	95.57***	3.21	2.60

[a]Levels of significance are pointed out (adapted from Ostrosky et al., 1997).
***$p < .0001$.

In conclusion, (1) calculation abilities present a very significant dispersion in the general normal population. (2) Even very simple arithmetical tasks are failed by a percentage of the normal population, including people with a high educational level. (3) Educational effect represents a robust effect in calculation tests. Lowest performance is observed in illiterate people. Interestingly, difficulties are observed not only in school-trained arithmetical abilities (e.g., arithmetical operations), but also in nondirectly school-trained numerical abilities, such as repeating digits backward. (4) Age effect is notoriously weaker than educational effect. In the range 20–69 no age effect is readily demonstrated (Deloche et al., 1994). Including people up to 85 and illiterate people, the age effect was disclosed only in some numerical tests, such as mentally subtracting 3 from 20. But the age effect cannot be interpreted as a score decrease associated with age, but rather as a tendency to score increase up to the 50s. Further, scores may remain stable or slightly decrease. And (5) there is a gender effect in calculation abilities demonstrated in tests such as mental calculation and solving simple numerical problems. Curiously, this gender effect is stronger in people with high educational level and weaker in illiterates.

Cultural and Educational Variables in Calculation Abilities

Some studies have approached the analysis of calculation abilities in different cultural contexts and in people with different educational backgrounds (e.g., Cauty, 1984; Grafman and Boller, 1987; Levy-Bruhl, 1947). Rosin (1973) analyzed the way in which illiterates perform arithmetical tasks. It was observed that calculation was laborious and strongly relied in memorizing each step. For counting, fingers were used with large numbers requiring representing the hands. Often, doubling and halving the figures were used for arithmetical operations (as observed in the initial Egyptian division and multiplication systems). For actual trading and marketing, the operations could be initially performed visually using physical entities, and the results retained in memory.

Posner (1982) analyzed the development of mathematical concepts in West African children aged 5–10. A mild effect of experiential factors on the ability to judge the magnitude of numerical quantities was observed; counting was noted in all children, usually relying on size cues for small qualities. To perform even simple arithmetical operations and to solve numerical problems was particularly difficult.

Casual observation of illiterates discloses that they can use simple numerical concepts and they easily handle money in daily activities (at least in a country such as Colombia, where bills of different value have different colors, albeit, not different sizes). Illiterates readily recognize the "bigger" and "smaller" bill, and can perform simple computations (e.g., a 5000-peso bill is equivalent to two 2000-peso bills plus one 1000-peso bill). However, to perform subtractions is particularly painstaking, and illiterates easily get confused (e.g., when shopping). They usually cannot multiply or divide, excepting by 10 (e.g., 3, 30, 300, etc.), and two, doubling and halving figures (e.g., 200, 100, 50, etc.). This frequent ability to multiply and divide by 10 and 2 is used to perform simple arithmetical operations. Illiterates also use a very important amount of everyday numerical facts: dates (e.g., "today is April 5, 1999"), time (e.g., "I work eight hours a day: from 8 AM to 4 PM"; "I am 52 years old"), weight (e.g., "the cow weights 350 kg"), distance measures (e.g., "from my house to the park there are five blocks"), and so on. Illiterates can also use simple fractions (e.g., half, quarter, tenth). In brief, illiterates can develop some calculation abilities (i.e., counting, magnitude estimation, simple adding and subtracting). More complex arithmetical skills evidently benefit and depend on schooling. Noteworthy for illiterate people it is notoriously easier to perform concrete mathematical operations than abstract arithmetical operations. In other words, for the illiterate person it is notoriously easier to solve the operation "If you go to the market and initially buy 12 tomatoes and place them in a bag. Later on you decide to buy 15 additional tomatoes. How many tomatoes will you have in the bag?" than the operation: "How much is 12 + plus 15?"

Grafman and Boller (1987) proposed that some arithmetical skills appear genetically linked (e.g., equivalence or certain counting skills), and some are educational linked (e.g., arithmetical calculation and the "tool" used to calculate: fingers, abacus, calculator, computer, or the brain). It is reasonable to expect that some basic numerical strategies will be found in different cultural groups. The best example is the use of fingers in counting.

Gender Differences in Calculation Abilities

Gender differences in calculation abilities have been recognized since long time ago (see Halperin, 1992). It is usually accepted that men outperform females not only in calculation tasks, but also in those tests requiring spatial manipulation. On the quantitative portion of the Scholastic Aptitude Test (SAT-M) there is a difference of about 50 points between males and females (National Education Association, 1989). SAT is a highly standardized test, which is administered nationally to college-bound high school seniors in the United States.

To account for these gender differences, however, has been quite polemic (McGlone, 1980). As mentioned previously, gender differences may be evident even in simple arithmetical operations and in solving arithmetical problems. Gender differences, nonetheless, are not found in all mathematical tests. Stones et al. (1982) analyzed gender differences at 10 different colleges. Ten different mathematical ability tests were administered. Gender differences were found in some individual tests. Females scored significantly higher on tests of mathematical sentences and mathematical reasoning, perhaps reflecting the use of verbal strategies in solving these problems. Males scored significantly higher than females in geometry, measurement, probability, and statistics, perhaps reflecting the use of visual–spatial strategies in these areas. Thus, the gender effect is not a homogenous effect, but varies according to the specific calculation tasks. Sometimes, the inverse pattern (females outperforming males) can be observed.

Gender differences are observed not only in normal but also in special populations. DD may be more frequent in boys than in girls. Gender differences are also found in mathematically gifted children (Bensbow, 1988). There is a significantly higher percentage of males than females in mathematically gifted

Table 11. *F* Values and Level of Significance for the Gender Differences in the Four Educational Groups Found in the Three Numerical Items of the NEUROPSI Neuropsychological Test Battery ($n = 800$) (Adapted from Ostrosky et al., 1997)

	Education							
	0 year		1–4 years		5–9 years		10–24 years	
	F	*p*	*F*	*p*	*F*	*p*	*F*	*p*
Digits backward	2.11	.14	0.98	.32	3.98	.05	6.23	.01
20 minus 3	5.89	.02	0.07	.91	2.41	.12	8.15	.01
Calculation abilities	2.76	.09	0.27	.60	2.22	.01	7.14	.01

children. Furthermore, gender differences in mathematical abilities are progressively higher when moving to more extreme scores: gender differences are minimal in those children one standard deviation above the mean, but higher in those children two standard deviations above the mean and even higher three or four standard deviations above the mean scores. This observation has been confirmed in different countries and remained stable over 15 years (Bensbow, 1988). We can assume it represents a quite robust observation.

Noteworthy, gender differences increase with age. No significant differences are observed in elementary school and middle school children. A moderate male superiority is found in high school, and a large and very significant advantage is observed in college male students (Hyde et al., 1990). It means, when numerical knowledge becomes more complex, gender differences become more significant.

It has been proposed that gender differences in numerical abilities are a consequence of differences in spatial abilities (e.g., Anderson, 1990). Indeed, a strong correlation has been demonstrated between a person's mathematical talent and his or her scores on spatial perception tests, almost as if they were one and the same ability (Dehaene, 1997). Fennema and Sherman (1977) report a correlation of .50 between scores on a spatial relations tests and achievement in mathematics. Hills (1957) found that score on spatial visualization and spatial orientation were correlated with performance in college mathematics courses in about .23. Ardila et al. (1998) found a correlation of about .25 between different arithmetical ability tests (Mental Arithmetic, Arithmetic Problems, WAIS Arithmetic subtest) and several spatial tests (Rarcliff's test, Perceptual speed, Block design, WAIS Picture completion). These three calculation ability tests correlated .35, .40, and .28, respectively, with a visuoperceptual factor. It means a visuoperceptual factor can account for a significant percentage of the variance in numerical ability tests. Interestingly, mathematically gifted children tend to have very high spatial abilities (Halperin, 1992). Thus, there is ground to suppose that numerical abilities and spatial abilities are sharing some common factor. Nonetheless, numerical abilities required more than spatial skills. Correlations between spatial and numerical abilities, even though highly significant, are usually in a moderate range.

Ostrosky et al. (1997) analyzed in Mexico the gender differences found in the NEUROPSI neuropsychological test battery. Only few differences were statistically significant. No Gender $\times$ Age interaction effect was found. Gender partially interacted with education. In the three NEUROPSI tests that include numerical information (Digits backward, 20 minus 3, and Calculation abilities) statistically significant gender differences were observed. Performance was higher in men than in women. Differences were particularly evident in the Calculation Ability subtest (to solve three simple arithmetical problems). Noteworthy, gender differences in calculation abilities were robust in participants with a high level of education, and minimal in illiterates or people with a limited education (Table 11).

Summing up, gender differences in numerical ability represents a solid observation, confirmed in different studies across different countries. The hypothesis that differences in calculation abilities are due to differences in spatial abilities has been usually supported. Nonetheless, correlations between both numerical and spatial abilities, even though highly significant, only account for a moderate percentage of the variance. It has to be assumed that not only spatial, but also other types of abilities, are also involved in numerical skills.

TYPES OF ACALCULIA

Several classifications have been proposed for acalculias (e.g., Ardila and Rosselli, 1990; Grafman, 1988; Grafman et al., 1982; Hécaen et al., 1961; Lindquist, 1936; Luria, 1976). The most traditional classification distinguishes between a primary acalculia and a secondary acalculia (Berger, 1926). This distinction became broadly accepted, and it is usually assumed that acalculia can result from either a primary defect in computational abilities (primary acalculia) or a diversity of cognitive defects (language, memory, etc.) impairing normal performance in calculation tests. Generally, it is considered that acalculia can be correlated with executive function defects (defects in planning and controlling the calculation sequence, impairments in understanding and solving arithmetical problems, etc.), and visuoperceptual recognition of numerical written information (defects in reading numbers, errors in reading arithmetical signs, etc.). In other words, acalculia can be observed in cases of anterior and posterior brain damage. Luria (1976) established a distinction between optic (visuoperceptual) acalculia, frontal acalculia, and primary acalculia, emphasizing that calculation disturbances can result from quite diverse brain pathology.

The most influential classification of acalculias was proposed by Hécaen et al. (1961). On the basis of the performance in different calculation tasks of 183 patients with retrorolandic lesions, they distinguished three major types of calculation disorders: (1) alexia and agraphia for numbers, (2) spatial acalculia (or acalculia of a spatial type), and (3) anarithmetia (primary acalculia). Alexia and agraphia for numbers would obviously induce calculation disturbances. It may or may not be associated with alexia and agraphia for words. Spatial acalculia represents a disorder of spatial organization where the rules for setting written digits in their proper order and position are disrupted; spatial neglect and number inversions are frequently found in this disorder. Anarithmetia (or anarithmetria or anarithmia) corresponds to primary acalculia. It implies a basic defect in computational ability. Anarithmetia does not suppose an isolated defect in numerical concepts and arithmetical operations, but excludes alexia and agraphia for numbers and spatial acalculia. Interestingly, in their analysis of acalculia Hécaen and colleagues only included patients with retrorolandic lesions; frontal-type acalculia was not considered.

Ardila and Rosselli (1990) proposed a new classification of acalculias. A basic distinction between anarithmetia (primary acalculia) and acalculia resulting from other cognitive defects (secondary acalculias) was included. Secondary acalculias can result from linguistic defects (oral or written), spatial deficits, and executive function (frontal) disturbances, such as attention impairments, perseveration, and disturbances in handling complex mathematical concepts (Table 12). There is, however, a certain degree of overlap among these acalculia subtypes. Thus, aphasic, alexic, and agraphic acalculias significantly overlap. Primary acalculia is frequently associated with aphasia, alexia, and agraphia.

Table 12. Classification of Acalculias

Primary acalculia:	Anarithmetia
Secondary acalculia:	Aphasic acalculia
	In Broca’s aphasia
	In Wernickes aphasia
	In conduction aphasia
	Alexic acalculia
	In central alexia
	In pure alexia
	Agraphic acalculia
	Frontal (executive dysfunction) acalculia
	Spatial acalculia

It is usually assumed that calculation ability represents a rather complex type of cognition requiring the participation of different cognitive abilities. Brain damage, nonetheless, may result in relatively restricted disorders in performing arithmetical operations. Benson and Denckla (1969) observed that verbal paraphasias may represent a source of calculation disturbances in aphasic patients. Ferro and Botelho (1980) found a case of limited alexia for arithmetical signs. Warrington (1982) reported a dissociation between arithmetical processing and the retrieval of computation facts. Benson and Weir (1972) described a patient who, following a left parietal lesion, was able to read and write numbers and arithmetical signs and maintained his rote arithmetical knowledge (e.g., multiplication tables), but was unable to “carry over” when performing arithmetical operations. Hittmair-Delazer et al. (1994) described a patient affected by an inability to recall and use “arithmetical facts” of one-digit multiplication and division. This impairment contrasted with the preservation of a wide range of complex notions (cardinality judgments, recognition of arithmetical signs, written calculations, solving arithmetical problems, additions, and subtractions). Cippotti et al. (1995) found a patient with an arteriovenous malformation in the left parietal region, who was able to read letters, words, and written number names, but was unable to read aloud single Arabic numerals. His ability to produce the next number in a sequence and answers to simple additions and subtractions was relatively spared when the stimuli were presented as number names but impaired when the stimuli were presented as Arabic numerals. Semenza (1988) reported a patient with a specific deficit for arithmetical procedures due to the systematic application of disturbed algorithms. This patient’s difficulty stemmed from an inability to monitor the sequence of operations that calculation procedures specify. Cipolotti et al. (1991) observed a patient with the classical signs of Gerstmann syndrome. A significant impairment in number processing and number knowledge was demonstrated. Nonetheless, the patient showed a largely preserved ability to deal with numbers below 4 in all tasks and all modalities, whereas she was totally unable to deal with numbers above 4. Lampl et al. (1994) described a patient with a selective acalculia for addition, multiplication, and division but with intact ability to subtract and distinguish mathematical signs. Patients with frontal damage may present selective impairments in using numerical information applied to temporal facts (e.g., "How many years ago did WWII end?") while normally performing arithmetical operations. Dehaene and Cohen (1997) described two patients with pure anarithmetia, one with a left subcortical lesion and the other with a right inferior parietal lesion and Gerstmann syndrome. The subcortical case suffered from a selective deficit of rote verbal knowledge (e.g., arithmetical tables), whereas the semantic knowledge of numerical quantities was intact. The inferior parietal case suffered from a category-specific impairment of quantitative numerical knowledge, with preserved knowledge of rote arithmetical facts. The potential dissociation of different calculation elements supports the assumption that numerical ability represents a multifactor skill, requiring the participation of different abilities and quite diverse brain areas.

Anarithmetia

Anarithmetia corresponds to primary acalculia. It represents a basic defect in the computational ability. The patients with anarithmetia present with a loss of numerical concepts, inability to understand quantities, defects in using syntactic rules in calculation (e.g., "to borrow"), and deficits in understanding numerical signs. However, they may be able to count aloud and to perform some other rote numerical learning (e.g., the multiplication tables). They may conserve some numerical knowledge but fail in comparing numbers (magnitude estimation) and performing arithmetical operations (Rosselli and Ardila, 1995). In primary acalculia the calculation defect must be found in both oral and written operations. That is, anarithmetia is a fundamental calculation defect, and is not restricted to a specific type of output (oral or written). Anarithmetia could be interpreted as an acquired defect in understanding how the numerical system works.

The patient with anarithmetia usually presents errors in the management of mathematical concepts and incorrectly uses arithmetical symbols. The patient also fails in solving arithmetical problems. Although it is uncommon to find cases of pure anarithmetia caused by focal lesions in the brain, it is routine to find some anarithmetia in cases of dementia (Ardila and Rosselli, 1986; Grafman et al., 1989; Parlatto et al., 1992). Hécaen et al. (1961) found a overlap between anarithmetia and alexia and agraphia for numbers. In a sample of 73 patients with anarithmetia, they found that 62% had aphasia, 61% constructional errors, 54% visual field defects, 50% general cognitive deficits, 39% verbal alexia, 37% somatosensory defects, 37% right–left confusion, and 33% ocumolomotor defects. Their sample, however, was too heterogeneous, and acalculia could easily be correlated with other neurological and neuropsychological defects.

Noteworthy, half of the Hécaen et al.'s acalculic patients also presented a general cognitive deterioration. A significant correlation between arithmetical abilities and general cognitive performance has been proposed (Ardila et al., 1998). Furthermore, arithmetical ability impairments have been postulated to represent an early sign of dementia (Deloche et al., 1995).

A memory defect (amnesia for quantities) has been conjectured in rendering acalculic patients unable to carry, borrow, and retrieve arithmetical facts (Cohen, 1971; Grewel, 1952). Patients with primary acalculia as a matter of fact present a decreased digit span. Their performance in the WAIS Digits subtest is usually abnormally low. They frequently state that they get confused with numbers, and quantities are difficult to understand. Blatant difficulties in manipulating and memorizing quantities in consequence can at least partially account for their computational defects. It is not easy to find cases of primary acalculia without additional aphasic, alexic, and agraphic defects. As a matter of fact, few cases of pure anarithmetia have been described to date. Some authors have even challenged the existence of a primary acalculia not associated with other cognitive deficits (e.g., Collington et al., 1977; Goldstein, 1948).

Anarithmetia is observed in cases of left angular gyrus damage. This localization for primary acalculia has been widely accepted since Henschen (Gerstmann, 1940; Grafman, 1988; Henschen, 1925; Levin et al., 1993; Luria, 1973). Rosselli and Ardila (1989) analyzed the errors made by a sample of patients with left parietal injuries. They found that these patients exhibited defects in oral and written calculations, most of the patients confused arithmetical symbols, and all presented errors in transcoding tasks, in successive operations, and in solving mathematical problems. It could be proposed that in case of primary acalculia, global quantification ability (discriminating between collections containing different number of objects) and probably correspondence construction (comparing elements in different collections) are preserved. However, the basic principles used in counting elements [viz., (1) one-one principle; (2) stable order principle; and (3) cardinal principle] may be impaired. Counting aloud as a numerical rote learning nonetheless can be preserved in

Table 13. Calculation Ability Test Performance in a Patient with Primary Acalculia Associated with a Left Angular Gyrus Infarct (Adapted from Ardila et al., 2000)

	Score	
Counting: forward	10/10	Normal
Backward	9/10	Abnormal
Reading number up to 3 digits	5/5	Normal
More than 3 digits	3/5	Abnormal
Writing numbers	10/10	Normal
"Greater" and "smaller" relations	5/5	Normal
Transcoding: verbal to numerical	5/5	Normal
Numerical to verbal	3/5	Abnormal
WAIS-R Digits	Scaled score = 4	Abnormal
Arithmetic	Scaled score = 5	Abnormal
Mental arithmetical operations		
Adding small quantities	5/5	Normal
Subtracting small quantities	3/3	Normal
Adding and subtracting larger quantities	0/4	Abnormal
Multiplications (2 digits)	0/3	Abnormal
Written arithmetical operations		
Adding and subtracting (3 digits)	4/4	Normal
Multiplying and dividing (3 digits)	0/4	Abnormal
Arithmetical signs: Reading	4/4	Normal
Interpreting	3/4	Abnormal
Successive operations: adding (1, 4, 7 . . .)	10/10	Normal
Subtracting (100, 87, 74 . . .)	4/5	Abnormal
Aligning numbers in columns	10/10	Normal
Solving arithmetical problems	0/5	Abnormal

primary acalculia. In aphasic patients, however, counting elements can be preserved (Benson and Ardila, 1996; Seron et al., 1992). In any event, computational strategies required in arithmetical operations (adding, subtracting, multiplying, and dividing) and mathematical problem-solving ability are severely disrupted in primary acalculia.

Illustration

Table 13 presents the performance in a Calculation Ability Test in a patient with anarithmetia. MRI images are presented in Fig. 2. This patient was a 58-year-old, right-handed male, with high-school-level education. Until his cerebrovascular accident, he worked as a successful businessman and prestigious politician. Twenty-eight months before the current evaluation he suddenly lost language production and understanding. Speech therapy was initiated and his language has generally improved, although he remained with significant word-finding disruptions. Substantial difficulties were present in discriminating antonyms; such as right–left, up–down, open–close, to go in – to go out, before–after, and over–below. In addition, he reported important impairments in understanding numbers and using numerical concepts. He was aware and critical of his deficits. Brain MRI showed a small ischemic lesion involving the left angular gyrus.

When formally tested, a primary acalculia was observed. Digit span forward and backward score was 3 for each (first percentile). Significant difficulties were observed in the WAIS Arithmetic subtest. Forward counting was flawless, but he made one omission (1/10) when counting backward. Reading numbers with three or fewer digits was normal. However he demonstrated inversions (4908 → 4098) and omissions (10003 →1003) reading numbers with more than three digits. Writing numbers under dictation was normal. Transcoding from verbal to a numerical code and vice versa, was nearly normal. Only few literal paragraphias (homophone-orthographic and non-homophonic errors) and some decomposition errors were observed in writing (10003 → *un mil cero tres*; one thousand zero three). He understood "greater" and "smaller" relations when comparing two quantities. Oral arithmetical operations were correct in adding or subtracting small quantities, but he confused adding and subtracting signs. Mental multiplications with two digit figures were impossible. Reading arithmetical signs was correct except for adding instead of subtracting when performing written operations. Addition and subtraction of three-digit quantities was normal, but multiplication and division were abnormal. Successive operations were correct except a corrected error when subtracting 13 from 100. Aligning of numbers in columns during the mathematical operations was normal. He failed in solving arithmetical problems, because he "got confused." Interestingly, this patient strongly complained of difficulties understanding antonymous words and calculating.

Aphasic Acalculia

Calculation difficulties are generally found in aphasic patients, correlated with their linguistic defects. As a result, patients with Wernicke's aphasia exhibit their verbal memory defects in the performance of numerical calculations. Patients with Broca's aphasia have difficulties handling the syntax when applied to calculations. In conduction aphasia, repetition defects may affect successive operations and counting backward, which, like repetition, require subvocal rehearsal. This means that, ultimately, the calculation defects could very well have originated and been correlated with general linguistic difficulties in aphasic patients (Grafman et al., 1982). Numerical defects are simply a result of the linguistic deficits in aphasic patients. Aphasic patients present in consequence an acalculia resulting from the language defect (aphasic acalculia).

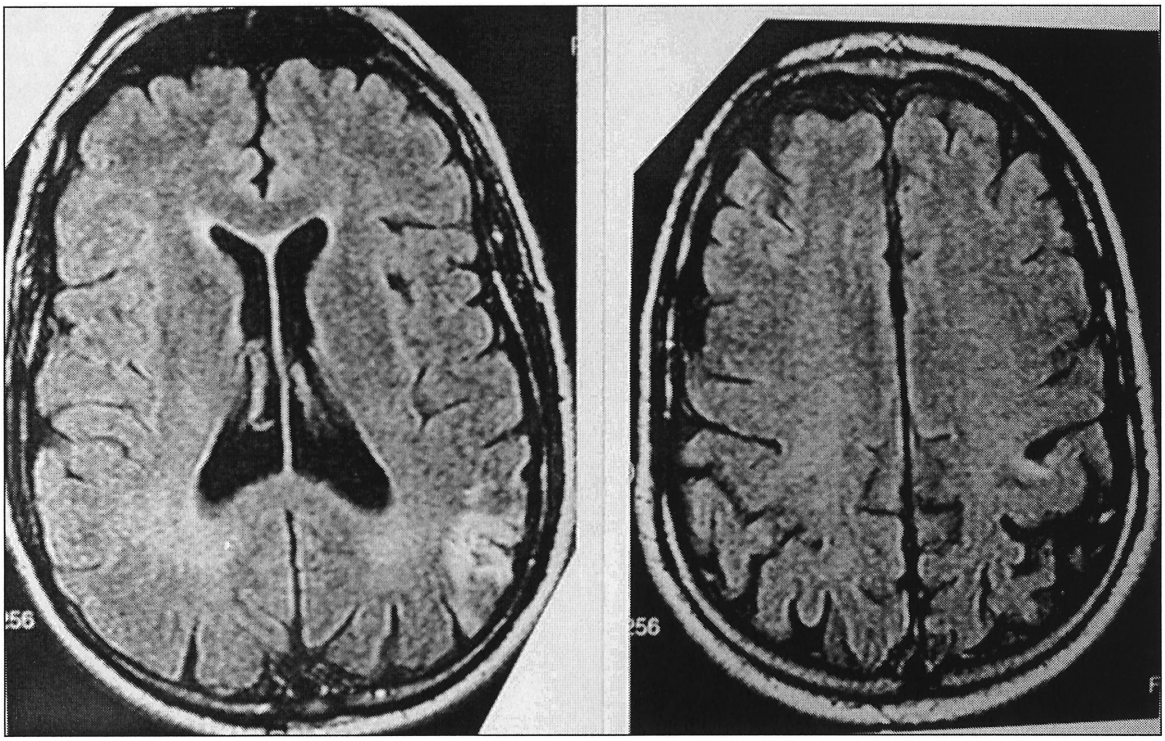

Fig. 2. Brain MRI in a patient with primary acalculia. A left angular gyrus infarct is observed.

The overall error rate in various calculation tasks is clearly correlated with the severity of the language deficit, global aphasics being the most impaired patients. The qualitative error pattern, however, may vary between different types of language disorders, and some numerical aspects, for example, multiplication facts, may be preferentially mediated by verbal processing (Delazer et al., 1999).

Acalculia in Broca's Aphasia

Dahmen et al. (1982) studied calculation deficits in patients with Broca's and Wernicke's type of aphasia. Using a factor analysis, they were able to identify two different factors: (1) numeric–symbolic and (2) visual–spatial. The milder calculation defects found in patients with Broca's aphasia are derived from the linguistic alterations, while with Wernicke's aphasia, defects in visual–spatial processing significantly contribute to calculation difficulties. The syntax of calculation is impaired in Broca's aphasics. These patients present "stack" errors (e.g., 14 is read as 4) that could be interpreted as an agrammatism in the numerical system. They also have difficulties counting backward and in successive operations (e.g., 1, 4, 7, or 20, 17, 14). Counting forward represents an automatic rote learning, whereas counting backward represents a controlled verbal sequencing activity similar to say backward the days of the week, and would thus mirror some of calculation difficulties of the conduction aphasics.

Use of morphology and syntax represents one of the central impairments in patients with Broca's aphasia (Delazer et al., 1999). This is clearly observed in transcoding tasks from a verbal (e.g., three hundred and forty-two, etc.) to a numerical code (342, etc.) and also from numerical to verbal. The patient has defects in interpreting grammatical elements pointing to the position of the number within the class (e.g., when reading "three hundred thousand two hundred" is difficult to understand that "hundred" in the first and in the second case do not mean the same; hierarchy errors are evident). Broca's aphasia at least partially be interpreted as a disorder in language sequencing and consequently, in calculation tasks, numerical sequencing will also be altered. Reading not only Arabic numbers but also number words, and writing number words is abnormal. Mental and written calculations are significantly defective (Delazer et al., 1999).

Acalculia in Wernicke's Aphasia

Lexical and semantic errors are most significant in Wernicke's aphasia. Patients with Wernicke's aphasia present semantic and lexical errors in saying, reading, and writing numbers. Luria (1973) suggests that calculation errors in patients with acoustic–amnesic aphasia (one subtype of Wernicke's aphasia according to Luria) depend on their defects in verbal memory. This is particularly noticeable in the solution of numerical problems, when the patient is required to remember certain conditions of a problem. The verbal memory span is limited, and the patient loses the thread and mixes the conditions of the problem.

Lexical errors also play a significant role in the acalculia found in Wernicke's aphasia. When the patient is asked about numerical facts (i.e., "How many days are there in a year?"), paraphasic errors become evident. The meanings of all words (including number words) are weakened. Lexical errors are abundant in different types of tasks. Benson and Denckla (1969) stressed the presence of verbal paraphasias as an important source of calculation errors in these patients.

Wernicke's aphasics present semantic errors in the reading and writing of numbers (Delazer et al., 1999; Deloche and Seron, 1982). When writing numbers by dictation, patients with Wernicke's aphasia may write completely irrelevant numbers (e.g., the patients are required to write the number 257; they repeat 820, and finally, write 193), exhibiting a loss of the sense of the language (numerical paragraphias). Lexicalization (e.g., 634 is written 600304) is frequently observed. In reading, they show numerical paralexias (e.g., 37 is read as 27). Decomposition errors are frequent (e.g., 1527 is read 15-27) in reading.

Mental operations, successive operations, and the solution of numerical problems appear equally difficult for these patients as a result of their verbal memory, lexical, and semantic difficulties (Rosselli and Ardila, 1989). Verbal memory defects are evident in mathematical problem solving when the patient has to retain different elements of the problem.

Illustration

A 46-year-old right-handed man, professional lawyer sustained a vascular accident involving the temporal branches of the left middle cerebral artery. In the Boston Diagnostic Aphasia Examination the profile of a typical Wernicke's aphasia was found: severe auditory comprehension disturbances, naming difficulties, paraphasias, and language repetition errors. Language was fluent, abundant, prosodic, and without articulatory errors. No grammatical omissions were found, but a significant empty speech was evident. If testing for calculation ability, it was observed that the patient could count forward, but when counting backward difficulties and errors were recorded (when counting backward from 80 to 70, the patients performed "80, 77, 78, 76, 75, 70, 80 . . ."). Errors in reading (e.g., 49 → 29) and writing numbers to dictation (e.g., 3041 → 3091) were also observed. In transcoding from numerical to verbal code 3/8 errors (order errors and letter omissions) were noted. In transcoding from numerical to verbal code 3/8 errors were also found (order errors and hierarchy errors). Aligning numbers in columns was correct. Simple mental arithmetical operations were errorless. Written operations were difficult and the patient failed in 50% of the cases. When reading arithmetical signs, he confused plus (+) and multiplication (×) signs, and stated that he does not know what the minus (−) sign means. Solving arithmetical problems was impossible because of the significant language understanding defect.

Acalculia in Conduction Aphasia

Patients with conduction aphasia (afferent motor aphasia) frequently present significant calculation errors. They may fail in performing both mental and written operations. They have serious flaws in performing successive operations and in problem solving. In reading numbers, errors of decomposition, order, and hierarchy can appear. They usually fail in "carrying over," in the general use of calculation syntax, and even in reading arithmetical signs (Rosselli and Ardila, 1989). Taken together all these potential errors, the calculation defect associated with conduction aphasia could be interpreted as anarithmetia. However, it should be addressed that the topography of the damage in conduction aphasia can be close to the topography of the damage in anarithmetia. Conduction aphasia, as well as anarithmetia, has been correlated with left parietal brain injury. The association between conduction aphasia and some degree of anarithmetia is not coincidental.

Acalculia in Other Types of Aphasia

Calculation disturbances are also observed in other types of aphasia (Benson and Ardila, 1996). In extrasylvian (transcortical) motor aphasia the patient may have difficulties initiating and maintaining numerical sequences. Problem-solving ability may be significantly impaired, and the patient may even fail in understanding what the problem is about. In extrasylvian (transcortical) sensory aphasia, significant calculation defects are usually found associated with the language-understanding difficulties and echolalia. Temporal–parietal damage results in a variety of language disturbances and significant calculation defects. Mental and written calculation can be difficult, and errors are observed in writing number words (Delazer et al., 1999).

Alexic Acalculia

Impairments in calculation may also be correlated with general difficulties in reading. This represents an alexic acalculia or alexia for numbers, and has been recognized since Henschen (1925). Four basic types of alexia have been described: central alexia, pure alexia, frontal alexia, and spatial alexia. Calculation errors observed in frontal alexia were analyzed when describing acalculia in the Broca's aphasia, and errors in calculation in spatial alexia will be analyzed when describing spatial acalculia.

Acalculia in Central Alexia

Central (parietal–temporal alexia, or alexia with agraphia) alexia includes an inability to read written numbers and numerical signs. Usually, the ability to perform mental calculation may be considerably better. Quite often, central alexia is associated with anarithmetia. Reading and writing difficulties plus computational disturbances may result in severe acalculia. Frequently, in these patients, reading digits and numbers may be superior to reading letters and words. Occasionally, the patient may be unable to decide if a symbol corresponds to a letter or a number. Written mathematical operations are seriously impaired, and mental execution is superior.

Although the distinction between alexia with agraphia for numbers and anarithmetia is conceptually valid, in reality it may be difficult to establish. The posterior brain topography of the two syndromes is similar as pointed out by Hécaen et al. (1961). Usually alexia for numbers and arithmetical signs is associated with alexia for letters, some agraphia, and some aphasic disorders.

Acalculia in Pure Alexia

Pure alexia (alexia without agraphia, or occipital alexia) is mainly a verbal alexia in which letter reading is significantly superior to word reading. As expected, these patients present greater difficulties reading numbers composed of several digits (compound numbers) than reading single digits. When reading compound numbers, the patient exhibits decomposition (27 becomes 2,7) (digit-by-digit reading) and hierarchy errors (50 becomes 5) as a result of the omission for the right-side information. When reading words, letters placed on the left are generally understood better than letters placed on the right. Likewise, in reading numbers, only the first or the first two–three digits are read correctly, and a certain degree of right hemineglect is observed (5637 becomes 563). Because of the alexia, performing written operations is painstaking and even impossible. As a result of the visual exploration defects, aligning numbers in columns and "carrying over" are tasks the patient usually fails. It is important to stress that reading is performed from left to right (in Western languages, at least), but the performance of arithmetical operations goes from right to left. This disparity may create problems in those patients with visual attention problems.

Illustration

A 45-year-old right-handed man with 8 years of education presented with severe chest pains followed by loss of consciousness. He was hospitalized with transient cardiac arrest. During the following days, right homonymous hemianopsia, right hemi-body extinction, difficulties in visual exploration, optic ataxia, verbal alexia, and inability to recognize objects and colors were observed. CT scans showed a small hypodensity lesion in the left occipital lobe.

When testing for reading, it was observed that the patient could read letters and syllables. However, he was unable to read any word composed of more than three letters. A letter-by-letter or syllable-by-syllable reading was observed. A very significant amount of morphologic verbal paralexias was observed (e.g., *casa* → *cascara*). No significant word-finding difficulties or language-understanding impairments were observed in conversational language. He could write by dictation, even though he partially overlaps words when writing. A few letter omissions were noted when writing. He could read numbers (Arabic and Roman) up to three digits. With longer numbers, he only read the initial part (e.g., 7528 was read as 75, 2). When reading arithmetical signs, he confused plus (+) and multiplication (×). Transcoding from numerical to verbal code was normal for numbers up to three digits. Transcoding from a verbal to a numerical code was extremely difficult, as a result of his inability to read long words. Performance of simple mental arithmetical operation was normal. Performance of written operations was slow and abnormal, because of the significant visual scanning difficulties. Counting forward and backward was normal. Solving simple arithmetical problems was errorless. Aligning numbers in columns was painstaking, slow, and poorly performed because of the visual scanning defects.

Agraphic Acalculia

Calculation errors may appear as a result of an inability to write quantities. The calculation deficit may be a function of the type of agraphia. In the agraphia associated with Broca's aphasia, the writing of numbers will

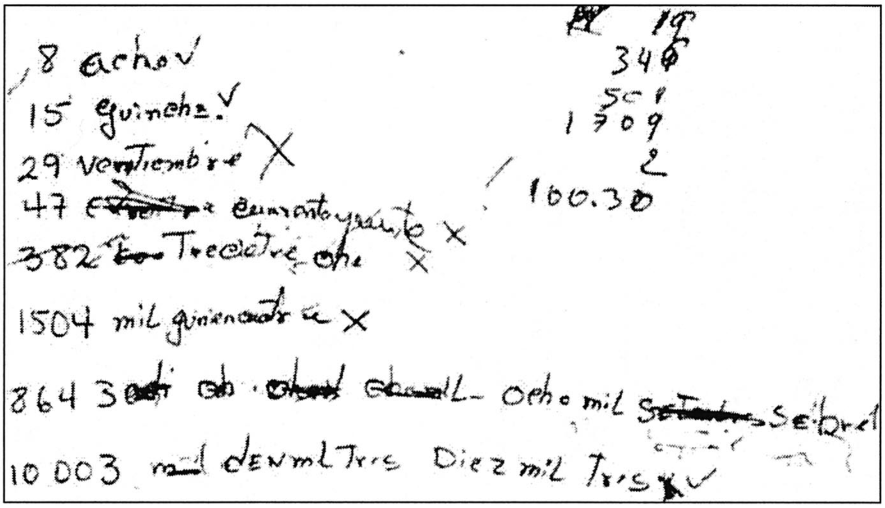

Fig. 3. Apractic agraphia illustration.

be nonfluent, with some perseveration and omissions. In transcoding tasks, from the numerical code to the verbal code, grammatical and letter omissions appear. The patient presents difficulties in the production of written numerical sequences (e.g., 1, 2, 3), particularly backward (e.g., 10, 9, 8) (Ardila and Rosselli, 1990).

In Wernicke's aphasia, there is a fluent agraphia for numbers. Because of verbal comprehension defects, the patient produces errors in writing numbers to dictation and even writes totally irrelevant numbers (428 becomes 2530). Lexical errors (numerical verbal paragraphias) and fragmentation (25 becomes 20 . . . 5) are observed. Language-understanding defects impair the ability to write quantities to dictation. Solving arithmetical problems is impossible because of the language-understanding defects and verbal memory defects.

In conduction aphasia, there may be a very significant agraphic defect in the writing of numbers by dictation. The patients may be unable to convert the number that they have heard and even repeated to themselves in graphic form. Order, hierarchy, and inversion errors are observed (Rosselli and Ardila, 1989). Some degree of apractic agraphia is frequently found in conduction aphasia, and some degree of apractic speech is frequently found in certain conduction aphasics.

Writing number defects are observed not only in aphasic but also in nonaphasic forms of agraphia. Apractic agraphia becomes evident in writing not only word but also quantities. Self-corrections and approximations are found, and frequently the patients fail to convert the numbers they hear in a correct graphic form. Apractic agraphia impairs the performance of motor sequences required to write letters. Writing numbers becomes slow and difficult, and permanent self-corrections appear.

In cases of motor agraphia, the difficulties observed in writing letters and words will be also found when writing numbers. In paretic agraphia, numbers are large and clumsily formed. In hypokinetic agraphia, difficulties in starting the motor activity are evident, as micrography and progressive narrowing of numbers appear. In hyperkinetic agraphia, numbers are usually large, hard to read, and distorted; frequently the patient is unable to write.

Illustration

A 33-year-old right-handed man with a high school level of education suddenly during the morning developed right hemiparesis and impossibility to speak. CT scans demonstrated a parietal–insular infarct. Aphasia and hemiparesis rapidly improved during the following days. After a formal language evaluation, a diagnosis of a conduction aphasia associated apractic agraphia was proposed. Counting forward and backward was intact. Reading numbers and magnitude estimation (what number is larger) was also correct. Transcoding from verbal to numerical code was correct even though the patient had significant difficulties writing some numbers. Transcoding from numerical to verbal was notoriously difficult because of the writing impairments (Fig. 3). Mental successive adding and subtracting was errorless. Finding the number or arithmetical sign lacking in an arithmetical operation (e.g., 12 + . . . = 19; 35 . . . 12 = 23) was appropriate. Solving simple arithmetical problems was normal. Written operations and aligning numbers in columns were difficult because of the defects in writing the numbers.

Frontal (Executive Dysfunction) Acalculia

Patients with prefrontal injuries frequently develop calculation difficulties that are not easily detected. Patients with damage in the prefrontal areas of the brain may display serious difficulties in mental operations, successive operations (particularly backward operations; e.g., 100–7), and solving multistep numerical problems. Written arithmetical operations are notoriously easier than mental operations. Difficulties in calculation tasks in these patients correspond to different types: (1) attention difficulties, (2) perseveration, and (3) impairment of complex mathematical concepts. Attention deficits are reflected in the patient's difficulty in maintaining concentration on the problem. Attention difficulties result in defects in maintaining the conditions of the tasks and impulsiveness in answers. Perseveration is observed in the tendency to continue presenting the very same response to different conditions. Perseveration also appears in writing and reading numbers. Perseveration results in incorrect answer (e.g., when subtracting 7 from 100: 93, 83, 73, etc.). Impairment in the use of complex mathematical concepts results in inability to analyze the conditions of numerical problems and developing an algorithm for its solution. When trying to solve mathematical problems, the patient may have difficulties simultaneously handling diverse information from the same problem and may even be unable to understand the nature of the problem. Instead of solving the mathematical problem, the patient with frontal acalculia may simply repeat it. The above defects are reflected in the abnormal handling of complex mathematical concepts.

The most profound defects are found in solving numerical problems, whereas elementary arithmetic is usually much better preserved. Mental arithmetic is significantly more abnormal than written operations, as in general, mental tasks are harder than tasks using external support, and using pen and pencil helps keep track of the material in operating memory.

Interestingly, patients with frontal lobe pathology may present notorious disturbances in the use of temporal measures. Time is measured using quantities (2 hr, 34 years, etc.). This specific type of numerical knowledge is significantly disturbed in this group of patients. They may be unsure if the discovery of America was carried out about 50 years ago, or about 100 years ago, or about 500 years ago. Patients may not know if their accident occurred 1 or 10 years ago. Of course, this deficit is related to the severe defects in temporal memory and in time concepts observed in this group of patients (Fuster, 1993).

Illustration

After a seizure, an anterior left hemisphere tumor was disclosed in a 52-year-old right-handed woman with a college-level education. No language abnormalities were found in the Boston Diagnostic Aphasia Examination. In testing for reading, she could read letters, words, sentences, and texts. Understanding of written language was normal. However, she read 4/10 pseudowords as real words, and 4/4 times she could not decide which of two words was correctly written. Spontaneous writing and writing by dictation was normal. Writing numbers with one or several digits was normal. Only one error was recorded when reading numbers (10003 was read "One million and three). Transcoding (verbal to numerical; and numerical to verbal) was also normal, except for one error (twelve thousand three hundred sixty-nine was written "2369"). Magnitude estimation (to decide which number is bigger, e.g., 189 and 201), reading of arithmetical signs, counting forward and backward, and simple arithmetical operations were performed without difficulty. Successive additions (1, 4, 7, . . . etc.) were errorless but successive subtractions (100, 87, 74 . . . etc.) were impossible. Written arithmetical operations with three digits (adding, subtracting, multiplying, and dividing) were normal. The patient could successfully find 5/10 times what was lacking in some arithmetical operations written on a card (e.g., $12 + \ldots = 15$; $93 \ldots 13 = 80$), and in 5/10 times she simply answered, "I cannot figure it out," or "For me, it looks OK." She failed 3/5 very simple arithmetical problems: "How many centimeters are there in two and half meters?"; "Paul receives 8 pesos per working hour. How much will he get working four hours?"; "Mary and John get 150 in a day. Mary receives twice as much as John. How much does each one receive?" Aligning numbers in columns was normal.

Spatial Acalculia

Spatial acalculia is observed in patients with right hemispheric damage, particularly parietal lobe pathology. It frequently coexists with hemi-spatial neglect, spatial alexia and agraphia, constructional difficulties, and other spatial disorders (Ardila and Rosselli, 1994; Hécaen et al., 1961).

In cases of spatial acalculia, mental calculation is superior to written calculation. No difficulties in counting or in performing successive operations are observed. A certain degree of fragmentation appears in the reading of numbers (523 becomes 23) resulting from left hemispatial neglect. In large quantities, the patient reads the last or two last digits, with a notorious tendency to omit left-sided information. Reading complex numbers, in which the spatial position is critical, is affected, particularly when the number includes several digits that are repeated (e.g., 1003 becomes 103). Inversions can be noted (32 becomes 23, or 734 becomes 43) (Ardila and Rosselli, 1990, 1994).

The difficulties observed in writing numbers are common across all written tasks. Such difficulties include exclusive use or simply overuse of the right half of the page; digit iterations (227 becomes 22277) and feature iterations (particularly when writing the number 3 extra loops are written); inability to maintain the horizontal direction in writing; spatial disorganization; and writing over segments of the page already used. When performing written arithmetical operations, the patient understands how much should be "carried over" (or "borrowed") but cannot find where to place the carried-over quantity. Also, the inability to align numbers in columns prevents such patients from performing written arithmetical operations. When performing multiplication, the difficulty in remembering multiplication tables becomes obvious, a defect correlated with the general difficulty in making use of automatic levels of language. These patients frequently mix procedures up (e.g., when they should subtract, they add). This is related to another frequently found defect: they do not seem surprised by impossible results (reasoning errors). For instance, the result of subtraction is larger than the original number being subtracted. This type of error in arithmetical reasoning has also been noted in children with DD.

Ardila and Rosselli (1994) studied calculation errors in a sample of 21 patients. Spatial defects that interfered with the reading and writing of numbers and with the loss of arithmetical automatisms (e.g., multiplication tables) were found. The processing system seems abnormal in these patients while the numerical calculation system is partially preserved. Difficulties in calculation procedures and problems in the recall of arithmetical principles were observed; however, arithmetical rules were intact. The authors concluded that the numerical changes observed in patients with right hemisphere injury are due to (1) visual–spatial defects that interfere with the spatial organization of numbers and mechanical aspects of the mathematical operations, (2) inability to evoke mathematical facts and remember their appropriate uses, and (3) inability to normally conceptualize quantities and to process numbers.

Spatial acalculia is most frequently observed in right hemisphere pathology (Ardila and Rosselli, 1994). Hemineglect, topographic agnosia, constructional apraxia, and general spatial defects are usually correlated with spatial acalculia. Patients with spatial acalculia perform much better in orally presented arithmetical tasks than in written ones.

Illustration

A 68-year-old right-handed woman with high school education was hospitalized because of a sudden loss of sensitivity in her left hemi-body. At the incoming neurological exam, left hypoesthesia in face and arm, left homonymous hemianopia, disorientation in time and place, and left hemi-neglect were found. CT scans revealed an ischemic lesion involving the temporal and parietal branches of the right middle cerebral artery.

Neuropsychological testing indicated a significant left spatial hemi-neglect. Neglect was observed in drawing, reading, writing, and performing spontaneous activities. Severe spatial and constructional defects were also found. A severe spatial acalculia was noted. The patient could not align numbers in columns (Fig. 4). Adding and multiplying were impossible because the patient could not find where to place the "carried" or "borrowed" quantities. She confused "plus" (+) and "multiplication" (×) signs and could not read numbers on the left (e.g., 9,231 was read as 31). Because of these defects, the patient failed in performing all the written numerical operations that were presented. Nonetheless, her ability to perform arithmetical operations orally and to solve numerical problems was nearly normal. A diagnosis of spatial acalculia associated with spatial alexia, spatial agraphia, hemi-spatial neglect, and general spatial defects was presented. During the following weeks neglect mildly improved. Still, she continues unable to perform any written arithmetical operation. Defects in spatial orientation continued to be significant. One year later she began to attend a rehabilitation program that lasted for about 1 year. Improvement was significant. At the end of this program, the patient could perform simple written arithmetical operations.

In summary, it is possible to find very different types of acquired disorders in calculation skills. Some of them represent disability derived from defects (oral and written, production and comprehension) in language. Others are closely correlated with either spatial defects (spatial acalculia), executive function deficits (frontal acalculia), or primary defects in the performance of arithmetical tasks (anarithmetia).

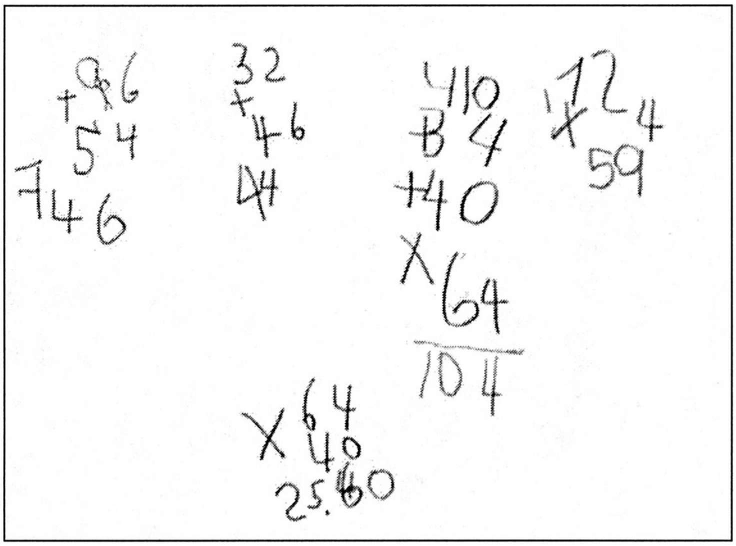

Fig. 4. Spatial agraphia illustration.

Commentary

Calculation ability might be interpreted as a type of cognition involving in their origins at least, some type of body knowledge (autotopognosis) (Gerstmann, 1940), spatial concepts, language, and executive function abilities. Finger agnosia (as a restricted form of autotopagnosia) supports the association between calculation abilities and body knowledge. The association between spatial knowledge mediated through language and calculation abilities have been emphasized by different authors (e.g., Ardila et al., 1989a,b; Hartje, 1987; Luria, 1966, 1976). Luria emphasizes that an inability to use verbally mediated spatial concepts (semantic aphasia) is always associated with acalculia.

The role of parietal lobe in body knowledge and the disorders of the body scheme in cases of parietal pathology have often been emphasized in the literature (e.g., Botez, 1985; Critchley, 1953). Parietal damages have been associated with asomatognosia in general, and hemiasomatognosia, alloesthesia, finger agnosia, autotopagnosia, asymbolia for pain, apraxia, and the so-called Verger–Dejerine syndrome (Hécaen and Albert, 1978).

Asymmetry in cerebral organization of cognition represents the most outstanding characteristic of the human brain. LeDoux (1982, 1984) proposed that the primary functional distinction between human hemispheres involves the differential representation of linguistic and spatial mechanisms. The right posterior parietal lobe is involved in spatial processing, whereas the left posterior parietal lobe is involved in linguistic processing. Spatial mechanisms are represented in both the right and the left parietal lobes in nonhuman primates. In humans, however, language is represented in a region (posterior parietal lobe) of the left hemisphere that, in the right hemisphere, is involved in spatial functions, and was involved in spatial functions in both hemispheres in human ancestors (Lynch, 1980). In consequence, the evolution of language involved adaptations in the neural substrate of spatial behavior (LeDoux, 1984). It is understandable that the left parietal lobe can play a significant role in understanding spatial concepts mediated through language. Boles (1991), presenting different tasks (recognition of words, products, locations, dichotic digits, etc.) and using a factor analysis, was able to identify different lateralized parietal functions: lexical functions (e.g., word numbers) were associated with left hemisphere, whereas primary spatial functions (e.g., locations of dots) were correlated with right hemisphere activity.

Luria (1966) emphasized that defects in spatial conceptualization underlying the acalculia observed in left-parietal-damaged patients. He proposed that left parietal–temporal–occipital damage could produce components of spatial apraxia, agnosia, semantic aphasia, and acalculia. Luria considered that the same cognitive defects were present in semantic aphasia and acalculia. In both syndromes, defects in understanding verbally mediated spatial concepts are evident. Therefore, acalculia is always associated with semantic aphasia. Spatial knowledge is crucial in understanding the numerical system. Cardinality requires the understanding of “after,” “larger,” and other spatially tinged relationships. Quantities and arithmetical operations are intrinsically spatial concepts. The numerical system, furthermore, assumes an spatial organization (units, tens, hundreds, etc.)

In brief, normal performance of calculation tasks requires different types of cognition. Language, spatial abilities, body knowledge, and executive (frontal lobe) function are necessarily included. Calculation abilities can be impaired as a result of language (oral and written), spatial, and executive function disturbances. General consensus in this regard can be found. The crucial unsettled point is whether acalculia can be found in isolation as a pure primary disorder with no other cognitive deficit. Or, is acalculia always associated with body knowledge disturbances (right–left disorientation, finger agnosia, and in general autotopagnosia), and disturbances in the linguistic use of spatial concepts (semantic aphasia)? The historical analysis of the evolution of numerical concepts and the clinical observation seems to provide some support to the second point of view.

ASSOCIATED DISORDERS

Calculation ability can be regarded as a multifactorial skill. An important association between calculation abilities and performance in different cognitive areas, including language, memory, constructional abilities, spatial skills, attention, and executive functions, has been found. Acalculia is quite often correlated with diverse disturbances in different cognitive domains.

Acalculia and Aphasia

Acalculia quite often overlaps with aphasia. Some authors have even stated that primary acalculia is always associated with a particular type of aphasia known as semantic aphasia (e.g., Luria, 1976). Frequently, acalculia is also associated with diverse types of language disturbances very specially, with conduction and Wernicke's aphasia (Rosselli and Ardila, 1989). According to Hécaen et al. (1961), 62% of the patients with anarithmetia (primary acalculia) also presented aphasia. Aphasia was demonstrated in 84% of the patients with alexic acalculia. The associated aphasia in Hécaen et al.'s study was a fluent type of aphasia.

Anomia, word-finding difficulties, language-understanding defects, and paraphasias are quite frequently correlated with primary acalculia. Aphasia is obviously not expected in spatial acalculia observed in cases of right hemisphere pathology. Frontal acalculia can be often correlated with an extrasylvian (transcortical or dynamic) motor aphasia.

However, the association between language disturbances and acalculia is not simple. Numbers are coded verbally (e.g., three, two hundred twelve) and in such regard, calculation requires language. But quantities are also coded numerically as Arabic digits (e.g., 3, 212). Further, language can be coded in two modalities (oral and written; i.e., phonologic and orthographic). Numbers in consequence can be coded in three different symbolic systems (the two language symbolic systems plus the Arabic numerical system). For small quantities, a second numerical system is sometimes used (Roman numerals). Performance of arithmetical operations using Roman numerals, however, is not evident.

Grewel (1952, 1969) considers that the three semiotic systems used for representing quantities (verbal–oral, verbal–written, and Arabic digits) are independent and possess their own distinctive features. Each numerical system includes several components: (1) the set of symbols used (lexicon), (2) the semantic representation of the numbers (semantics), and (3) the syntactic rules used for combining and manipulating numbers (syntax). Grewel proposes that the three semiotic systems may be independently affected in cases of brain pathology because they represent different symbolic systems with a different syntax. Clinical observation demonstrates that sometimes acalculic patients may fail in transcoding tasks (transcoding from the numerical to the verbal system and vice versa) (Deloche and Seron, 1982, 1984, 1987). At least the verbal and numerical system may become dissociated. Further, it seems to exist a general relationship between lexical/syntactical language processing preserved/impaired abilities in Broca's or Wernicke's aphasics, and their types of transcoding impairment (Deloche, 1993). In Broca's aphasia, lexical processing is better preserved, whereas syntactical processing is impaired. In Wernicke's aphasia the opposite situation is usually found.

Deficits in very specific linguistic elements have been often demonstrated in acalculia. Benson and Denckla (1969) observed that verbal paraphasias may represent a source of calculation disturbances in aphasic patients. A selective disturbance in the oral–verbal system was responsible for the calculation defect in this case. As mentioned previously, it has been also observed that the verbal processing system of number can become dissociated from the numerical calculation system (e.g., Dagenbach and McCloskey, 1992; Pesenti et al., 1994). Dehaene and Cohen (1997) described a patient with a selective deficit of rote verbal knowledge (e.g., arithmetical tables), whereas the semantic knowledge of numerical quantities was intact. They also described a second patient with preserved knowledge of verbal rote arithmetical facts and disturbances at the semantic level of quantities. No question, the semantic knowledge of numerical quantities depends on different brain areas than the knowledge of verbal rote arithmetical facts (multiplication tables, simple additions and subtractions, etc.).

Rossor et al. (1995) described a patient with severe language problems and good calculation abilities. The patient answered correctly simple subtraction, additions, multiplications, and multidigit operation without significant difficulty. This patient seemingly did not rely on verbal abilities in any of the operations, or compensated impaired verbal abilities with nonverbal skills. This report supports the assumption that, even though verbal abilities are involved in numerical tasks, at least in some numerical tasks, there is a functional independence between numerical and verbal abilities. The opposite pattern has been also observed: intact language function and impaired calculation ability (e.g., Warrington, 1982). All these dissociations support the assumption that different elements participate in normal calculation ability.

At the semantic level, it may be argued that the different number representation systems (oral, orthographic, numerical) share a single semantic representation. It may be conjectured that indeed all the three representation systems have in common a single semantics. Quantity conceptualization may be proposed to be the semantic core of the different numerical system representations. Deloche (1993), however, has argued that the assumption of a unique semantic representation for each number seems unrealistic, considering the constellations of usages and meaning of numbers (e.g., items of cardinality, prices, private bank account number, room numbers in a hotel, bus line numbers in a city, nursery rhymes, idiomatic expressions, etc.). Accordingly, there is not a single semantics for numbers, but semantics depends upon the specific usage of the numbers. Alternatively, it might be conjectured that numbers contain two different semantic aspects: (1) the cardinality of the number and (2) the "bigger" and "smaller" relationships (magnitude comparison). Cardinality is invariant, but "bigger" and "smaller" relations are relative and depend upon the objects that are counted, or they are related with. Two pages is "smaller" than one book. Five dollars is "big" money for a candy, but it is "small" money for a shirt. Bus line numbers has not any "big"–"small" semantic for the passenger, but may have some technical ("big"–"small" or "before"–"after" or whatever) semantic for the city traffic organization. We are proposing that cardinality may represent the invariant semantics of all notational numerical systems. Cardinality simply means "before"–"after." The specific semantic of the cardinal depends upon the particular object it is applied to.

The syntactic rules used for combining and manipulating numbers suppose the understanding about the organization of the numerical system (Grewel, 1952, 1969). The syntactic rules are required for the mathematical thought and the comprehension of the underlying operations (Boller and Grafman, 1983). In primary acalculia, disturbances in using syntactic rules and impairments in understanding how the numerical system is organized are assumed.

Delazer et al. (1999) analyzed the pattern of errors in a large sample of aphasic patients. They found as expected that most severe calculation impairments are observed in global aphasics. Broca's and Wernicke's aphasics scored similarly at the quantitative level, and amnesic aphasics showed only mild calculation difficulties (unfortunately, conduction aphasia patients were not included in this study). Calculation procedures were mainly impaired in Wernicke's aphasia. Syntactic errors were more frequent in Broca's aphasia whereas lexical errors were mainly observed in Wernicke's aphasia. The authors propose that the retrieval of multiplication facts is preferentially mediated by verbal processing. In cases of aphasia, it will be most impaired.

Because of its left parietal topography, primary acalculia may be associated not only with aphasia but also with alexia and agraphia. The association with parietotemporal alexia (central alexia or alexia with agraphia) is evident. The association with apractic agraphia is also evident (see the next section Acalculia and Apraxia). However, unusual cases of acalculia with unexpected localizations have been sometimes reported. For example, Tohgi et al. (1995) described a patient with agraphia and acalculia associated with a left frontal (F1, F2) infarction. Although the patient could add and subtract numbers, he could neither multiply nor divide because of a difficulty in retrieving the multiplication tables and calculation procedures. Lucchelim and De Renzi (1993) report the case of a 22-year-old man who had an infarct in the left anterior cerebral artery that destroyed the medial cortex of the frontal lobe. The patient manifested primary acalculia. Accounting for these unusual acalculia localizations is not easy. Disturbances in some brain circuitry required for normal numerical operations may be conjectured.

Acalculia and Apraxia

The association between acalculia and apraxia is not frequently mentioned excepting in the dementia syndrome (e.g., Cummings and Benson, 1992) and in traumatic brain injury cases. In those instances, apraxia and acalculia appear together as a result of an extended brain involvement and wide disturbances in cognition. This association is not particularly informative in understanding brain organization of calculation abilities, because acalculia and apraxia appear amid a whole array of diverse cognitive disturbances.

However, there is a twofold association between acalculia and apraxia: (1) primary acalculia quite often is associated with ideomotor apraxia, and (2) spatial acalculia significantly correlates with constructional apraxia. The frequent association between ideomotor apraxia and acalculia can be easily overlooked. As a matter of fact, many patients with primary acalculia also present ideomotor apraxia (Rosselli and Ardila, 1989). This is not a coincidental association. Ideomotor and ideational apraxia are observed in cases of left parietal damage, and primary acalculia too. Nonetheless, it is difficult to know how frequent ideomotor apraxia is associated with acalculia because of the lack of large samples of acalculic patients. Hécaen et al. (1961), however, reported that ideational or ideomotor apraxia was observed in 36.5% of the acalculic patients. Interesting to note, agraphia observed in the Gerstmann syndrome is an apractic agraphia apraxia, not an aphasic agraphia (Benson and Cummings, 1985; Strub and Geschwind, 1983), and hence, it represents a segmentary ideomotor apraxia. In consequence, ideomotor apraxia should be regarded as a frequently associated impairment in cases of acalculia. Unfortunately, ideomotor and ideational apraxia can be easily overlooked in a routine neurological or neuropsychological exam.

The association between spatial acalculia and constructional apraxia is quite evident. Patients with right hemisphere pathology, particularly right parietal damage, present general spatial and visuoconstructive defects. Spatial acalculia is just a single manifestation of the general spatial defects observed in these patients. Hécaen et al. (1961) observed visuoconstructive impairments in 95% of the patients presenting spatial acalculia. Spatial agnosia and general visuospatial defects were recorded in 62.5% of their patients. Ardila and Rosselli (1994) found a significant correlation between spatial acalculia, spatial alexia, spatial agraphia, and constructional apraxia.

Acalculia and Dementia

There is a significant correlation between calculation abilities and general intellectual performance (Ardila et al., 1998). Consequently, it is understandable that calculation deficits have been reported as an early sign of Alzheimer's Disease (AD) (Grafman et al., 1989; Deloche et al., 1995; Mantovan et al., 1999; Parlatto et al., 1992). Interestingly, mental calculation impairments represent an important factor in predicting cognitive deficits in these dementia patients (Roudier et al., 1991).

Nonetheless, acalculia in dementia has not been a significant research topic, and usually acalculia is not mentioned as a diagnostic criterion of dementia. The most frequently used clinical criteria in the diagnosis of AD (e.g., NINCDS; American Psychiatric Association, 1994; McKhann et al., 1984) do not list the numerical ability defects as a clinical symptom of dementia. Clinical criteria of dementia usually emphasize memory disturbances, language deficits, attention defects, and visuoperceptual impairments. As a matter of fact, very little research concerning the functions of the mathematical abilities in AD is found. Considering, however, the significant frequency of calculation disturbances in case of brain pathology, it may be assumed a high frequency of acalculia in cases of dementia.

Recently, a few papers have been published pointing out that calculation abilities represent indeed a very important defect in AD. Marterer et al. (1996) found a significant correlation between arithmetical impairment and the degree of dementia. Deloche et al. (1995) reported that calculation and number processing scores significantly correlate with the MMSE (Folstein et al., 1975) scores and language performance tests. However, their case analyses indicated heterogeneous patterns of preserved/impaired abilities with regard to other cognitive areas. Heterogeneous patterns of calculation deficits in brain-damaged patients have been also reported by different authors (Ardila and Rosselli, 1990; Boller and Grafman, 1985; Delazer et al., 1999; Grafman et al., 1982; Rosselli and Ardila, 1989).

Rosselli et al. (1990) analyzed the calculation abilities in AD. Twenty right-handed patients meeting the *DSM-IV* (American Psychiatric Association, 1994) criteria for AD were studied. Age ranged from 64 to 88 years. A neuropsychological test battery including language, memory, constructional abilities, attention, mathematics, and abstraction tests was administered. In addition, the MMSE was also used. Mathematical subtests correlated higher than the MMSE with the scores in the different neuropsychological tests (Table 14). Highest correlations of the mathematical subtests were observed with language repetition, nonverbal memory, and attention tasks. It was proposed that mathematical ability tests represent an excellent predictor of general intellectual performance in AD. It was further proposed that disturbances in arithmetical ability should be included as a diagnostic criterion for AD. These results support the hypothesis that a significant association exists between arithmetical impairments and the severity of dementia (Deloche et al., 1995;

Table 14. Correlations Among the Different Neuropsychological Tests, the MMSE, and Two Arithmetic Tests in the Dementia Group (Adapted from Rosselli et al., 1998)

Test	MMSE	Arithmetic	Additions
General cognitive functioning			
MMSE		.715	.751
Mathematics			
WAIS-R: Arithmetic	.715		.520
Mental Additions	.751	.520	
Executive functioning			
Similarities	.167	.197	.307
Trail Form B	−.492	−.315	−.148
Phonological fluency	.287	.113	.451
Attention			
Trail Form A	−.805	−.566 to.557	
Digits	.548	.580	.585
Language			
Boston Naming Test	.361	.413	.150
Reading Comprehension	.273	.009	.368
Sentence Repetition	.768	.600	.731
Semantic Fluency	.280	.342	.462
Visuoconstructive			
Rey–Osterrieth Figure: Copy	.354	.279	.307
WAIS-R: Block Design	.623	.431	.405
MEMORY			
WAIS-R: Information	.351	.486	.395
WMS: Logical Memory	.423	.393	.149
WMS: Visual Reproduction	.751	.727	.546

Marterer et al., 1996). Calculation abilities in Rosselli et al.'s research were assessed using a problem-solving test (Arithmetic subtest of the WAIS-R) and an arithmetical mental task (Consecutive Additions by 3s). Both tests presented strong correlations (.715 and .751, respectively; $p < .001$) with the MMSE scores. These very robust correlations with the MMSE may suggest that arithmetical abilities represent very good predictors of general cognitive performance in AD. Besides, correlations between arithmetical abilities and neuropsychological test performance were in general higher than correlation between the MMSE and neuropsychological test performance. Arithmetical ability tests may in consequence be considered to be even better predictors of general cognitive performance than the MMSE.

Deloche et al. (1995) found in patients with mild AD a correlation of .74 between total calculation and number processing scores, and the MMSE score. They observed that patients with calculation and number processing deficits also showed impaired language performance. By the same token, in the Rosselli et al. study, AD patients who performed significantly lower on arithmetical tests also did poorly on attention tests, sentence repetition, semantic verbal fluency, information, visual reproduction, block design, and line orientation tests. It is noteworthy that repetition is one of the language skills well preserved in patients with AD, and in consequence, repetition impairments are considered significantly abnormal (Benson and Ardila, 1996; Cummings and Benson, 1992). Repetition of sentences involves preserved verbal memory span. An important relationship between mathematical scores and short-term memory such as visual reproduction and immediate memory measured with the digit span subtest was also observed. Short-term visual memory (Visual Reproduction subtest from the Wechsler Memory Scale—Revised) had a stronger correlation than did short-term memory (Logical Memory from the Wechsler Memory Scale—Revised) with arithmetical ability tests and MMSE.

Mantovan et al. (1999) observed that patients with AD present significant difficulties in complex written calculation, but not so severe in the retrieval of arithmetical facts. Low consistency and high variability in the error types suggest that difficulties of patients with AD in complex calculation arise from a monitoring deficit and not from incomplete or distorted calculation algorithms. Deficits in monitoring procedures may be an early and common symptom of AD.

In summary, arithmetical ability tests seem to be excellent predictors of general cognitive performance. It may be even proposed that arithmetical ability disturbances should be included as an additional criterion of AD.

Gerstmann Syndrome

In 1940 Gerstmann described a syndrome associated to lesions in the left angular gyrus that characteristically included deficits of finger agnosia, right–left disorientation, agraphia, and acalculia (Gerstmann, 1940). Ever since, the existence of a Gerstmann syndrome has not been free of debate and questioning in the literature (Benton, 1977; Botez, 1985; Poeck and Orgass, 1966; Strub and Geschwind, 1983). In part this debate emerges because this syndrome usually unfolds either as an "incomplete" tetrad or in association to other cognitive deficits, particularly, aphasia, alexia, and perceptual disorders (Frederiks, 1985). The presence of Gerstmann syndrome, complete or incomplete, suggests a left posterior parietal damage and more specifically, a damage to the left angular gyrus. Even earlier the name "angular syndrome" was proposed in lieu of the more widely recognized "Gerstmann syndrome" (Strub and Geschwind, 1983). The appearance of a Gerstmann syndrome with electrical stimulation of the cerebral cortex in the posterior parietal area supports its angular localization (Morris et al., 1984). Mazzoni et al.

(1990) described a case of a "pure" Gerstmann syndrome associated with an angular gyrus traumatic damage.

According to Strub and Geschwind (1983), the angular localization in Gerstmann syndrome lesion would not have an extension toward the occipital lobe, as Gerstmann proposed, but rather toward the supramarginal gyrus and the inferior parietal gyrus. In this case the agraphia would correspond, then, to an apractic agraphia and not to an aphasic one, and consequently the agraphia is not necessarily be associated with alexia (Benson and Cummings, 1985). In fact, in cases of "incomplete" Gerstmann syndrome agraphia is usually the missing element, a possible reflection of the fact that apractic agraphia is not exactly angular, but inferior parietal instead. Furthermore, sometimes agraphia without alexia is observed, again as a result of an inferior parietal lesion.

Right–Left Orientation

All known languages distinguish different words to refer to right and left; all of them also include some other spatial relationship words, such as up and down (Greenberg, 1978; Hagége, 1982). Defects in understanding and using these words with a spatial meaning are observed in cases of left angular gyrus damage. Usually, right–left disorientation is mentioned in cases of Gerstmann syndrome. Nonetheless, these patients do not only present difficulties in recognizing right–left, but also other spatial relationships (e.g., up–down, over–below). Right–left disorientation is usually more evident, because it represents a relatively hard distinction. Even normal people sometimes confuse right and left. Right–left discrimination is also relatively late acquired during child language development. As a matter of fact, the difficulty observed in children during language development, as well as in Gerstmann syndrome, is not limited to discriminating right and left, but also other spatial relationships. Spatial concepts are strongly reflected in some language elements (e.g., place adverbs, prepositions). In contemporary languages the underlying spatial content of prepositions is evident (e.g., to, from, for). These spatial concepts mediated through language may be disrupted in cases of brain damage.

Pathogenesis of right–left disorientation is not completely understood. Patients with left posterior damage present more evident difficulties than do right posterior damaged patients (Ratcliff, 1979). Right–left disorientation implies difficulties in the application of spatial concepts in the body's lateral orientation.

Gold et al. (1995) observed a patient with Gerstmann syndrome whose ability to name or point to lateralized body parts using verbal labels "right" and "left" was not defective, but whose performance was always poor when mental rotation to a command was required. The authors suggested that a defect in horizontal translation, that is, mental rotation, accounted for the right–left disorientation in their patient. Furthermore, that acalculia and other signs associated with Gerstmann syndrome could also evolve from a deficit in the performance of these mental rotations. Similarly, this deficit in mental rotations could potentially be reflected in the impaired understanding of comparisons, for example, time and place adverbs, found in semantic aphasia. One could infer that a single underlying deficit, defective mental rotations, may account for right–left disorientation, finger agnosia, acalculia, and semantic aphasia. Their simultaneous appearance in a single clinical syndrome is not coincidental. Notwithstanding, agraphia would still remain unexplained by this unifying underlying mechanism. It is of interest and perhaps not surprising that it is precisely agraphia the missing sign in the Gerstmann syndrome.

Finger Agnosia

Finger agnosia, as initially described by Gerstmann in 1924 (Gerstmann, 1940), includes the inability to distinguish, name, or recognize the fingers not only in the own hands but also in examiner's hand or in a drawing of a hand. The patient presents difficulties to selectively move the fingers, both by verbal command and by imitation. Most evident errors are observed in the index, middle, and ring fingers. Usually the patient has difficulties recognizing his/her errors and consequently does not try to correct them. Later on, Gerstmann (1940) included finger agnosia, plus right–left disorientation, agraphia, and acalculia into a single syndrome. Interestingly, finger agnosia is associated with toe agnosia (Tucha et al., 1997).

Some authors have proposed that finger agnosia represents a mild form of autotopagnosia (e.g., Hécaen and Albert, 1978). However, it has been reported that autotopagnosia and finger agnosia can appear dissociated, and consequently would represent different defects (De Renzi and Scotti, 1970). Finger agnosia is a relatively frequent defect, whereas autotopagnosia represents a quite unusual syndrome. It has been proposed that finger agnosia might be a polymorphic phenomenon that includes apraxic, agnostic, and aphasic aspects. In consequence, different subtypes of finger agnosia can be distinguished: visual finger agnosia, finger constructional apraxia, apractic defects in finger selection, and finger aphasia (anomia) (Schilder and Stengel, 1931). The role of parietal lobe in body knowledge and the disorders of the body scheme in cases of parietal pathology have been particularly emphasized in the literature (e.g., Botez, 1985; Critchley, 1953). Parietal damage has been associated with asomatognosia in general, hemiasomatognosia, alloesthesia, finger agnosia, autotopagnosia, asymbolia for pain, apraxia, and the so-called Verger–Dejerine syndrome (Hécaen and Albert, 1978).

Historically, calculation abilities seem to develop from counting, and in child development this begins with the sequencing of fingers (correspondence construction) (Hitch et al., 1987). Finger nomination is usually sequenced in a particular order and this represents a basic procedure found in different cultures worldwide, both ancient and contemporary (Ardila, 1993; Cauty, 1984; Levy-Bruhl, 1947). In fact, in many contemporary languages a 10- or 20-based system is evident. From the Latin *digitus*, *digit* can mean both number and finger. Accordingly, a strong relationship between numerical knowledge and finger gnosis begins to become evident and some commonality in brain activity or anatomy can be expected. Right–left discrimination and finger gnosis are strongly interdependent and can even be interpreted as components of the autotopagnosia syndrome.

Semantic Aphasia

Commonly, investigators have reported the presence of Gerstmann syndrome without aphasia as one of its components (Roeltgen et al., 1983; Strub and Geschwind, 1983; Varney, 1984). However, the existence of a possible semantic aphasia has not been specifically explored nor ruled out (Ardila et al., 1989a). In 1926, Head described a language alteration whereby he defined an inability to recognize simultaneously the elements within a sentence, and he called it semantic aphasia. In the following years only a small group of researchers referred to this type of aphasia (e.g., Conrad, 1932; Goldstein, 1948; Zucker, 1934). Nearly 40 years later, Luria (1966, 1973, 1976) retook this concept and extensively analyzed it. Since then, however, only a few more authors have shown special interest in studying semantic aphasia, and just a handful of references have appeared in the literature over the last two decades (Ardila et al., 1989a; Benson and Ardila, 1996; Brown, 1972; Hier et al., 1980; Kertesz, 1979).

Luria (1973, 1976) considered that language deficiencies observed in semantic aphasia included the following: (1) sentences with a complex system of successive subordinate clauses; (2) reversible constructions, particularly of the temporal and spatial type; (3) constructions with double negative; (4) comparative sentences; (5) passive constructions; (6) constructions with transitive verbs; and (7) constructions with attributive relations. He also stated that these spatial disorders not only incidentally accompany semantic aphasia, but that semantic aphasia itself was a defect in the perception of simultaneous structures transferred to a higher symbolic level (Luria, 1976). In other words, patients with semantic aphasia have difficulty understanding the meaning of words tinged with spatial or quasi-spatial meaning.

There seems to exist a rationale for finding a common brain area and activity for acalculia, finger agnosia, right–left disorientation, and semantic aphasia. Ardila et al. (1989a, 2000) proposed to replace agraphia for semantic aphasia as a part of the angular gyrus syndrome; or simply to consider semantic aphasia as a fifth sign of the Gerstmann syndrome. Thus, Gerstmann (or angular) syndrome would include acalculia, finger agnosia (or a more extended autotopagnosia), right–left disorientation (and also difficulties with other spatial words), and semantic aphasia. Sometimes, agraphia without alexia will be observed, but agraphia would result from an inferior parietal lesion, not exactly from an angular pathology.

TESTING FOR ACALCULIA

Testing for calculation abilities in brain-damaged patients has four main purposes:

1. To find out abnormal difficulties in calculation tasks. If difficulties are severe enough, the diagnosis of acalculia may be proposed.
2. To distinguish the specific pattern of difficulties the patient presents, and the particular type of acalculia it corresponds to. Disturbances in different elements of the calculation system can be proposed.
3. To find out associated deficits. Acalculia is usually associated with diverse cognitive disturbances.
4. To describe the types of errors observed in the patient. This information will be particularly useful in developing rehabilitation procedures.

Despite the general agreement that a comprehensive cognitive evaluation should include testing for calculation abilities, there is a significant limitation in the available testing instruments. Lezak (1995) overtly states that “An assessment of cognitive functions that does not include an examination of calculation skills is incomplete” (p. 647). The WAIS-III Arithmetic subtest (Wechsler, 1997) is probably the most widely used instrument when testing for calculation abilities in neuropsychology. This subtest, however, has two significant limitations: (1) it is assessing just a single aspect in numerical processing (to mentally solve arithmetical problems) and (2) it is very difficult to administer to patients with language and memory defects.

A few models of testing for calculation abilities have been developed (Ardila and Rosselli, 1994; Deloche et al., 1994; Grafman et al., 1982; Harvey et al., 1993; Luria, 1966; Rosselli and Ardila, 1989; Warrington, 1982). However, norms regarding the effects of age, gender, and very special education are not easily available. Even the WAIS-III Arithmetic subtest does not control educational level.

There is, however, a kind of general agreement about how to test for calculation abilities. Testing for acalculia is rather similar everywhere. There are certain tasks that are supposed to be most important for targeting calculation abilities: reading and writing numbers, performing arithmetical operations, aligning numbers in columns, solving arithmetical problems, and so on. That is the way calculation abilities are tested in neuropsychology worldwide.

Lezak (1995) considers the following calculation ability tests as most frequently used in assessing numerical skills:

1. WAIS Arithmetic subtest.
2. Arithmetical problems. For example, those problems used by Luria (1976) in assessing frontal lobe pathology. A typical example is as follows: “There are 18 books in two shelves. In one shelf there is the double of books than in the other. How many books are there in each one?” There are different arithmetical problems included in psychological test batteries, such as the Stanford-Binet Intelligence Scale (Terman and Merril, 1973).
3. Wide Range Achievement Test (WRAT) Arithmetic subtest (Jastak and Wilkinson, 1984).
4. Woodcock–Johnson Psycho-Educational Battery—Revised (WJ-R) (Woodcock and Johnson, 1989).

The best-designed and almost only acalculia standardized neuropsychological test battery was developed by a group of European neuropsychologists headed by Deloche (Deloche et al., 1989). Preliminary normative results in 180 participants stratified by education, age, and gender were further available (Deloche et al., 1994). This battery was named as EC301.

In this chapter, a general description of the EC301 test battery will presented. Later, a model for testing calculation abilities in clinical neuropsychology will be proposed. In the final part of this paper, an analysis of the errors found in calculation abilities disturbances will be introduced.

EC301

Deloche et al. (1989, 1994) attempted to develop a calculation ability testing instruments, with norms by age, educational level, and gender. This instrument, the EC301 calculation ability test battery, is composed of 31 subtests that cover 8 compound arithmetical functions and 5 single functions. Up-to-dately, the EC301 can be considered as the best instrument in testing for calculation abilities. The following sections are included in the EC301:

1. *Counting* (three subtests).
   - 1.1. Spoken verbal counting (four items): by ones from 1 to 31, by tens from 10 to 90, by threes from 3 to 33, and counting backward by ones from 22 to 1.
   - 1.2. Digit counting requires counting by ones from one to 31 (1,2,3, . . . 31).
   - 1.3. Written verbal counting has two items: counting by ones from 1 to 16 (one, two, three . . . sixteen), and counting by tens from 10 to 90 (ten, twenty . . . ninety).
2. *Dot enumeration* (five subtests). In all subtests, responses have to be produced in Arabic digit form. In Subtests 1 and 2, items contain some dots and are either canonical (dominoes) or non-canonical patterns, organized in subgroups with no more than five dots. In Subtests 3, 4, and 5, participants have to point to the dots while counting. The arrangement of the dos varies according to spatial organization.
3. *Transcoding* (seven tasks). Considering that numbers can be represented in three different codes (spoken verbal, written verbal, and Arabic) the six possible transcoding plus repetition are included. Each subtest contains six items, and the structure of the items is similar (thousands, hundreds, tens, units).
4. *Arithmetical signs* (2 subtests).
   - 4.1. Reading aloud arithmetical signs.
   - 4.2. Writing from dictation.
5. *Magnitude comparison* (two subtests). Eight pairs of numbers are included in each subtest asking the participant to indicate which number is larger.
   - 5.1. Arabic digit form.
   - 5.2. Written verbal form.
6. *Mental calculation* (two subtests). The four basic operations are presented with two items of medium difficulty.
   - 6.1. Spoken verbal.
   - 6.2. Arabic digit condition.

7. *Calculation approximations* (1 subtest). The participant is required to estimate the result of different arithmetical operations. The response has to be selected from a multiple choice of four-number array.
8. *Placing numbers on an analogue line* (two subtests). Participants have to point to the place of a number among four alternatives (correct and three distractors) indicated by ticks on a vertical scale with 0–100 as marked end points. Five items are included in each subtest.
   8.1. Arabic digit form.
   8.2. Spoken verbal numbers.
9. *Writing down an operation* (one subtest). The ability to organize two numbers (two-digit and three-digit) spatially is tested. The participant is required to organize on a sheet of paper the numbers to perform the four basic arithmetical operations, but without the operation is actually done.
10. *Written calculation* (three subtests). There are two items in each subtest. The three subtests include addition, subtraction, and multiplication.
11. *Perceptual quantity estimation* (one subtest). Participants are shown six photographs of real objects or sets of elements. They have to estimate either weights, lengths, or number of elements.
12. *Contextual magnitude judgment* (one subtest). The task contains five items. Participants have to indicate of a number of objects or persons represent a small, medium, or large quantity, according to some context (e.g., 35 persons in a bus is a medium number of persons, whereas 20 pages in a letter has to be rated as a large letter, despite the fact that 35 is larger than 20).
13. *Numerical knowledge* (one subtest). There are six questions for each of which there is only one correct answer (such as the numbers of days of the week).

The EC301 is currently in normalization process in different countries.

A Proposed Model for Testing Calculation Abilities

We are proposing a model for testing calculation abilities, that is just and extension of the EC301 (Deloche et al., 1989, 1994). Global quantification (or numerosity perception) represents the most elementary quantification process. However, global quantification does not represent yet a truly numerical process, because it does not suppose a one-to-one correspondence. As a matter of fact, it is a purely visuoperceptual process, usually not included in testing for calculation abilities. Enumeration represents the most elementary type of truly numerical knowledge. Counting is a sophisticated form of enumeration: a unique number name is paired with each object in a collection, and the final number name that is used stands for the cardinal value of that collection. Testing for calculation abilities can begin at the counting level.

1. *Counting.*
   1.1. Counting real objects. A card with 10 randomly distributed dots can be used. The participant is required to count aloud the dots.
   1.2. Counting forward. To count from 11 to 20 can be used because this is usually the most irregular segment in different numerical systems.
   1.3. Counting backward. Even though counting forward is clearly verbal rote learning, counting backward does not represent a rote learning. Attention plays a significant role to count backward from 30 to 20.
2. Cardinality estimation corresponds to the task named as "Placing numbers on an analogue line" by Deloche et al. (1994). The participant has to point to the place of a number among four alternatives (correct and three distractors) indicated by ticks on a vertical scale with 0–100 as marked end points. Some five items can be included in each subtest.
   2.1. Arabic digit form (10, 75, 40, 9, 87).
   2.2. Spoken verbal numbers (eleven, ninety, twenty-five, sixty, thirty-five).
3. *Reading numbers.*
   3.1. Arabic digits. To read numbers with different levels of complexity: 3, 27, 298, 7,327, 10003.
   3.2. Roman numerals. Usually general population people can read Roman numeral up to 12, because they are frequently included in clocks. Roman numerals heavily rely on the spatial position III, IV, VI, XI, IX.
4. *Writing numbers.* Different levels of complexity can be used (7, 31, 106, 1,639, 50,002,). Quantities are dictated to the patient: one-digit numbers, two-digit numbers, numbers requiring the use of a positional value, and so on.
5. *Transcoding.*
   5.1. From a numerical to a verbal code. Numbers using a numerical code are written (e.g., 7, 23, 109, 9,231, 1,000,027), and the patient is asked to write them with

letters (e.g., seven, twenty-three, one hundred nine, etc.).

5.2. From a verbal to a numerical code. Numbers using a verbal code are written (e.g., three, forty-nine, three hundred seventy-six, nine thousand two hundred seventeen, seven millions seven hundred nine), and the patient is asked to write these quantities with numbers (e.g., 3, 49, 376, etc.).

6. *Reading and writing arithmetical signs.*

6.1. Reading arithmetical signs written on a card (+, −, : , ×, =). If the patient answers correctly, the question "And what does it mean?" can be presented.

6.2. Writing arithmetical signs by dictation (plus, minus, divided, by, equal).

7. *Numerical rote learning*. Usually it is accepted that multiplication tables represent a numerical rote learning, required for multiplications. Nonetheless, numerical rote learning includes as well adding and subtracting one-digit quantities, required in adding and subtracting. Numerical rote learning can be preserved in cases of parietal acalculia, but may be impaired in cases of subcortical damage and right hemisphere pathology.

7.1. Multiplication tables (3, 7, and 9).

7.2. Adding one-digit quantities (3 + 2, 5 + 3, 7 + 6, 9 + 4, 8 + 6).

7.3. Subtracting one-digit quantities (7 − 5, 5 − 2, 9 − 6, 8 − 5, 9 − 5).

8. *To complete an arithmetical operation.* This is a very traditional test in evaluating calculation abilities in patients with brain pathology. Different arithmetical operations are written on a card, but one number or arithmetical sign is missing. The patient must say what number or arithmetical sign is missing.

8.1. Numbers (15 + ... = 37; 15 × ... = 45; 12 − ... = 4 . . . /5 = 3).

8.2. Arithmetical signs (12 . . . 18 = 30; 17 . . . 6 = 11; 60 . . . 4 = 15; 9 . . . 6 = 54).

9. *Magnitude comparisons.* The patient is asked to judge which number is larger and which is smaller. An error in this task does not mean that the patient does not understand what number is larger. Numbers due to flaws in differentiating between tens, hundreds, and so on. Errors may also be due to the difficulties understanding the words "larger" and "smaller" (e.g., 24 − 42, 29 − 61, 103 − 97, 1,003 − 795, 428 − 294).

10. *Successive arithmetical operations.* Successive operations can be failed as a result of calculation difficulties, but also because of attention defects and perseveration. These successive arithmetical operations should be performed aloud.

10.1. Adding (e.g., 1, 4, 7 . . . 37).

10.2. Subtracting (100, 93, 86 . . . 16).

11. *Mental calculation.* Adding, subtracting, multiplying, and dividing two-digit quantities.

11.1. Adding: 12 + 25, 31 + 84, 27 + 16, 76 + 40, 89 + 36.

11.2. Subtracting: 35 − 14, 74 − 65, 44 − 29, 91 − 59, 70 − 39.

11.3. Multiplying: 14 × 3, 17 × 8, 29 × 3, 14 × 13, 27 × 12.

11.4. Dividing: 15/3, 75/5, 60/15, 89/16.

12. *Written calculation.* Adding, subtracting, multiplying, and dividing two- and three-digit quantities.

12.1. Adding: 398 + 724.

12.2. Subtracting: 721 – 536.

12.3. Multiplying: 127 × 89.

12.4. Dividing: 465/27.

13. *Aligning numbers in columns.* A series of numbers (e.g., 27, 2, 2,407, 12,057, 1,421,967) are dictated to the patient. The patient is required to place them in a column, as for adding.

14. *Arithmetical operations using a different numerical base.* Arithmetical operations generally use a decimal base. There is, however, a type of everyday numerical knowledge that does not use the decimal system, namely, time measures. Hours use a 12-base and minutes and seconds a 60-base. Potentially, this is an excellent calculation ability test. Sometimes, however, this task may appear hard for normal people ("How long time is there between 8:30 and 11:15?" "How long time is there between 10:45 AM and 2:10 PM?" "How long time is there between 3:55 and 7:10?").

15. *Fractions.* The use of fractions represents a relatively difficult numerical ability. The numerical value of fraction is inverse to the absolute number value (i.e., one sixth is larger than one seventh). Adding and subtracting fractions may be difficult even for normal people.

15.1. To compare fractions. ("What is larger between one-half and one-third?"; "one-third and one-fourth?"; "one-seventh and one-eighth?").

15.2. Adding and subtracting ("How much is one-half plus one-half?"; "How much is one-half plus one-third?"; "How much is one-fifth plus one-third?").

16. *Digit span.* It has been demonstrated that patients with acalculia have significant difficulties

memorizing digits. In cases of brain pathology, some specific types of amnesia are observed: patients with aphasia have difficulties memorizing words, patients with apraxia have difficulties memorizing sequences of movements, and patients with acalculia have difficulties memorizing numbers. For the acalculic patient, numbers are confusing and the meaning of digits is weak. In consequence, digit repetition represents a good task to test for numerical representation.

16.1. Digits forward. (Series of digits are presented at 1-s interval. The initial series has three digits –4, 2, 7. If the patient fails, a different 3-digit series is presented –9, 3, 6. If the patient repeats it errorless, a 4-digit series is presented. When the patient fails two consecutive times in repeating a list with certain number of digits, the test is stopped.)

16.2. Digits backward. (Beginning with a 2-digit series, the patient is required to repeat it backward. Two opportunities are allowed for each series. When the patient fails two consecutive times a list with a certain number of digits, the test is stopped.)

17. *General numerical knowledge. (*e.g., "How many days are there in one week?"; "How many weeks are there in one year?"; "At what temperature the water boils?"; "How many continents are there?"; "What is the population in this country?")
18. *Personal numerical knowledge.* Everybody uses a diversity of personal numerical knowledge that can include the own phone number, address, personal identification, car tag, social security number, and so on. Ask about three items (e.g., phone number, address, social security number).
19. *Quantity estimation.* To estimate the weights, lengths, or number of elements. Interestingly, this is a task that presents a significant dispersion in scores when administered to normal population people. ("How much an egg weights?"; "How long an average car is?"; "How many people can normally travel in a bus?").
20. *Time estimation.* Time estimation is an ability not usually including in testing for acalculia. Some brain-damaged patients, particularly patients with frontal lobe pathology, may present very significant difficulties estimating time. ["How long time ago did the WWII end?"; "How long time ago was the discovery of America?"; "How long time ago the current president was elected?"; "How long a regular movie lasts?"; "How long time does it take to walk one block (100 m) at a regular walking speed?"].
21. *Magnitude estimation.* The relative value of numbers depends upon the context. Ask the patient to state if an object or a person represents a "too much" or "too little" quantity according to a specific semantic context. (e.g., "A book with 20 pages, is it too much or too little?"; "The weight of an adult person is 70 pounds, is it too much or too little?"; "One hundred people in a bus, is it too much or too little?"; "To spend two hours brushing the teeth, is it too much or too little?").
22. *Numerical problems.* Problems requiring the use of one or several numerical operations are presented to the patient (e.g., "How many oranges are there in two and half dozens?"; "John had 12 dollars, received 9 and spent 14. How much money does he have now?"; "Mary and John get 150 in a day. Mary receives the double than John. How much each one receives?"; "The sum of the ages of a father and a son is 48. If the father has the triple of age of the son, how old are each one of them?"; "There are 18 books in two shelves; in one of them there is double number than in the other. How many books are there in each one?").
23. *Using money.* Performing arithmetical operations with money represents one of the most important everyday uses of arithmetical operations. Low-educated people and elderly people with mild dementia can do better performing arithmetical operations with money than in abstract. Hence, the use of money may represent a potentially useful calculation test. Different bills and coins can be provided to the participant (e.g., five $1 bills, two $5 bills, one $10 bill, and one $20 bill; five pennies, five nickels, three dimes, and three quarters). The patient can be required to select certain quantity ("Give me 17 dollars and 77 cents," "give me 23 dollars and 18 cents," "give me 39 dollars and 94 cents").

Table 15 summarizes the different tests and subtests that can be used when testing for calculation abilities.

Analysis of Errors

A diversity of errors potentially can be observed in performing calculation tasks (Table 16). Nonetheless, six major errors groups can be separated: (1) errors in digits, (2) errors in "carrying over," (3) borrowing errors, (4) errors in basic principles, (5) errors in algorithms, and (6) errors in arithmetical symbols.

Table 15. Summary of the Areas that Potentially Can Be Included in a Test for Calculation Abilities

1. Counting
   1.1. Counting real objects
   1.2. Counting forward
   1.3. Counting backward
2. Cardinality estimation
   2.1. Arabic digit form
   2.2. Spoken verbal numbers
3. Reading numbers
   3.1. Arabic digits
   3.2. Roman numerals
4. Writing numbers
5. Transcoding
   5.1. From a numerical to a verbal code
   5.2. From a verbal to a numerical code
6. Reading and writing arithmetical signs
   6.1. Reading arithmetical signs
   6.2. Writing arithmetical signs
7. Numerical rote learning
   7.1. Multiplication tables
   7.2. Adding one-digit quantities
   7.3. Subtracting one-digit quantities
8. To complete an arithmetical operation
   8.1. Numbers
   8.2. Arithmetical signs
9. Magnitude comparisons
10. Successive arithmetical operations
    10.1. Adding
    10.2. Subtracting
11. Mental calculation
    11.1. Adding
    11.2. Subtracting
    11.3. Multiplying
    11.4. Dividing
12. Written calculation
    12.1. Adding
    12.2. Subtracting
    12.3. Multiplying
    12.4. Dividing
13. Aligning numbers in columns
14. Arithmetical operations using a different numerical base
15. Fractions
    15.1. To compare fractions
    15.2. Adding and subtracting
16. Digit span
    16.1. Digits forward
    16.2. Digits backward
17. General numerical knowledge
18. Personal numerical knowledge
19. Quantity estimation
20. Time estimation
21. Magnitude estimation
22. Numerical problems
23. Using money

Errors in Digits

Errors in digits represent a fundamental type of error in cases of acalculia. The patient may have difficulties deciding the larger of two numbers (magnitude comparison). This type of error may result in impairments in appreciating the tens, hundreds, and so on, or difficulties understanding and confusions between "larger" and "smaller." This defect in magnitude comparison is observed in cases of primary acalculia.

Patients with Wernicke's aphasia frequently present lexicalization errors: Two hundred fifty is written 20050. This type of errors is observed in writing. In reading, a frequent type of errors observed in these patients, but also in patients with pure (occipital) alexia is decomposition. The number 15 is read as "one five" and the number 537 is read "fifty-three seven." In pure alexia this situation can result in a digit-by-digit reading. Inversion errors (12 is read or written as 21) is observed in spatial acalculias, but also can be found in fluent aphasia.

Substitution errors in oral language, writing, or reading simply mean that a number is substituted by a different one. The errors refer to the number hierarchy (e.g., 30 becomes 3), order (5 becomes 6), or the so-called "stack" errors (e.g., 15 becomes 50).

Omissions and additions are not unusual types of errors in cases of brain pathology. Omissions can be observed right-sided (most frequently in pure alexia) or at the left (most frequently in left hemi-spatial neglect). Additions are found in digits (23 becomes 233) but also in strokes when writing a digit, most typically, the number 3 is written with extra loops.

In transcoding tasks (from numerical to verbal or from verbal to numerical) different types of errors may be found, including decomposition, additions, omissions, and substitutions. Transcoding is a relatively difficult calculation task.

Errors in "Carrying Over"

Primary difficulties in "carrying over," that is, inability to understand the calculation principle for "carrying over" is observed in anarithmetia. Nonetheless, most patients with calculation defects may fail in carrying because of language difficulties, spatial difficulties, and so on. In spatial acalculia, difficulties in correctly placing the carried over quantity represent the most important difficulty. The patient knows what should be carried over but does not know where to place it.

Borrowing Errors

Correctly borrowing supposes an understanding of the arithmetical rules of permutability. Evidently, in anarithmetia, not only carrying over, but also borrowing

Table 16. Classification of the Types of Errors in Calculation Tests (Adapted from Ardila and Rosselli, 1994; Levin et al., 1993; Rosselli and Ardila, 1989, 1997 Spiers, 1987)

Type of error	Description	Type of acalculia
Errors in digits		
Number value	Inability to distinguish the larger of two numbers because of flaws in differentiating between tens, hundreds, etc. or defects in understanding "larger," "smaller"	Anarithmetia
Lexicalization (Wernickes)	Numbers are written as they sound without integrating tens, hundreds, etc.	Aphasic
Decomposition	Numbers are read without considering the number as a whole	Anarithmetia; aphasic (Wernickes); alexic (pure)
Inversions	Numbers are copied, written, and repeated in the reverse order. This rotation can be partial	Spatial posterior aphasias
Substitutions	A number is substituted by another due to paralexia, paraphasia, or paragraphia, affecting the result of an operation	
	1. Hierarchy errors: The number is substituted by another from a different position of series (e.g., 5 → 50)	Posterior aphasics
	2. Order errors: The substitution is by another from the same series (e.g., 5 → 6)	Aphasia (posterior and anterior); anarithmetia
	3. Stack errors (e.g., 14 → 40)	Aphasic; anarithmetia
Omission	One or several digits are omitted (can be on the left or right side)	Spatial; pure alexia
Additions	A digit is inappropriately repeated upon writing it. Addition of traits to a digit (usually 3)	Spatial
	Addition of numbers previously presented can be a perseveration	Frontal
Errors in Transcoding	When numbers are passed from one code to another code (numerical to verbal or vice versa), decomposition, order, omission, or addition errors are noted	Aphasic; anarithmetia
Errors in "carrying over"		
Omission	The patient does not "carry over"	Anarithmetia
Incorrect "carrying over"	Any "carrying-over" error with acalculia	All acalculias
Incorrect placement	"Carries over" correctly but adds in the wrong column	Spatial
Borrowing errors		
Borrowing zero	Difficulties or confusion if the arithmetical operation has a zero	Anarithmetia
Borrowing	The last digit on the left is not reduced despite verbalizing the loan	Frontal acalculia
Defective borrowing	Adding the borrowed quantity incorrectly; borrowing unnecessarily	Anarithmetia
Errors in basic principles		
Multiplication tables	Incorrect recall; usually the patient tries to correct it with additions in series	Spatial
Zero	Fundamental errors are observed when a zero is present	Anarithmetia
Errors in Algorithms		
Incomplete	Initiates operation correctly but is not capable of finishing	Frontal
Spatial	1. Inappropriate use of space on the paper limiting the correct response	Spatial
	2. Inappropriate use of columns in arithmetical operations	Spatial
Incorrect sequence	Initiates the operation from left to right	Anarithmetia
Inappropriate algorithms	Numbers are organized spatially on the page for a different operation than the desired one (e.g., in multiplication, the numbers are placed to be divided)	Anarithmetia; Spatial
Mixed procedures	Different operations are used in the same problem (e.g., Adds in one column not multiplies in another)	Spatial
Reasoning errors	The subject does not realize that the result is impossible (e.g., The result is larger than what was subtracted)	Spatial
Errors in symbols		
Forgetting	The patient cannot remember nor write the four arithmetical signs	Anarithmetia
Substitution	The sign is substituted by another that is different from what was asked	Spatial Acalculia

is defective. It is not infrequent that patients with frontal lobe pathology omit to borrow. To borrow and to carry over, suppose a normal working memory, an internal processing of the numerical information, and an appropriate level of attention.

Errors in Basic Principles

There are certain basic principles that have to be used all the time to successfully perform an arithmetical task. Patients with right-hemisphere damage associated with spatial acalculia quite frequently have defects in recalling the multiplication tables. This defect is also observed in some cases of subcortical pathology. Using the zero represents a relatively complex numerical concept. Abnormalities in using the zero are found in primary acalculia. Some other patients can also fail when the zero is present. For example, in spatial acalculia the patient can get confused in that there are several zeros (e.g., 100003), but they also have difficulties when any digit is repeated several times (e.g., 122225).

Errors in Algorithms

Patients with frontal damage typically have difficulties finishing an arithmetical operation or solving a problem, even though they can begin to perform the arithmetical operation or solving the arithmetical problem. This difficulty in ending a task, as a matter of fact, is observed in different areas. For instance, when writing, the patient can begin to write, but does not end the writing.

In cases of spatial defects, an inappropriate use of the space on the paper limits the possibility of obtaining a correct response in solving an arithmetical task. Patients with right-hemisphere pathology and spatial acalculia can present as well difficulties in spatially organizing the numbers for performing a particular operation. The distribution of the numbers is different, for example, when multiplying and dividing. An abnormal spatial distribution of the quantities impedes a successful performance.

Arithmetical operations follow a sequence from right to left, that is the opposite sequence to that used in writing and reading. This is a fundamental principle when performing an arithmetical operation. Errors in the directionality are associated with misunderstanding of the positional value of units, tens, hundreds, and inability to understand the whole numerical system. This type of directionality error is found in primary acalculia.

There is a very interesting type of error sometimes found in cases of right hemisphere pathology. When subtracting, for example, the result is larger than the minuend. This is simply impossible and absurd, and this type of error is known as "reasoning error." The reason for this type of error is unclear.

Errors in Arithmetical Signs

Errors in reading and using arithmetical signs are associated with anarithmetia. For even some types it may be the major manifestation of primary acalculia. Patients with a spatial type of acalculia may confuse the directionality of the "plus" (+) and "times" (×) arithmetical signs. This confusion will obviously result in a failure in performing the correct arithmetical operation.

REHABILITATION OF CALCULATION DISORDERS

The majority of brain-injured patients (especially in vascular and traumatic injury cases) present some spontaneous cognitive recovery during the first months after the injury. Afterwards, the spontaneous recovery curve becomes slower and requires the implementation of rehabilitation programs to achieve some additional improvement (Lomas and Kertesz, 1978).

Two strategies have been proposed to explain the rehabilitation of cognitive difficulties: the reactivation of the lost cognitive function and the development of an alternative strategy that achieves the same result through an *alternative* cognitive procedure. The majority of the rehabilitation models developed for aphasias, alexias, and agraphias have emphasized the second strategy, which implies a cognitive reorganization (Seron et al., 1992) or a "functional system" reorganization (Luria, 1973). A solid model applicable to calculation rehabilitation still does not exist. The rehabilitation of calculation abilities is frequently neglected in the neuropsychology literature and ignored in most rehabilitation programs. Acalculia is usually evaluated and rehabilitated as a language-dependent function. There are few investigative efforts directed at studying the rehabilitation of calculation deficits.

The remediation of arithmetical facts has been reported using a case study design (McCloskey et al., 1991b; Girelli et al., 1996). The patients in both studies presented a selective multiplication deficit. McCloskey et al. (1991b) reported a selective training effect for repeated multiplication problems that extended to the problems' complements but did not improve performance in general (e.g., the training of 3×6 would improve 6×3 but not 3×8). Girelli et al. (1996) described two acalculic patients who totally lost multiplication facts. After the remediation procedure both patients presented a decrease in the error rate of trained and untrained multiplication problems. The performance in the trained set, however, improved more than the performance in the untrained one. In the course of the training the error types changed drastically from unsystematic errors to errors clearly related to the correct answer.

The cognitive model of number processing has also been used in the development of a training program in a 13-year-old with a severe difficulty in number-transcoding tasks (Sullivan, 1996). The transcoding of numbers from written verbal to Arabic and from spoken verbal to Arabic was impaired whereas numeral comprehension was preserved. After the training program the participant displayed significant gains in the Arabic transcoding tasks. The lack of consistency in the type of syntactic errors made by the patient and the control participants made the authors unable to interpret the generalization of their results in terms of the cognitive model used.

A good neuropsychological assessment is the first step in the rehabilitation of acalculia. A test battery developed specifically to evaluate acalculia should be used to analyze the disturbances of calculation in a patient with brain injury. Variables such as the patient's educational level and occupational activity should be carefully considered. Once the presence of acalculia is determined, a quantitative and qualitative analysis of the patient's errors in different sections of the test battery should be performed. The justification of the rehabilitation plan should be based on the limitation that acalculia has in the patient's occupational and social life. The patient should preserve sufficient cognitive capability that will allow the development of a new compensatory behavior or an alternative strategy (De Partz, 1986; Seron, 1984).

In the following sections, techniques that have been developed to rehabilitate patients with primary and secondary acalculias are presented. The majority of the methods described have been implemented in individual cases. Until now, no study exists that evaluates the effectiveness of these techniques in large samples of patients.

Primary Acalculia Rehabilitation

Primary acalculia or anarithmetia is associated with parietal or parietal–occipital injuries (Ardila and Rosselli, 1992). Tsvetkova (1996) considers that underlying primary acalculia is an alteration in the spatial perception and representation of numbers along with defects in verbal organization of spatial perception. The alteration in spatial coordination systems constitutes a central underlying defect in this type of acalculia (Luria, 1973; Tsvetkova, 1996). These patients present defects in numerical concepts, in understanding number positions, and in the performance of arithmetical procedural sequences and often make mistakes in recognizing arithmetical symbols.

Patients who have primary acalculia combined with semantic aphasia (and, according to some authors, this combination is constant) (Ardila, 1993; Ardila et al., 1989a; Luria, 1973), the comprehension defects extend themselves to logic–grammar relationships in language. In cases of semantic aphasia, numbers lose their relationship with the conceptual system and are perceived in a concrete and isolated manner (Tsvetkova, 1996). These patients present numerous errors in the "larger than" and "smaller than" tests, perceiving the number 86 as larger than 112, because they consider the independent value of each number. Although the concrete denomination of digits (reading digits) is preserved, it is impossible for them to use abstract numerical concepts. They are unable to recognize the number of tens and hundreds included in the number (e.g., in 800) or of understanding the content of relationships such as $30 = 10 + 10 + 10$. Accordingly, Tsvetkova proposes a structured rehabilitation plan for anarithmetia aimed at recovering the understanding of the composition of numbers and their positional value. Initially, the patient relearns the concept of numbers by performing tasks consisting of putting together real objects (tokens or sticks) and illustrations that contain the corresponding number. The tasks consist of dividing the objects into groups (initially, alike, and afterwards, different), counting the number of objects in each group, finding the illustration that represents the corresponding number, placing it in each group, deciding how many of these numbers are found in the given amount, and finally, writing the number on a sheet of paper.

Once the patient has reacquired the concept of digits and tens, he/she moves on to developing the concept of numerical composition, interrelationships between numbers, and the possibility of operating with them. Then, numerical denomination exercises (beginning with the second tenth) and comprehension exercises between the name of the number and its position are initiated. The patient begins to understand that the name of the number indicates its positional value and the left-to-right reading indicates to him/her a decreasing positional value (e.g., 154: one hundred and fifty-four). During this time, the positional composition of the numbers and their quantitative significance depending on their place in the series is worked on. Tsvetkova emphasizes the importance of using concrete mediators like tokens. When the patient also presents anomia for numbers, one should work on the reestablishment of the naming of numbers.

The understanding of numbers constitutes a recurring learning process to relearn the arithmetical operations. This relearning should always be initiated in the most explicit and concrete way possible, using external aides such as outlines, drawings, and so on. Verbalizing aloud the steps that should be followed is generally useful. As patients improve, they go from speaking aloud to murmuring, then to "speaking to themselves." Training for particular problems (i.e., 9×0) can lead to the recuperation of arithmetical rules ($n \times 0 = 0$) (McCloskey et al., 1991a). The training for particular problems or operations within the rehabilitation sessions leads then to the comprehension of arithmetical principles and rules (Girelli et al., 1996).

Understanding calculation direction should be worked on simultaneously with the relearning of arithmetical operations. If the patient with anarithmetia also presents aphasic problems, language should be rehabilitated first. Calculation rehabilitation can be implemented once an appropriate level of linguistic comprehension and production is achieved (Tsvetkova, 1996).

In conclusion, the patient with anarithmetia should relearn the basic concepts that underlie the numerical system. These basic concepts range from knowing numbers to handling them within the system of operations.

Rehabilitation of Secondary Acalculias

In patients with difficulties in recognizing numbers as a result of a perceptual deficit, the rehabilitation process is directed at the recovery of steadfastness and the generalization of visual perception. These patients frequently present low scores on reading numbers tests and on transcoding numbers from one code to another, with numerous rotation errors. (Rosselli and Ardila, 1989). The visual–perceptual difficulties affect the execution of written numbers tasks, in contrast to an adequate performance of mental arithmetical operations. When writing is preserved in these patients (as is the case of alexia without agraphia), writing numbers in the air helps their recognition.

Tsvetkova (1996) proposes using the "number reconstruction" technique with these patients. The technique includes number reconstruction by starting from certain visual elements (e.g., completing eight, starting from the number 3), looking for certain elements within a number (e.g., looking for the number 1 in the number 4), and finally, performing a verbal analysis of the similarities and differences that can be observed between numbers. At the same time that the "number reconstruction" technique is used, spatial orientation exercises, comprehension of the right-to-left relationship, and visual analysis of geometrical objects and forms should be developed.

Patients with alexia without agraphia generally present spatial integration difficulties (simultanagnosia) and inaccuracy in visual motor coordination (optic ataxia). Treatment should then include exercises that permit spatial analysis and visual motor ability training. Rehabilitation tasks are implemented following a program that progressively increases difficulty, beginning with simple movements designed for reaching for or indicating objects followed by copying figures in two dimensions, and concluding with the construction of three-dimensional figures (Sohlberg and Mateer, 1989). The training in the reproduction of designs of different forms, colors, and sizes can be initiated with aides. For instance, the patient is asked to finish a design already started until he/she can finally perform the task completely and independently (Ben-Yishay, 1983). Sohlberg and Mateer (1989) propose, as a procedure to evaluate the generalization of the task, obtaining a base line over the performance of 10 designs, noting the accuracy, time of execution, and the number of aides required. The therapist can choose 5 out of 10 designs for training. When the execution desired from these five designs is achieved, the performance in the five unused designs during training is evaluated with the goal of observing the effects of training. This generalization should be looked for in untrained visual motor tasks that require the same underlying skill (Gouvier and Warner, 1987). When a visual search defect (ocular apraxia) exists, visual pursuit tasks may help to compensate.

Rosselli and Ardila (1996) describe the writing and reading rehabilitation of a patient with Balint's syndrome, with severe ocular apraxia. They used visual movement exercises such as (1) demonstrating the visual pursuit of objects; (2) placing the index fingers at a distance of 15 cm from the sides of the face and requiring the patient to look toward the left and right index fingers 10 times consecutively, and practicing convergence exercises; and (3) from a central point at a distance of 30 cm, the patient must bring the right or left index finger toward his/her nose, permanently maintaining visual contact. In addition, visual–kinesthetic exercises were included in the rehabilitation plan; the patient was shown letters he had to reproduce in the air, and, later, he had to say the name of the letters. Likewise, when following words, the patient should simultaneously perform the movements of writing these words. In place of letters, numbers may be used. Within the visual searching exercises described by Rosselli and Ardila (1996), looking for words and letters in letter groups that progressively become more complex was included. Time and precision were recorded.

Patients with aphasic acalculia that receive therapy for their oral disorder usually improve significantly and in parallel fashion with the improvement of the calculation disturbance (Basso, 1987). Acalculia rehabilitation in these patients parallels language rehabilitation using denomination techniques, auditory verbal memory techniques, and semantic conceptual classification techniques. When acalculia is fundamentally derived from defects in phonological discrimination, prominent errors in oral numerical tasks are found. Therefore, within the rehabilitation program, visual stimuli should be used initially (Tsvetkova, 1996).

Patients with frontal lesions generally present perseverations and attention difficulties that prevent an adequate performance on calculation tests. These patients usually do not present errors in naming or recognizing numbers. Tsvetkova (1996) proposes the idea of providing control strategies to patients that will allow them to direct their attention and reduce perseveration. These control strategies refer to descriptions of the steps that the patient should follow to satisfactorily complete the task. When faced with the problem of forming the number 12, by starting from other numbers, the following steps can be described to the patient: (1) forming the number 12 by starting from other numbers with the help of addition, the patient is asked to use the maximum number of combinations; (2) achieving the same number by starting from other numbers with the help of subtraction, the patient is asked to use the maximum number of combinations; and (3) achieving the same number starting from other numbers with the help of multiplication. The maximum number of combinations should be used. The patient is trained to verbalize and follow the necessary steps. Because these patients do not generally present defects in mathematical procedures, it is not necessary to provide them with special instructions for the execution of each operation.

The use of permanent verbalization is a useful technique with patients with visual–perceptual difficulties. Spatial acalculia is associated with hemi-inattention (unilateral spatial neglect), which can be observed in right as well as left injuries (Rosselli et al., 1986). Although it is notoriously more frequent and severe in cases of right brain injury, unilateral spatial neglect or hemi-inattention refers to the inability to respond (attend to stimuli) presented in the contralateral visual field to the brain injury. These patients tend to present number omissions on the opposite side of the brain lesion. The hemi-spatial neglect constitutes one of the factors that interferes most with an adequate cognitive recovery. Although hemi-spatial neglect is frequently associated with hemianopsia (visual loss in the contralateral field to the lesion), it should be evaluated independently. Frequently, cancellation tasks, copies of drawings, visual search tasks, bisection of a line, and a drawing of clock are used, as well as tasks that help to overcome the neglect. On the basis of the hypothesis that patients with unilateral spatial neglect present difficulties in adequately exploring their environment, several rehabilitation programs have been directed to develop this ability (Weinberg et al., 1977). Within the rehabilitation techniques for hemi-inattention during reading, the following is discussed: (1) placing a vertical line on the left margin of the paragraph to be read, and (2) numbering the beginning and end of each line. As the treatment advances, the clues are eliminated until the patient is finally capable of reading without help. Upon diminishing the hemi-spatial neglect in general, spatial defects in reading diminish simultaneously (Ardila and Rosselli, 1992). In the recovery of spatial agraphia, it has been suggested using sheets of lined paper, which limit the writing space. It is also suggested to draw vertical lines that mark spaces between letters and words.

Rosselli and Ardila (1996) describe the rehabilitation of a 58-year-old woman with spatial alexia, agraphia, and acalculia associated to a vascular injury in the right hemisphere. The rehabilitation process was based in the rehabilitation of unilateral spatial neglect and associated spatial difficulties. The patient could adequately perform oral calculations but was completely incapable of performing written arithmetical operations with numbers composed of two or more digits. In a special test of written arithmetical operations (additions, subtractions, multiplications, and divisions), an initial score of 0/20 was obtained. Left hemi-inattention, a mixing up of procedures and the impossibility of adequately orienting the columns were observed. The rehabilitation techniques implemented included the following:

1. Using short paragraphs with a red vertical line placed on the left margin and with the lines numbered on the left and right sides, the patient, using her index finger, had to look for the numbers corresponding to each line. The clues (vertical line and numbers) were progressively eliminated.
2. In a text with no more than 12 lines, the patient had to complete the missing letters (i.e., perform sequential and ordered spatial exploration).
3. Letter cancellation exercises were repeated constantly, and clues to facilitate their execution were included. Time and precision were recorded.
4. In spontaneous writing exercises using lined paper with a thick colored line in the left margin, the patient had to look for the vertical line when finishing each line. Later, the line was eliminated, but the patient had to verbalize (initially aloud and later to herself) and explore to the extreme left before beginning to read the next line.
5. To facilitate the relearning of numbers through dictation, squares were used to place the

numbers in space, and the concepts of hierarchy were practiced permanently (units, tens, hundreds, etc.).

6. To provide training in arithmetical operations, she was given in writing additions, subtractions, multiplications, and divisions with digits separated in columns by thick colored lines and the tops of the columns were numbered (from right to left). The patient had to verbalize the arithmetical procedures and, with her right index finger, look for the left margin before she could pass to the next column. Later, the patient herself would write the operations she was dictated.

The techniques described previously were proven useful 8 months after the treatment was started. The patient presented significant improvement but in no way a complete recovery.

Rehabilitation of Developmental Dyscalculia

Strang and Rourke (1985) recommend that the remedial programs for children with dyscalculia include, when possible, systematic and concrete verbalizations of the operations and arithmetical procedures. The operations that involve mechanical arithmetic should be converted into verbal tasks that permit the child to take apart the operations, and, in this way, facilitate his/her learning. The teaching method should be clear, concrete, precise, and systematic.

Once the child has developed an adequate recognition of the numbers, one should begin to work with calculation difficulties. Initially, one should choose an arithmetical operation that presents a problem, and describe it verbally in such a way that the child can repeat the description independently of whether or not he/she understands the underlying mathematical concept. Later, the child should verbalize the steps that should be followed to perform the operation in question (e.g., Step 1, name the mathematical sign; Step 2, direct eyes toward the right, etc.). Once the different steps have been verbalized, the child should write them and repeat them orally as many times as needed. Then, the instructor should use concrete aides (table, equipment, and places) to explain the mathematical concept.

Squared sheets of paper should always be used. At times, the use of colors helps the discrimination of the right–left. Each time the child is presented with an arithmetical problem, it should be read out to minimize the possibility of his/her forgetting visual details. It is useful to have a pocked calculator at hand so that the child can revise the results of the operations. It is very important that the instructor record all errors committed by the child, with the purpose of analyzing the cognitive processing steps that have problems (Rosselli, 1992).

Counting by tens starting at a number other than 10 is the first step in developing a technique that utilizes the basic structure of the decimal system in teaching addition (Neibart, 1985). In children with dyscalculia, the training to count by tens should start at 10 (10, 20, 30 . . .) and after several repetitions move to another number (3, 13, 23). According to Neibart, the use of block is important for the child to “discover” the concept that is then internalized. Once the student has mastered counting by 10 at any number, the addition of 10 ($23 + 10$) is developed.

Sullivan (1996) developed a training program for a child with transcoding number difficulties. He was unable to transcribe a number given in written verbal or in spoken verbal (six hundred forty thousand sixty-four) into an Arabic code (640, 064). The training program consisted of introducing a syntactic frame (H = hundreds; T = tens, O = ones) for Arabic numeral production. It was shown how to use the syntactic frame. For example, for the numeral six hundred eighty-seven, he was told that because the number contains a hundred quantity the frame must have three slot. He was then shown how to fill the frame with the Arabic number (6/H, 8/T, 7/0). After he had understood how to fill a syntactic frame successfully, he was able to generate the syntactic frames on his own. After the training he showed a significant improvement in transcoding number tasks.

The child’s parents should be taught about the learning strategy that is being used so that in family activities (e.g., shopping), they practice the same arithmetical activities, and the generalization of the remedial program can be promoted. Because these children with DD present attention and visual–perceptual discrimination difficulties, it is convenient to involve the child in tasks that require a detailed description of visual stimuli. In addition, one should work on the parts in an organized manner. The difficulties in the interpretation of social situations can be improved by creating artificial situations (e.g., movies, photographs, etc.) so that the child can interpret the images and the context. The instructor can give clues to perceive the fictional circumstances and the meanings of the gestures (Ozols and Rourke, 1985).

The prognosis of DD depends on variables such as the severity of the disorder, the degree of the child’s deficiency in the execution of neuropsychological tests, the promptness of the initiation treatment, and the collaboration of the parents in the remedial program.

CONCLUSION

Calculation ability represents a very complex type of cognition, including linguistic (oral and written), spatial, memory, body knowledge, and executive function abilities. Considering its complexity, it is not surprising to find how frequent it is impaired in cases of focal brain pathology and dementia. Neuropsychology has usually recognized the importance of developmental and acquired calculation disturbances. However, there is some paucity in the research devoted to the neuropsychological analysis of calculation disturbances. The difficulty in finding cases of isolated acalculia may have played some role in the relative paucity of research in this area.

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