

Sequence analytical and vector geometry at teaching of solid geometry at secondary school.

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Resumé

Problémom v súvislosti s vyučováním matematiky na strednej škole je náväznost obsahu vyučovania matematiky a obsahu vyučovania zvyšných predmetov. Preto v článku chcem venovat pozornosť predovšetkým vzťahu stereometrie z pohľadu vektorového počtu a syntetickému prístupu k vyučovaniu stereometrie. Závery uskutočneného experimentu mi pomôžu predovšetkým upresniť formuláciu úlohy, ale aj niektoré metodické aktivity priebehu experimentu.

Abstract

The problem of teaching mathematics at secondary school is the continuity of mathematics content and the content of other subjects. The abstract mostly focuses on the relation between solid geometry from the vector calculus point of view and the synthetic approach to teaching of solid geometry. Findings from the experiment will enable I have carried out to specify the formulation of the tasks but also some methodical activities of the running experiment.

Résumé

Dans l'enseignement des mathématiques au secondaire, le problème d'articulation entre les différents chapitres se posent avec acuité. L'article propose l'articulation pour ce qui est de la géométrie dans l'espace. L'expérience réalisée dans un lycée a beaucoup contribué dans la formulation du texte des exercices et dans la méthode des expériences proposée.

Riassunto

Nell'insegnamento delle matematiche nelle scuole secondarie vi é un grande problema che tocca il concatenamento dell'insegnamento delle differenti parti interne delle matematiche. Questo articolo é dedicato alla geometria dello spazio, dalla parte del calcolo vettoriale (la geometria analitica) e dalla parte della geometria sintetica. I risultati di una esperienza realizzata in un liceo aiutano a precisare la formulazione del testo dell'esercizio e in ugual misura il metodo delle esperienze successive.

Zusammenfassung

Ein Problem des Mathematikunterrichts an der Mittelschule (Gymnasium) besteht darin, dass dieser thematisch mit dem Inhalt anderer Lehrfächer vernüpf ist. Der Aufsatz widmet sich insbesondere dem Bezug zwischen der Stereometrie unter dem Aspekt der Vektorrechnung und einer synthetischen Methode bei der unterrichtlichen Vermittlung der Stereometrie. Die aus den durchgeführten Experimenten sich ergebenden Schlüsse tragen vor allem dazu bei, die Aufgaben sowie einige methodische Vorgehensweisen präziser zu formulieren.

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1. Introduction

One of the long – lasting problems in teaching mathematics at secondary school is the problem of relations among subjects, respectively the continuity of mathematics content and the content of other subjects. However, a specific problem is the continuity or the co-ordination of individual parts of mathematics in the process of education. In this abstract I want to pay attention to this part of teaching mathematics, teaching specifically geometry - focus on the relation between solid geometry from the vector calculus point of view and the synthetic approach to teaching of solid geometry.

The pupils are taught the basics of vector algebra and analytic geometry already in the 8th or 9th grade at the primary school. Later on they get other information with the notion of vector and its operations (sum, odds, ...) in the first grade at secondary school, and this part is mostly used as a tool for knowledge analytic geometry, for example equations of the straight lines, planes, ... and later, for the analytic geometry of conics, balls, and so on.

Another problem is little attention paid to application of the vector calculus to solve the tasks of solid geometry. These tasks are taught in other parts only by the synthetic geometry, as well as little time spent for application tasks in other subjects (physics, geology, geography, etc.).

The aim of the work is to attest the above mentioned hypothesis, which we can formulate as follows:

H1: The solid geometry is taught at secondary school separately, i. e. students are separately taught axiomatics, separately deal with synthetic geometry, separately analytic geometry, and so the sequence of various approaches is minimal or any.

H2: In math textbook there aren't examples with unification character, which would promote elimination limitations (to fault) listed in H1.

H3: Students of secondary school with the established educational schedule haven't enough ability to apply their knowledge of the vector calculus in other areas of mathematics, apart from analytic geometry, and that in this case only formally.

H4: Students of secondary school are not able in ample measure to be aware of the continuity of synthetic and analytic geometry (vector calculus) in solution of particular problem situations. Similar situation applies to University students who will be teachers of mathematics.

The aim of my experiment will be to attest and confirm the hypothesis.

2. Preparation of the experiment

In accordance with the tenets theory of didactic situations frame: within the frame of the didactic situation S_3 (noosferic didactic situation) we made an analyse of math textbooks for secondary schools, an analyse of various mathematical materials, where the goal was to choose a useful problem for students and which would help us to find out reply to already formulated hypothesis H1, H2, H3 and H4 in the introduction. Our goal was to seek such a problem, which the students were not able to solve with the learnt simple algorithms.

The task for the students in this experiment was to try give an example from the solid geometry with exploitation knowledge out of analytic geometry and vector calculus. Results of this experiment had to show how the students can use attainments from these units.

The task was: *Given is a cube ABCDEFGH and K-point, L-point, M-point, N-point, so that K-point is centre of upper surface EFGH, L- point is centre of the AB, M- point belong*

to AE, where $\vec{AM} = \frac{1}{3}\vec{AE}$ and N- point belong to BG: $\vec{BN} = \frac{1}{3}\vec{BG}$. Are the points K, L, M, N complanary?

Based on the above mentioned criteria the final sentence of the given task can be interpreted in several different ways, their experimental attesting will be the part of our following research of this field.

3. Possible strategy of students solution - analyse a priori of problem designation to experiment

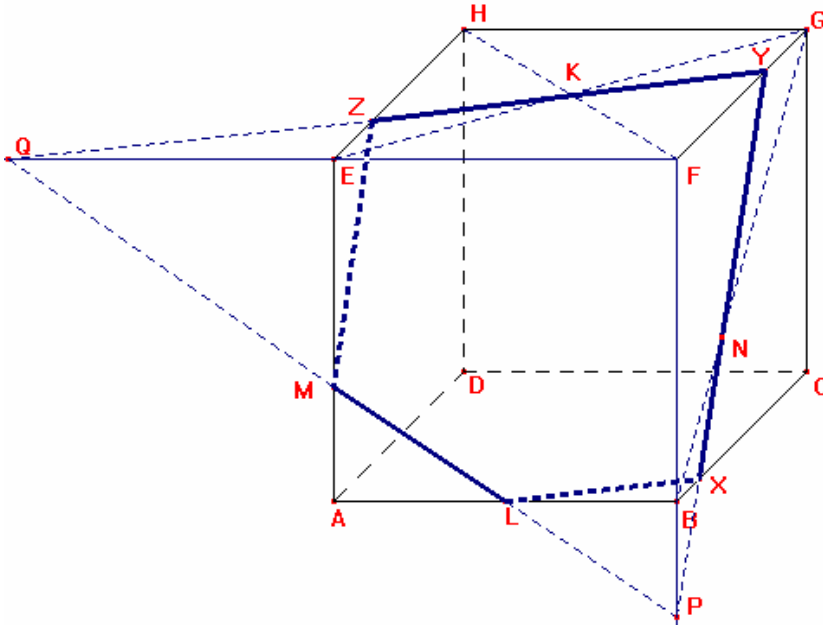
Q₁: Synthetic approach

Q₂: Analytic solution

Q₃: Vector calculus – exploitation collinearity of vectors or complanarity of vectors

Q₃': Vector calculus – exploitation “barycentre”

Q₁: Synthetic approach



$$1) \overline{ML}; M \in \overline{ABF}; L \in \overline{ABF}; \overline{ML} \in \overline{ABF}$$

$$2) P, Q; \overline{ML} \in \overline{ABF}; \overline{BF} \in \overline{ABF}; \overline{EF} \in \overline{ABF}$$

$$P = \overline{ML} \cap \overline{BF} \wedge Q = \overline{ML} \cap \overline{EF}$$

$$3) \overline{PN}; P \in \overline{BCG}; N \in \overline{BCG}; \overline{PN} \in \overline{BCG}$$

$$4) X, Y; \overline{PN} \in \overline{BCG}; \overline{BC} \in \overline{BCG}; \overline{FG} \in \overline{BCG}$$

$$X = \overline{PN} \cap \overline{BC} \wedge Y = \overline{PN} \cap \overline{FG}$$

$$5) \overline{LX}; L \in \overline{ABC}; X \in \overline{ABC}; \overline{LX} \in \overline{ABC}$$

$$6) \overline{QY}; Q \in \overline{EFG}; Y \in \overline{EFG}; \overline{QY} \in \overline{EFG}$$

$$7) Z; \overline{EH} \in \overline{EFG}; \overline{QY} \in \overline{EFG}$$

$$Z = \overline{EH} \cap \overline{QY}$$

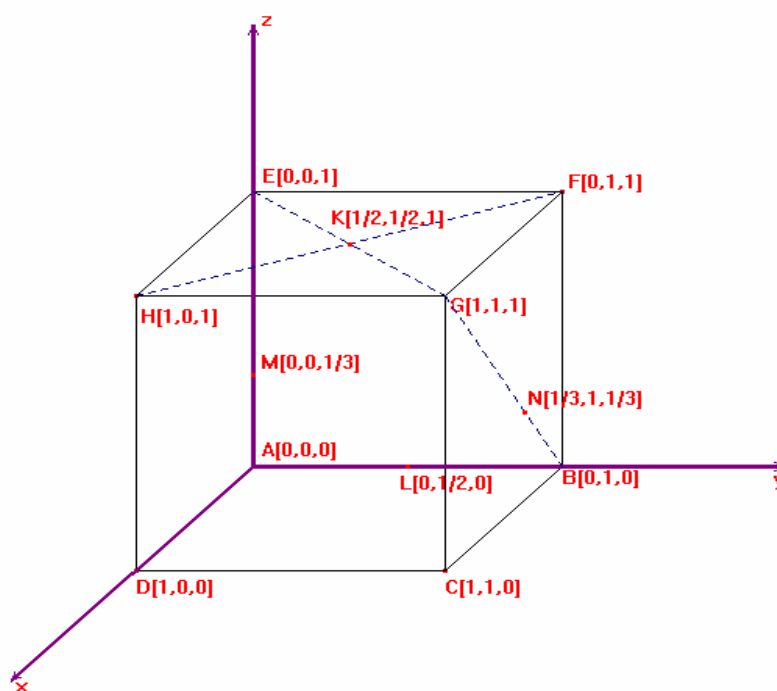
$$8) \overline{MZ}; M \in \overline{ADH}; Z \in \overline{ADH}; \overline{MZ} \in \overline{ADH}$$

$$9) \overline{MLXYZ}$$

a) we construct the section plane \overline{LMN} of cube $ABCDEFGH$ and than

b) we attest: $? K \in \overline{LMN} ?$ (that they are single calculations).

Q₂: Analytic solution



This is parametric equation of plain \overline{MLN} ($M, \overline{ML}, \overline{MN}$): $x = \frac{1}{3}t$

$$y = t + \frac{1}{2}s$$

$$z = \frac{1}{3} - \frac{1}{3}s; \quad t, s \in \mathbb{R}$$

And then the question is: $K\left[\frac{1}{2}, \frac{1}{2}, 1\right] \in \overline{MLN}$?

We solve the system of equations : $\frac{1}{2} = \frac{1}{3}t$

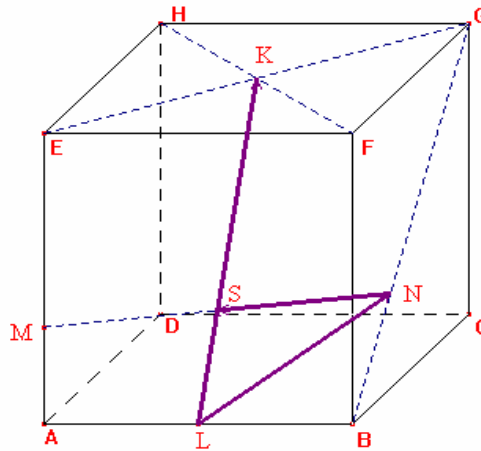
$$\frac{1}{2} = t + \frac{1}{2}s$$

$$1 = \frac{1}{3} - \frac{1}{3}s.$$

For parameters $t = \frac{3}{2}$ and $s = -2$, the equations have a solution, thereout resulting:

$$K \in \overline{MLN}.$$

Q₃: Vector calculus (exploitation collinearity of vectors or complanarity)



The problem is: Are the vectors \overrightarrow{LS} and \overrightarrow{LK} collinear? (point S is a centre of line segment MN). If the vectors are collinear, so the K-point, L-point, M-point, N-point are complanar.

For the vector \overrightarrow{LS} resulting these terms: $\overrightarrow{LS} = \overrightarrow{LB} + \overrightarrow{BN} + \overrightarrow{NS}$; $\overrightarrow{LS} = \overrightarrow{LA} + \overrightarrow{AM} + \overrightarrow{MS}$

$$2\overrightarrow{LS} = \overrightarrow{LB} + \overrightarrow{LA} + \overrightarrow{BN} + \overrightarrow{AM} + \overrightarrow{NS} + \overrightarrow{MS}.$$

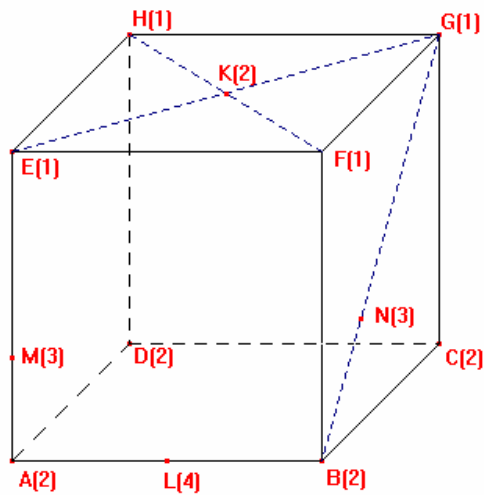
By means of substitution relations and simple reforms resulting term: $\overrightarrow{LS} = \frac{1}{6}(\overrightarrow{BG} + \overrightarrow{AE})$.

In like the manner for the vector $\overrightarrow{LK} \Rightarrow \overrightarrow{LK} = \frac{1}{2}(\overrightarrow{BG} + \overrightarrow{AE})$.

And from the relationship of vectors \overrightarrow{LS} , \overrightarrow{LK} following: $\overrightarrow{LS} = \frac{1}{3}\overrightarrow{LK}$ and so, the vectors

\overrightarrow{LS} and \overrightarrow{LK} are collinear.

Q₃': Vector calculus – exploitation “barycentre”



K -point is a barycentre of $E(1)$, $G(1)$, L -point is a barycentre of $A(2)$, $B(2)$, M -point is a barycentre of $A(2)$, $E(1)$ and N -point is a barycentre of $B(2)$, $G(1)$.

And then, the question is: What is barycentre G of this points set $\{A(2), B(2), G(1), E(1)\}$?

From the facilities of barycentre $\Rightarrow G$ is barycentre of $\{M(3), N(3)\}$ and $\{K(2), L(4)\}$.

So, the points K, L, M, N are complanary, because $\overrightarrow{MG} = \frac{3}{6}\overrightarrow{MN}$ and $\overrightarrow{LG} = \frac{2}{6}\overrightarrow{LK}$.

4. Analyse a priori problem had been formulated and teacher's activity in a didactic situation

Analyse of teacher's work

S_3 – **noosferic situation** – on this stage we analyse the math textbook for secondary school, analyse various mathematical materials, specifically study of solid geometry, vector algebra and analytic geometry. Goal was to choose a useful problem for students which would help him to find reply to already formulate hypothesis H1, H2, H3 and H4. Finish of noosferic situation will be the milieu for the next situation.

S_2 – **constructional situation** – teacher will try to find examples, that were defined in noosferic situation S_3 and on the other side in situation S_1 , in which they will be able to realize. They are examples which students can abet in examples solution Q_1, Q_2, Q_3 a Q_3' .

S_1 – **project situation** – in situation S_1 , teacher writes a text of the example and he “projects” his solution. Student is one on teacher's consciousness. This is a situation which involves student's activity, too. The student can solve problem in a way that he constructs the section

plane of cube, and then he finds out if other point is point of plane; or the student solves a problem that he writes parametric equation of plain and he finds out if the fourth point is the point of the plane; or he applies exploitation collinearity of vectors or complanarity or exploitation “barycentre” (but this solution assuming nothing).

S_0 – didactic situation – in this situation we analyse and do institucionalition of the new knowledge and we formulate the problem. We follow student’s solution, too. It is a situation where the analyse of teacher’s work and analyse of student’s work meet, and the didactic situation will be the result of the teaching process.

Analyse of student’s work

I introduce analyse of problem Q_1 (synthetic approach).

S_3 – objective situation – the student gets acquainted with the problem and with the material milieu. Material milieu are a cube $ABCDEFGH$ and K, L, M, N – points, cognitive component of milieu are knowledge about incidence of points, lines, planes, geometrical construction new section of body, basis of vertical projection, the notions as skew lines, intersecting lines, etc.

S_2 – modelling situation – the student solves the problem in milieu S_3 , i. e. he constructs the section plane \overline{LMN} of cube $ABCDEFGH$ (for example). The student makes use of the knowledge from solid geometry, he works with material milieu, he applies visions and plot, he makes use of known relations and practices.

S_1 – situation of learning – in this situation the student takes the teacher’s role. He solves the formulated problem by means of question: „ Are the given points complanar? “, or he verify to validity $K \in \overline{LMN}$. The student obtains information from reading text of example and he formulates his own results. The teacher is a scrutator and he tries to help, if the student has some absurdity, alternatively if he fails to solve it. In such a case he falls into position S_0 – didactic situation.

S_0 – didactic situation – in this situation the work of students is affected by the teacher and takes his advice in form institucionalition, which can help to student by solving given example, but teacher takes into consideration student’s solution, too. The teacher can help

for example with individual elements of section cube or with the correct registration of solving.

M_3 constructional milieu		P_3 teacher - didactic	S_3 noosferic situation
M_2 project milieu		P_2 teacher - constructor	S_2 constructional situation
M_1 didactic milieu	E_1 reflective student	P_1 teacher - designer	S_1 project situation
M_0 milieu of learning	E_0 student	P_0 teacher	S_0 didactic situation
M_{-1} modelling milieu	E_{-1} cognizant intellect st.	P_{-1} teacher - scrutator	S_{-1} situation of learning
M_{-2} objective milieu	E_{-2} activ student		S_{-2} modelling situation
M_{-3} material milieu	E_{-3} objective student		S_{-3} objective situation

5. Conclusion

In the first stage of experiment an example was taken to students at grammar-school in Nitra (17-18 years old). The students had a review lesson on solid geometry in their final grade. They had 25 minutes on problem solution. 14 students solved the problem and all the solutions had a synthetic approach.

Even though the realized experiment is insufficient from statistical point of view, they show correctness of the formulated hypothesis to the intent that students applicated only one of more solutions.

In other stage of the experiment I will try to change formulation of task in two levels:

- a) in the example there will be none used vectors
- b) I won't use the formulation with the form: “prove that . . .”.

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