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CONTENT

PREFACE

This booklet contains papers accepted by the program committee of Discussion group 12 *Rethinking doctoral programs in mathematics education* of the eleventh International Congress on Mathematical Education, ICME11 in Mexico in July 2008.

The discussion group will deal with three overarching themes, one in each of the three sessions. Each session will have a short introduction, where background and frameworks are presented. Participants will then work in small groups and discuss a set of more elaborated issues and questions under each theme. Session 1 will focus on *The Goals and Processes of Doctoral Programs in Mathematics Education*, Session 2 will focus on *Participants of doctoral programs* and session 3 on *A vision for doctoral programs in mathematics education*. Questions will include 1) What are the goals of different programs? 2) Who are the participants? and 3) Is there a central core of knowledge that doctorates in mathematics education should possess?

In the booklet we present the background document for the work, the nine written contributions and an overview of them and the contact information needed about the contributors to the group.

The booklet has been printed and produced in Riga and we express our sincere thanks to the University of Latvia.

On behalf of the organising team for DG12 in ICME11

Agnis Andžāns, Dace Bonka, Barbro Grevholm
Editors

RETHINKING DOCTORAL PROGRAMS IN MATHEMATICS EDUCATION

Discussion group 12 at ICME11

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Abstract

The is the background document. On the basis of it the papers were selected and the work of the discussion group will be organized. The discussion group will deal with three overarching themes, one in each of the three sessions. Each session will have a short introduction, where background and frameworks are presented. Participants will then work in small groups and discuss a set of more elaborated issues and questions under each theme. Session 1 will focus on The Goals and Processes of Doctoral Programs in Mathematics Education, Session 2 will focus on Participants of doctoral programs and session 3 on A vision for doctoral programs in mathematics education. Questions will include 1) What are the goals of different programs? 2) Who are the participants? and 3) Is there a central core of knowledge that doctorates in mathematics education should possess?

Session 1, coordinator Peter Sullivan

The Goals and Processes of Doctoral Programs in Mathematics Education

This component of DG 12 will facilitate sharing of approaches to doctoral education in mathematics education across in various countries. It will be analogous to the TIMSS lesson study approach, with the focus being doctoral programs, and the researchers being participants in the DG. This comparison of approaches aims to: - increase understanding of the diversity of goals and processes for doctoral study in mathematics education; - allow reflection on common elements of the doctoral programs, and critical consideration of features that differ; - facilitate identification of the best features of various programs and support participants in reviewing their own approaches; and - provide commonality of understandings that will provide the background for sessions 2 and 3 of this DG. A range of perspectives on the focus questions below will be sought. We welcome participants from all countries. All participants will be invited to prepare, in advance, written responses to the following questions. The following are some focus questions, with some indicative issues that could be addressed):

What are the goals for doctoral programs in your University? (e.g., is priority given to candidates learning to research, contributing to new knowledge, being inducted to academician?) Are particular perspectives privileged on the nature of knowledge, argument, theory, and methodology? (e.g., are some methods favoured over others, are there cultural perspectives on what constitutes evidence?) What are the expectations for candidates' background for entry to doctoral programs? (e.g., what mathematics studies are expected, what practical education experience is required, are there pre-requisites for prior research?) What is the content, and what is the demand for coursework? (e.g., are coursework studies core, and if so what are they, are they elective, if so from what range of courses?) What are the expectations for supervision, of both the supervisor(s) and the candidate? (e.g., how many times would the supervisor(s) read Chapter x, how many minutes would supervisors meet with full time students each month?) What are the requirements for the thesis? (e.g., what is the word length, are there specifications for quality, are there alternatives to a thesis, what is the minimum full time equivalent time for study?) What is the process for examination, and what guidelines are given to examiners? (e.g., is there a viva presentation, can examiners confer, is it possible to "fail"?) To what extent is professional and practical knowledge valued? (e.g., are curriculum or resources development projects considered as an alternative to conventional research?) Are there differences between mathematics education and other education theses, and other doctoral theses? (e.g., are students directed to a particular topic or can they choose, do requirements for entry vary between programs?) What are the expectations for candidatures to participate in the life of the Faculty and University? (e.g., are there expectations for tutoring, attendance at non required seminars, mentoring of other candidates?) It is noted that goals and processes vary between institutions, and we are only asking participants to report for their University, not their country The co-ordinator of the session, Peter Sullivan, will analyse and synthesise some of the responses for the first session, and Barbro Grevholm will synthesise others for the second session.

Session 2, coordinator Barbro Grevholm

Participants of doctoral programs in mathematics education

In this session we will focus on the people who participate in the programs, the doctoral students, the supervisors and teachers of doctoral courses. What academic and professional backgrounds should individuals admitted to graduate studies aiming at mathematics education research have? - What is the case today and how could it be changed? - What kinds of problems are linked to recruitment of doctoral students? - What influence do the backgrounds have on the outcome of the education?

The doctoral student's ability to write is crucial for success. How can this ability be developed systematically during the program?

Doctoral students often come with ideas about what to do research on. The choice of research problem is crucial, its limitations and precision is an important and difficult process. The importance of having a burning interest for what you are investigating is often critical for the doctoral student. What experiences do we have about these issues?

How can we define new areas for research internationally? Is there any common consensus about these new areas? How do we ensure that the research problems doctoral students choose are relevant for mathematics teaching and learning in school or other educational institutions? What research problems are supervisors prepared to work with?

Do we have any experience from systematic exchange programs for doctoral students? How can such programs be built up?

How are supervisors educated and how can they develop their skills? What education for supervisors do we know about? What are the demands for supervisors in order to be accepted as such?

What do we know about the subtle relation and work between student and supervisor? What degree of freedom do doctorates have in choosing a supervisor for their degree? What variables influence them to choose their supervisor? If there is any barrier of lack of freedom in making the decision to have her/his supervisor, what is it that caused that to happen?

What is the role of supervision and how do we offer competence development for the supervisors? What are the responsibilities of the supervisor?

Session 3, coordinator Robert Reys

A vision for doctoral programs in mathematics education

Background Doctoral programs in mathematics education vary greatly within and across countries. Some doctoral programs require K-12 teaching experience prior to admission. Others require collegiate teaching experience. Still others require no prior teaching experience. Some institutions require full-time residence for multiple years in order to complete a degree, other programs can be done on a part-time basis and a doctorate be completed while working full-time in another position. Programs also vary greatly in the range and depth of mathematics content required, as well as the manner in which research competence is acquired. Some view this diversity in programs as a strength, others as an area of concern. It certainly raises at least one important question: Is there a central core of knowledge/experiences that doctorates in mathematics education possess? An equally important question is: Should there be a common core of knowledge for graduates with doctorates in mathematics education? That is, when someone says they have a doctorate in mathematic education, what is reasonable to assume about the knowledge they possess with respect to mathematics education?

If the answer to this question “Is there a central core of knowledge that doctorates in mathematics education possess?” is **Yes**, then several natural questions follow, including: What should constitute this common core of knowledge? Who should decide what constitutes this common core? How should it be delivered? How should competence in mathematics education be assessed? Should there be an accreditation of doctoral programs in mathematics education?

One could argue that answers to these questions would provide useful guidance to doctoral granting institution. Others may argue that such information would be too prescriptive, and therefore run the risk of curtailing creativity and uniqueness currently associated with doctoral programs in mathematics education.

One vision for the future A vision for the future is that doctoral programs in mathematics education become more convergent. Does this mean that all doctoral programs in mathematics education would be alike? No, definitely not. Such convergence does not exclude interdisciplinary experiences, but it would insure that doctorates in mathematics education would share a common core of knowledge. Unless a common core of knowledge exists, it is hard to justify mathematics education as a discipline of study.

The Association of Mathematics Teacher Educators has developed a document entitled Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education that included the identification of core knowledge areas. At the least this effort provides some talking points regarding a ‘common core of knowledge’. If there is agreement that some refinement of this type of effort would be of value internationally, then perhaps some plans could be made to move at ICME12 in that direction.

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A SPECTRUM OF DOCTORAL PROGRAMMES AROUND THE WORLD

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For the sessions of discussion group 12 about Rethinking doctoral programs we have received nine written contributions, which have been distributed on the web page of DG12 in advance and they are now presented in this booklet for DG12. They represent an interesting spectrum of doctoral programmes around the world and this chapter is a short summary of the nine papers.

From the University of Latvia professors Agnis Andžāns and Līga Ramāna report experiences from a program entitled Modern elementary mathematics and didactics of mathematics. In 1995 elementary mathematics was recognised as an independent branch of mathematics by Latvia's Council of Science. Doctoral student must fulfil the requirements for future doctors of mathematics. The authors exemplify what kind of questions that are recommended for the students. Since 2006 the formal requirements for receiving a doctoral degree in Latvia include two reports at international conferences and that main results must be published in internationally reviewed journals, proceedings or paper collections. Six examples of findings of doctoral students are presented and references given to them. The conclusion is that the close integration of doctoral studies in didactics of mathematics with modern elementary mathematics has made a good service for both and has lead to improvements in the education at several school levels.

The doctoral program in Korea in mathematics education, presented by Sang Sook Choi-Koh, was first created in 1996 at the Graduate School of Dankook University of Education. Now there are 10 national and 8 private universities that run programs in mathematics education. The purpose of the programs is to provide society with professional educators. The curriculum of the program is described. The prerequisites for entrance to the program are a masters' degree and it is also desirable to have teaching experience. Students need to take courses in education, mathematics education, mathematics, and some optional areas to qualify. The degree must be finished within 10 years from enrollment. The thesis is evaluated by a committee with five members who are experts in mathematics education.

Barbro Grevholm presents the only existing doctoral program in mathematics education in Norway and places it among programs in the other Nordic countries and in relation to the Nordic Graduate School in Mathematics Education. An evaluation of the program, which started in 2002, has taken place but is not yet reported and a number of important issues are mentioned that concern the quality and development of the program. Among worries is the fact that most students need more time to finish the studies than the expected three years. One year of course work is normal in this program and the thesis work leads to a published dissertation, which is defended in public with opponents from the international community. The rethinking of the program will be based on the evaluation report and the experience gained so far from the 24 students in the program.

Vena Long, Theresa Hopkins and Geri Landry present a successful alternative to the traditional doctoral program. A distance model is used for the delivery of courses in order to reach the targeted rural population. All types of assignments are possible using the technology in a strategic way. An innovative residency includes two summers of coursework each at one of the participating institutions. This program allows the student to stay on the job, why completing an advanced degree, still in the reality of the classroom. Geographically isolated, disabled and place-bound populations can be reached with the high quality opportunities that are used in the program. In this case creativity, flexibility and external funding have helped to overcome tradition and the hesitation of professors. This innovative program is now ready to be duplicated and replicated by others.

Robert Mayes and Patricia McClurg use complexity and uncertainty as drivers for programs in mathematics and science education. They claim that ‘The proposed Ph.D. in Mathematics Education incorporates cognates and apprenticeships that will engage the students as practitioners in a community of STEM scientists, mathematicians, and educators. The primary drivers of complexity and uncertainty motivate an integrated science approach based in modeling real-world phenomena using mathematics and technology. Graduates of such a program are uniquely poised to address pressing needs in K-12 STEM education. There is a pressing need to move curricula from the current silo approach to teaching mathematics and science as a collection of isolated facts, to an integrated approach that coalesces STEM disciplines around real-world problems. There is a pressing need to provide preserve and inservice teachers with professional development that prepares them to teach mathematics and science through a problem/project based pedagogy that engages and motivates students by demonstrating the utility of science. There is a pressing need to develop teacher educators that are enculturated into the STEM communities way of knowing (what does it mean to DO science or mathematics) and reflect the central concepts of scientific inquiry and mathematical problem solving/proof in their practice. Finally, there is a pressing need to bring educational research in the area of cognition to the classroom in a way that impacts teacher's practice and student learning.'

Michaela Regecová introduces us to Doctoral programs in the Slovak Republic. Among the 19 doctoral programs in Comenius University we find since 2006 a program in Theory of mathematics education. The entrance interview consists of the two parts written test that examines the basic knowledge in the field and a personal interview by the entrance committee. Two to five doctoral students will enter each year according to plans. The duration of the program is 3 years. The supervisors are professors or docents. The study part is described with examples of courses given. The scientific thesis work can be done individually or in teams. The student is also expected to develop abilities to lead professionally teaching and learning projects and to organize research events. The degree qualifies for work with research at university or the Slovak Academy of Science and at research institutions, leader of team in various problem fields and in educational management. There is a wish for the program to compare range and depth of mathematics content required and the manner in which research competence is acquired and to improve international cooperation.

Challenges and a vision for doctoral programs are offered by Robert Reys in his contribution. He points out, that doctoral programs in mathematics education vary greatly within and across countries and refers us to reports of such variations. The question he asks is if there is a common core of knowledge that a person with doctorate in mathematics education should possess. If the answer is yes, then a number of sub questions emerge:

What should constitute this common core of knowledge? Who should decide what constitutes this common core? How should it be delivered? How should competence in mathematics education be assessed? Should there be an accreditation of doctoral programs in mathematics education? The vision for the future that Robert Reys is presenting consists in a wish that doctoral programs in mathematics education become more convergent. Finding a core of knowledge, which can prepare doctors in mathematics education for diverse careers is challenging.

Filippo Spagnolo describes a program in History and Mathematics Education, History and Physics Education and History and Chemistry Education, which is an international doctoral program offered by a consortium of 14 departments in Italy, Slovakia, Cyprus and Spain. The construction of the program through courses, seminars, workshops and visits abroad is described and related to the positions in the time scale of the program. The aim of the education is to prepare future inspectors in the educational sector and to prepare researchers in the history and didactics of mathematics, physics or chemistry. Yearly reports to the Italian Ministry of Universities are based on four quality parameters: students' publications, reports of experts involved in the project activities, students' lectures in national conferences, and in international conferences. The examination process of the thesis is described in the paper in detail. Finally an overview is given of how many students are involved and where they are in the program. The first Ph Ds in this program will graduate in the end of 2008.

Peter Sullivan describes some aspects of doctoral programs at two Australian universities. The goals of the two universities for the doctoral programs are presented and both emphasise knowledge creation and research training. The entry requirements are detailed with minimal discipline specific requirements but the overall requirements are substantial. The key responsibilities for supervisors at Melbourne University are to facilitate the completion of the graduate research, monitor the quality, and assist graduates to develop transferable skills and prepare for their careers. In both universities the length of the thesis must be substantial. Six different options for the Ph D education are presented in addition to the traditional Ph D by thesis. An advisory committee meets regularly to follow the progress of the students. Finally the examination process is discussed. Peter Sullivan summarises:

'The two universities are young in comparison to other major world universities. The two programs are distinctive in emphasis. There is not a strong culture of programmatic research, and where there are strong groups of mathematics education doctoral candidatures, this is usually the result of an active supervisor rather than a coherent program.'

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HOW TO MAKE THE FAMILY HAPPY

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Keywords: *Modern elementary mathematics, mathematical learning theory.*

Introduction

The question mentioned in the title is as old as the mankind itself, and almost everybody has tried to solve it in his own way. The "mathematics education" is also a kind of family, and it also can't be happy if both members aren't equal in their rights and duties. Unfortunately, this is hard to achieve.

There are 4 types of educators:

	In mathematics	In didactics
A	Strong	Strong
B	Strong	Weak
C	Weak	Strong
D	Weak	Weak

In our opinion, those of type C are the most dangerous: they can convince their students of absolutely wrong things so effectively that the situation becomes totally unimprovable. So the role of mathematics should in no way be underestimated while preparing the educators; unfortunately, this often happens in traditional study programs (for bachelors, masters, doctors). Moreover, at our opinion mathematics is such a well – organized discipline, that the didactics of it is almost predetermined by the inner logic of the subject; so recognizing this logic in each separate case is crucial for successful teaching.

As for teaching/learning methods, we should remember that people themselves are learning almost exceptionally from examples, and this is how the teaching/ learning process usually is organized. However, it has been proved in the form of exact theorems in theoretical computer science (see, e.g., [1]) that the possibilities of obtaining general rules from series of experiments are very limited. So other models of inference must be developed and used.

All this creates the necessity of research and study programs for educators in mathematics with strong theoretical components both in mathematics and computer science. One of possible attempts has been made in University of Latvia, where a doctoral program “Modern elementary mathematics and didactics of mathematics” has been established.

The concept of modern elementary mathematics

It is a tradition that the words "elementary mathematics" are connected with school only. It's not quite correct. Of course, no definition in the mathematical sense is possible. Trying to list the parts of elementary mathematics we include Euclidean planimetry and stereometry, linear operations with plane and space vectors, scalar, pseudoscalar and vectorial products, the greatest part of combinatorial geometry, elementary number theory, equations and systems solvable in radicals, algebraic inequalities, elementary functions and their properties, the simplest properties of sequences and the combinatorics of finite sets. There are many mathematicians, however, who include also elements of graph theory, simplest combinatorial algorithms, simplest functional equations in integers, etc. There are parts of mathematics which definitely should not be included: we can mention the methods which are effectively used only by a small amount of mathematicians as well as methods which, though used widely, demand a specific and advanced mathematical formalism.

We can give the following approximate description of elementary mathematics. Elementary mathematics consists of: 1) the methods of reasoning recognized by a broad mathematical community as natural, not depending on any specific branch of mathematics and widely used in different parts of it, 2) the problems that can be solved by means of such methods.

Evidently, such a concept of elementary mathematics is historically conditioned.

Many new areas of mathematics, especially in the discrete and algorithmic parts of it, are still today exploring elementary methods as the main tool. Obviously it can be explained at least partially with the fact that the natural questions there have not yet been exhausted, and natural approaches are therefore effective.

The movement of mathematical contests, especially of mathematical olympiads, has made an important service to elementary mathematics. Becoming a mass activity, the system of math competitions created a large and constant demand for original problems on various levels of difficulty. Clearly school curricula couldn’t settle the situation, and the organizers of the competitions turned to their own research fields where they found rich and still unexhausted possibilities.

One of important results that originated from the "olympiad mathematics" was the identification of the so called general combinatorial methods (mean value method, invariant method, extremal element method, interpretation method) (see, e.g., [2]).

Elementary mathematics was first officially recognized as an independent branch of mathematics in 1995 when the Latvia's Council of Science published the formal structure of science in Latvia, "Modern elementary mathematics and didactics of mathematics" (further MEM/DM) being one of 12 parts of mathematics. Since then, master and doctoral degrees are awarded in this area.

We stress especially that with this decision the didactics of mathematics is also recognized as a part of mathematics.

The formal structure of the doctoral program

From the formal point of view there is only one doctoral program in mathematics in the University of Latvia (the leading scientific/ educational institution in the country with high international reputation); it has 8 branches (algebra; differential equations; geometry and topology; …; MEM/DM). So doctoral students must fulfill the general requirements for future doctors in mathematics. At first, it means the examination in general mathematics; each student has to select (together with his supervisor) 20 questions out of the list of 92 questions. We give 10 characteristic examples of questions usually recommended for doctoral students in MEM/DM:

- Most important axiom systems of set theory: Zermelo-Fraenkel and Goedel-Bernays systems. Corollaries from the axioms. The axiom of choice and its equivalents: Zorn's lemma, Kuratowski's principle etc.
- Elements of algebraic and analytical number theory. Field of algebraic numbers and its basic properties. Basics of ideal theory. Classical transcendence proofs. Central results on prime distribution. Riemann ζ-function and Dirichlet L-function. Fast algorithms for factorization and for primality.
- Combinatorial structures. Elements of enumerative combinatorics. Transversals, latin squares, block-schemas, finite geometries. Generating functions and their algebra. The method of recurrence relations. Moebius inversion function. The orbit method.
- Ramsey theory. Classical Ramsey numbers and their generalizations. Ramsey-type structural theorems in number theory. Ramsey type results in geometry, algebra, mathematical analysis, combinatorics. Classical minimax theorems.

- Classical fast algorithms. Sorting algorithms. Algorithms for arithmetical operations. Algorithms for computing polynomials. Algorithms for operations with matrices.
- Main formal concepts of algorithm and their basic properties. Turing machines, normal algorithms, recursive functions and their equivalence. Reducibility and its formalizations. Algorithmically unsolvable problems, Kleene-Mostowski hierarchy. The characterization of recursively enumerable sets through Diophantine predicates.
- The concept of probabilistic algorithm. Probabilistic Turing machines, their principal possibilities in set recognition compared with those of deterministic Turing machines. Advantages of probabilistic Turing machines and various types of automata over their deterministic counterparts from the complexity point of view.
- Elements of dimension theory. Small inductive (Menger-Urison) dimension *ind*. Large inductive (Brauer-Čech) dimension *Ind*. Layer (Lebesque) dimension *dim*. Dimension of the subsets of Euclidean space. Basic properties of the dimensions of separable metric spaces. Inequalities between various dimensions. Zero-dimensional spaces.
- Main methods of proving algebraic inequalities. Classical inequalities and their generalizations. Isoperimetric problem and its variations. Fast algorithms in the analysis of systems of inequalities.
- General combinatorial methods and their applications. A concept of invariant method, mean value method, extremal value method, interpretation method. Formal deductive systems. Bases of the systems of functions in the algebra of logic. Impossibility proofs in automata theory. Lower complexity bounds for combinatorial algorithms. Elements of the complexity theory for computations.

As we see there is a strong stress on the discrete/ algorithmic side of mathematics. This is explained by the growing role of it in science and applications, which must be reflected also in the education on all levels.

Doctoral students must pass also 1) the examination in their “narrow speciality” that requires studies in didactics, 2) the examination in the “second” foreign language.

Since 2006 the formal requirements for receiving the doctoral degree in Latvia include only 2 reports on international conferences and a quite smooth demand that the “main results” must be published in internationally reviewed journals/ proceedings/ paper collections etc. Nevertheless, for doctoral students in MEM/DM the unofficial standard is not less than 5 conferences and 10 publications of the above type. Also at least one published teaching aid based on the performed research is considered as very welcome.

The main directions of the research

During the last 30 years – both in Soviet period and after regaining the political independence – the education in Latvia is reformed almost without interruption. Generally speaking, it has become more "colorful" but also, unfortunately, more sketchy. The great aim to make the education more **human** than it was in the Soviet period was substituted by the aim to make it more **humanitarian**. So significant harm was done to the teaching of exact disciplines at middle and high school, and serious efforts must be applied now to turn the wheel back. The main task of researchers in the area of mathematical education in Latvia is to find the ways how to do it as fast as possible. It seems that there is no hope to return to the previous number of lessons for mathematics (at least at this moment); so something principally new must be found.

Three main directions that are chosen for the research are as follows:

1) the integration of teaching various topics on the basis of unifying ideas discovered in the area of modern elementary mathematics,
2) the possibilities provided by ICT and deeper understanding of the nature and role of algorithmics in mathematics,
3) the development of the system of out-of-class activities, especially mathematics contests.

Although each possible doctoral student is offered to select a topic from the abovementioned areas, he has a full right and is even urged to come up with his own proposals.

Some results

There are some findings made by doctoral students and confirmed by the praxis that have found serious applications in the educational system of Latvia:

1) the classification of the contest problems and solution methods within Latvian-Icelandic project LAIMA (see, e.g., [3] and [4]),

2) the classification of Internet resources in school mathematics and creating a structured survey of them within the Latvian Education Informatization System project during 1997-2005 (see, e.g., [5]),

3) the investigation of the possibilities to use the method of invariants as the unifying factor in teaching some hard topics (see, e.g., [6]),

4) the investigation of the possibilities to use the method of interpretations as the unifying factor in teaching seemingly different topics (see, e.g., [7]),

5) the investigation of the principal questions about problem setting for math competitions (see, e.g., [8]),

6) the investigation of the possibilities to integrate elements of combinatorics into the curricula for middle grades (see, e.g., [9]).

Conclusions

The close integration of doctoral studies in didactics of mathematics with modern elementary mathematics has made a good service for both and has led to real improvements in the education on middle and high school levels.

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THE DOCTORAL PROGRAM IN KOREA

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Korea underwent 7 times the reforms of school curricula since the establishment of the Republic of Korea in 1948, after World War II. They were controlled by the ministry of education. According to Korean history of education, the first teachers´ training school was founded to provide agriculture teachers to schools in 1927 through modernization of society. However, in old days, teachers were grown through the traditional way of apprenticeships.

In further, **GDU** means The Graduate School of Dankook University.

GKNUE means The Graduate School of Korea National University of Education.

1. History of the doctoral program

(1) GDU: March, 1996, the first doctoral program in mathematics education among private universities in Korea. Now there are about 10 national and 8 private universities that run a doctoral program in mathematics education.

(2) GKNUE: March, 1987 (Especially, KNUE was founded only for teacher education in 1985. It played a role of the model as a specialized college for the teacher education program in Korea.)

2. Purpose

(1) GDU: The Graduate School in Mathematics Education was designed to provide professional educators for society, not only with the latest in educational theories and research techniques, but also with the deepest sense of responsibility as dedicators to mankind - and the followers of DKU's founding mottoes of 'Truth' and 'Service' were based on the spirit of humanitarian ideals.

(2) GKNUE: The graduate school for a doctorate program in mathematics education was founded to educate capable people who were able to play pivotal roles in enhancing the quality of our education through in-depth academic training and scholastic research in theory and practice of mathematics education.

3. Curricula (GKNUE)

(1) Common Curriculum in Secondary & Elementary Mathematics Education

	Course code	Course	Credit
Math Education	10-501	Research Methodology in Mathematics Education	3
	10-504	Current Issues in Mathematics Education	3
	10-505	Mathematics Education Technology	3
Math	10-602	Topics in History of Mathematics	3
	10-604	Statistics	3
	10-605	Mathematical Analysis	3
	10-607	Modern Algebra	3

(2) Curriculum in Elementary Mathematics Education

	Course code	Course	Credit
Math Education	11-501	Elementary School Mathematics Curriculum	3
	11-505	Problem Solving in Elementary School Mathematics	3
	11-518	Psychology for Teaching Elementary School Mathematics	3
	11-519	Methods for Teaching Elementary School Mathematics	3
	11-520	Assessment for Elementary School Mathematics	3
	11-521	Study of Topics for Teaching Elementary School Mathematics	3
	11-522	Developing Creativity & Elementary Mathematics Education	3
	11-523	Seminar in Elementary Mathematics Education	3
	11-524	Technology & Elementary Mathematics Education	3
	11-525	Advanced Topics in Teaching Elementary Mathematics Education	3
Math	11-601	Linear Algebra	3
	11-602	Abstract Algebra	3
	11-603	Advanced Analysis	3
	11-604	Topics in Applied Mathematics	3
	11-605	Point set Topology	3
	11-606	Topics in Modern Geometry	3
	11-633	Combinatorial Topology	3
	11-644	Topics in Analytic Geometry	3
	11-652	Probability Theory	3
	11-653	Statistical Methods	3
Research	11-801~803	Individual Research in Elementary Mathematics Education	3

(3) Curriculum in Secondary Mathematics Education

	Course code	Course	Credit
Math Educa-tion	12-501	Introduction to Mathematics Education	3
	12-506	Evaluation in Mathematics Education	3
	12-507	Research on Mathematics Teaching Materials	3
	12-509	Topics in History of Mathematics Education	3
	12-511	Philosophy of Mathematics Education	3
	12-515	Problem Solving in Mathematics	3
	12-516	Topics in Mathematics Teaching Method	3
	12-517	Seminar in Mathematics Education	3
	12-519	Theory of Mathematics Curriculum	3
	12-520	Mathematics Education Psychology	3
Math	12-601	Topics in Algebra	3
	12-602	Real & Complex Analysis	3
	12-603	Functional Analysis	3
	12-604	Advanced Discrete Mathematics	3
	12-605	Topological Space	3
	12-606	Algebraic Topology	3
	12-613	Advanced Linear Algebra	3
	12-627	Advanced Numerical Analysis	3
	12-643	Advanced Modern Geometry	3
	12-644	Topics in Differential Geometry	3
	12-652	Advanced Probability Theory	3
	12-653	Advanced Statistical Method	3
	12-801-803	Individual Research in Mathematics Education	3

4. Entrance Qualifications for a Doctorate

(1) People who have a master's degree from inside or outside of the country, or people whom the minister of education approves as being qualified in the same field of study.

(2) Highly desirable qualifications: Math teachers who have more than 4 years experiences in schools, school inspectors, educational researchers, educational public service employees over level 5, and educational administrators in private schools who are approved to have similar qualifications by the minister of education, the president of a university, the principal of their affiliated school, or the superintendent of their own state or city.

5. Graduation Qualifications for a doctorate

(1) GDU: They should take at least 45 credits in majors along with 9 credits in research after passing the foreign language test and the preliminary test.

(2) GKNUE: They are supposed to earn 60 credits in major areas with more than 6 research credits, after passing the foreign language test and the preliminary test.

Education	Mathematics Education	Mathematics	Electives	Total
6	12	15	27	60

6. Admitted Period for Graduation

(1) GDU: They should graduate within 10 years from their first enrollment.

(2) GKNUE:

They should graduate within at most 6 years of course-works, otherwise the dean of the graduate school decides according to the committee's findings when candidates turn in a certain statement giving the reason for delay.

7. Foreign Language Test and Preliminary Test

(1) GDU: They are qualified to take the foreign language (mainly English) test starting from the second semester after their enrollment and they are able to take the preliminary test after they have earned 45 credits in their major.

(2) GKNUE: They are able to take the foreign language test only after they finish at least 3 semesters of attendance and only after they have earned 51 credits in their major.

The preliminary test for both universities is composed of 3 fields: education, mathematics, and mathematics education. The Council of the graduate school provides specific instructions.

8. The period of Course-works and Attendance

(1) GDU: They have at least 2 years for course-work

(2) GKNUE: They have usually 3 years for course-work and they can not exceed 5 years for attendance at school.

9. Class Dates

(1) GDU: More than 15 weeks per semester

(2) GKNUE: 30 weeks per year

10. Limit of Credits & Valid GPA

(1) GDU: They cannot take more than 9 credits per semester; however if a student transfers from a school that allows 10 credits per semester, then he

or she may take up to 10 credits per semester. They should maintain at least a B average for graduation.

Score	100-95	94-90	89-85	84-80	79-75	74-70	69-...
Grade	A+	A	B+	B	C+	C	F

(2) GKNUE: They take at most 10 credits each semester. If they took prerequisite courses related to the course, then they are allowed to take 12 credits. The courses with credits below C+ may be taken repeatedly. Previously taken credits are excluded from their GPA. In screening for graduation, the average GPA of total credits should be over a B0 (see the table below for GPA).

성적등급	실 점	평 점
A+	97 - 100	4.5
A0	94 - 96	4.1
A-	90 - 93	3.8
B+	87 - 89	3.4
B0	84 - 86	3.1
B-	80 - 83	2.8
C+	77 - 79	2.4
C0	74 - 76	2.1
C-	70 - 73	1.8
F	70점 미만	0.0

11. Doctoral Thesis

A total of members in Committee is 5 : They should be experts about mathematics education ; 3 from student's own university and 2 from other universities. Some theses are more about didactics through experimental research and some about theory-based research to enhance the quality of education.

References

1. GDU : http://www.dankook.ac.kr/Department/index.jsp?p_NoticeKindId=139&p_MenuHead=b01

2. GKNUE : http://grad.knue.ac.kr/

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A NORWEGIAN DOCTORAL PROGRAMME IN DIDACTICS OF MATHEMATICS – DOCTORAL PROGRAMMES IN THE NORDIC RESEARCH COMMUNITIES

Barbro Grevholm

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Introduction

In the Nordic countries doctoral research studies within the area of didactics of mathematics have long been carried out in doctoral programmes for general education or pedagogy. As recent as in the 1990-ies programmes in didactics of mathematics were created in Sweden and Norway. Finland and Iceland still do not have specific programmes in didactics of mathematics. The first doctoral programme in Norway in didactics of mathematics was founded in 2002 at University of Agder. Earlier (under the name Agder University College) this institution had functioned as a networking point in mathematics education for all of the academic institutions in Norway. As the programme in University of Agder has now been working for a little more than five years an evaluation has taken place this May done by Anna Sierpinska from Concordia University in Montreal. In the process of preparing this evaluation the Mathematics Education Research Group at Agder (MERGA) had an opportunity to rethink the doctoral programme based on the experiences since 2002.

The time for revision has come and after we receive the evaluation report careful considerations will be made and the changes implemented. In this paper I will discuss some of the issues we found important in the development and for possible future changes.

About terminology

In Sweden and the Nordic countries mathematics education is called ‘matematikdidaktik’ (or similar words in the different languages), didactics of mathematics, thereby following the German and French tradition rather than the Anglo-Saxon, when it comes to the notion. In Sweden mathematics education is translated to ‘matematikutbildning’, which means education in mathematics, including school level and other levels. Thus there is a risk of misinterpretations when using the term ‘mathematics education’ as a name for the research field. Here I will use both these notions interchangeably.

The structure of a doctoral programme in Norway

In this section I offer a short description of some characteristic features of the doctoral programme in didactics of mathematics in University of Agder. Although Norway is not a part of the European Union the country has adopted the so called Bologna system for structuring the academic education. This means that the bachelor degree should take 3 years, the masters' degree 2 years and the doctoral education 3 years.

Thus in Norway the doctoral education in general is a three year study with course work and research leading to a written thesis. The course work in general is one semester but in the programme in didactics of mathematics at University of Agder the course work covers one year of fulltime study. The prerequisites for study are a master's degree in mathematics or mathematics education and teaching experiences. The research education leads to a thesis that will be examined by three examiners, and two of them are external. Examiners must be professors or on professor's level in the academic system. The examiners read the thesis and produce a written document, where they classify the work as acceptable for defence or not. If the work can be accepted after minor revisions a student can get some months to revise the work and then have it evaluated again. After the acceptance the doctoral student is allowed to defend the thesis in a public viva (disputation), where also external persons can criticise and discuss the content. The examiners act as opponents during the public defence (disputas). The dissertation are published in a university series. There is no national Graduate School in Mathematics Education in Norway yet, but there are plans to start one. On the other hand, the Nordic Graduate School in Mathematics Education (see more below) is situated in University of Agder. University of Agder is at the moment the only university that offers doctoral courses in mathematics education at a regular basis in the Nordic countries.

Exemplifying with the programme of University of Agder, it can be noticed that each doctoral student will get at least two supervisors (a main supervisor and a co-supervisor) and an individual study plan is made up each year, followed by a yearly report to the board about the outcome of each study year. Also supervisors' reports to the board are handed in and carefully followed up. Two courses are compulsory, Theory of science from a didactics of mathematics perspective (5 study points) and Research methodology in Mathematics Education course (15 study points). If the student does not have a course in History of mathematics in the masters education such a course is compulsory at doctoral level. One or two courses are running each semester and they normally attract doctoral students from the whole of Scandinavia.

It is possible for a student to distribute the stipend time over 4 years if the student takes on a teaching load of 25 % parallel to the doctoral studies. The administrative process for the student starts with applying for a stipend, which is actually a position as a doctoral student at the university. The student becomes employed by the university and gets all rights of an employee. The monthly salary is rather good and the position includes all societal rights if the students gets sick or has to take parental leave. After having received the stipend, the student has to apply to be taken up in the doctoral programme and this includes writing a research proposal (in collaboration with the two supervisors). There is a board for the doctoral programme to advice the faculty board in issues concerning the doctoral programme. This board evaluates the proposal and if it finds it good the student is taken up in the programme.

In the programme six doctoral courses have been developed and they are offered to students on a regular basis according also to the wishes of the students. A majority of the students work at UiA and the rest of them have a stipend at some other university or university college and do their work there. The courses are thus constructed so that they can be taken as distance courses with limited time for presence at UiA. A student working at another university normally has one of the supervisors there and the other at UiA. Students can take courses at other universities after agreement with the supervisors.

The programme has 24 students taken up in 2008 and there are about 15 professors working as supervisors.

In addition to what is offered at UiA the students have profited from extra resources offered by the Nordic Graduate School in Mathematics Education, which started in 2004.

Collaboration in Graduate Schools

National graduate schools

Research areas that are small with only one or two students and one or two faculty members are vulnerable and it is tempting to create cooperation between institutions. The idea to build National Graduate Schools has been developed in Finland, Sweden, and Denmark. Finland was first in 1995, followed by Sweden in 2000 and Denmark in 2005. Finland has repeated the initiative once, in Sweden it is so far a one time activity between 2000 and 2006. In 2008 UiA together with three partner institutions has applied to get a Norwegian Graduate school. Many reasons have been presented for having national graduate schools. There is a wish to increase the number of students finishing in time, a wish to shorten the actual study time (which normally can be longer than the formally expected time), to offer a richer study environment for the students and to offer competence development for the supervisors. In Sweden an evaluation of 16 national graduate schools was published in 2006 (Persson, 2006). The report points out that there have been some problems, such as lack of knowledge about and experiences in didactical research, inadequate planning and organisation of the activities, in some cases insufficient supervisor competence, difficulties to cooperate and antagonism between different fractions in the subject fields, financing problems and so on. The evaluation of the outcomes is more positive. Most doctoral students have finished in reasonable time and supervisors' competence has developed well. National and international networks have been established and are strengthening the opportunities for further development and improvement of research in subject didactics. The relations to the core subjects have been highly improved. There seems to be a promising labour market for the new doctors. Persson points out that when establishing new graduate schools these experiences must be taken into account. There must be adequate supervisor's competence from the beginning and resources for competence development of supervisors must be set aside. Not too many institutions should be involved and very good preparations are necessary for a graduate school to function well from the beginning. All participating institutions must agree about the aim and goals of the activities. Common guidelines for students' conditions and financing must be agreed upon.

The Nordic Graduate School in Mathematics Education

The Nordic Graduate School in Mathematics Education is based on funding from the Nordic Research Academy (NordForsk). It is a five year activity during 2004-2009 with the idea that after five years the cooperation built is strong enough to survive by support only from the involved institutions.

I will present the Nordic Graduate School in Mathematics Education, its aims and some of its activities. The Graduate School is a network of about 40 Nordic and Baltic research environments with graduate education in mathematics didactics. Around 120 supervisors and 86 doctoral students are part of the network in 2008. An account will be given of doctoral courses so far, of seminars for supervisors and of workshops and summer schools that have taken place.

The aim of a Nordic Graduate School in Mathematics Education - NoGSME

The aim of the Nordic Graduate School as it was decided by the application to NordForsk in 2003 (The Nordic Research Academy) is to

- support and develop the education of researchers in mathematics education in the Nordic and Baltic countries,
- create constructive cooperation in order to raise the scientific quality of research in mathematics education,
- give all doctoral students in mathematics education an access to the activities of the Graduate School
- create cooperation among a greater group of doctoral students and supervisors in order to share experiences and opportunities to improve the education of researchers.

The utmost aim is to create a network of cooperating partners, who can continue to collaborate after the five years of the Graduate School (Grevholm, 2004a).

Activities in the Nordic Graduate School in Mathematics Education

The activities in the Graduate School can be summarised in the following points (Grevholm, 2004b, 2005a):

- Common courses are created with the added competence from all researchers in the Nordic countries and international partners (Grevholm, 2004c)
- Seminar-series in specific research areas are offered as a complement to local series and workshops on subjects or issues of main importance (Grevholm, 2005b)
- Competence development for supervisors and exchange of experience is offered
- Partnerships and collaboration with distinguished international scholars are built
- Creating a database for ongoing work, theses and greater development work in mathematics education
- Mobility stipends and special financial support for doctoral students are given.

Courses that have been offered since 2004

The courses offered are of two kinds. Courses that are given on a regular basis at some of the participating universities are open to all doctoral students in the network. They are advertised each semester. Other courses are initiated by the board of NoGSME. The board collaborates with some interested colleagues in one of the participating universities and the course is constructed and given at that place, with financial support from NoGSME (Grevholm, 2004d, 2005c). The regular courses so far have been given at University of Agder in Norway. The courses that have been initiated by NoGSME have taken place in Copenhagen University (Winsløw, 2006), Denmark Pedagogical University, and Norwegian University of Technology, Roskilde University, Umeå University and Helsinki University. One course in Malmö is under construction. Here are the courses given or ongoing so far:

- Theory of science from a mathematics education perspective, UiA
- Meta-perspectives on mathematics and the learning of mathematics in a technological environment, UiA
- History of mathematics with emphasis on modern mathematics, UiA
- Theoretical aspects of mathematics education with emphasis on the French School, Copenhagen University
- Problem solving in mathematics education, UiA
- Theories of learning and teaching mathematics, UiA
- Research design and research methods in mathematics education, UiA
- Views of knowing and learning: Constructivism and socio-cultural theory, Denmark Pedagogical University
- Gender and mathematics education, Norwegian University of Technology
- Justification of research in mathematics and science education with special emphasis on the role of theory in such justification, Roskilde University Centre
- Research on assessment in mathematics education, Umeå University
- Conceptions in mathematics, Helsinki University
- Mathematical literacies, Malmö University College

Students get travel support to come to the courses and they can also apply for mobility stipends if they want to spend one or two months at another Nordic university. The mobility stipend covers real costs for travels and accommodation.

Summer schools

Summer schools have been offered each year and are much appreciated by the doctoral students. The main part of the programme is taken up by work in groups, where each student can get her research project discussed and commented on. The groups are lead by international experts in the field, which is highly appreciated by the participants. Among these experts we have had excellent and well known researchers who have inspired the students. The friendship and mutual understanding that is built in these summer schools are expected to be the foundation of long-standing cooperation of the students in their coming careers (Grevholm, 2004b, 2006b).

Seminars for supervisors

A crucial component of doctoral education is an access to good and experienced supervisors. In order to assist the environments in strengthening the competence of supervisors NoGSME is organising seminars and competence development programmes for supervisors. They have focussed much on quality issues in research education and publications (both papers and theses) (Grevholm, 2006c). NoGSME has built a close cooperation with the journal Nordic Studies in Mathematics Education (Nomad) in order to enrich these programmes. Here are some of the themes of the seminars given so far:

Quality in research in mathematics education, Quality of theses in mathematics education,

Supervision of doctoral students, Reviewing of papers in mathematics education, Research programmes in mathematics education, Critical situations in supervision of doctoral students in mathematics education, Trondheim, Review process of papers for scientific journals, Lund, Outcomes of research in mathematics education, and Scientific profile and characteristics of journals in mathematics education.

The seminars most often have between 20 and 30 participants and quite an important network of researchers is growing from the meetings that take place there. International scholars have been invited and generously offered from their expertise. Some of the invited researchers so far have been Frank Lester, Diana Lambdin, Uri Leron, Erkki Pehkonen, Gunnar Gjone, Carl Winslöw, Morten Blomhöj, Paola Valero, and Gabriele Kaiser.

Workshops

NoGSME organises workshops on central research issues of interest for the participants in the Graduate School (Grevholm, 2007a). The activity involves both doctoral students and supervisors. The first workshop dealt with classroom research in mathematics education. The second workshop focussed on research on mathematics textbooks. The experts here were Birgit Pepin and Linda Haggarty. Here a Nordic network for research on mathematics textbooks was created. A third workshop on research on use of ICT in mathematics education took place with 25 participants and two invited experts, Luc Trouche and John Monaghan. The fifth workshop was on mathematics and language with Heinz Steinbring and Candia Morgan as invited guests. A workshop on Justification of research in mathematics and science education with special emphasis on the role of theory in such justification was lead by Mogens Niss and it was closely linked to the corresponding doctoral course. Patricio Herbst was one of the invited lecturers. In the latest workshop about Use of ICT in mathematics education

– neither salvation nor catastrophe several Nordic researchers contributed, such as Mette Andresen, Per Eskil Persson and Christer Bergsten.

The board of the Nordic Graduate School in Mathematics Education

The board consists of the director, one member from each of the five Nordic countries and a representative for the Baltic countries. Board members currently are Barbro Grevholm, director, Christer Bergsten, Sweden, Trygve Breiteig, Norway, Ole Björkqvist, Finland, Gudny Gunnarsdottir, Iceland, Madis Lepik, Estonia, and Mogens Niss, Denmark.

The members of the board are not paid for their work, but contribute for idealistic reasons and as part of their positions at the home university. The board meets about three times a year in connection to other NoGSME-activities. The board is responsible for the initiatives and work and has to report to The Nordic Research Academy once a year. Most of the board members are also active in their national society for research in mathematics education and in national graduate schools.

Cooperation with Nomad

NoGSME has close cooperation with the journal Nomad, Nordic Studies in Mathematics Education. Doctoral students and supervisors are invited to publish in Nomad, and in each issue of Nomad a few pages are devoted to the NoGSME programme and activities (Grevholm, 2006a). Here they can publish in their Scandinavian mother tongue or in English.

International centres of excellence are working partners

To get support for the application to NordForsk in order to get financing for NoGSME we turned to a number of important international centres of excellence and asked them to write letters of support for us. Leaders from these centres have then been involved in our plans and activities in different ways. The centres we collaborate with are Institute of Advanced Study, La Trobe University, Gilah Leder; Concordia University, Anna Sierpinska; University of Michigan, Hyman Bass and Deborah Ball; University 7, Paris, Michele Artigue; and University of Klagenfurt, Didaktik der Mathematik, Willibald Dörfler.

Another important discussion partner has been Jeremy Kilpatrick, who is well informed about Nordic conditions relating to mathematic education. He has among other things been a guest professor at Gothenburg University and the supervisor of some Swedish doctoral students. All the above mentioned features of the Nordic Graduate School has been used to argue for a Norwegian national Graduate School in ME in the application sent in recently.

Results and outcomes of the Nordic Graduate School

The activities of the Nordic Graduate School are building strength in Nordic research for the future (Grevholm, 2006e). The knowledge and contacts that doctoral students and supervisors are getting from the events together offer insights that can not be achieved from reading books or by other means. In the future these links will be important and valuable for the field of mathematics education. Models of organising research education and supervision can be compared and developed and fruitful ideas from one university can be spread to other places (Grevholm, Persson & Wall, 2005). It is especially important for the Nordic Graduate School to build the contacts with colleagues in the Baltic countries.

Some features in order to strengthen the quality of researcher education

Ninety percent seminars

Mathematics education as a field of research is developing in the Nordic countries but it is still a young area and there is a need to assure the quality of the work and to live up to international expectations and standards. A number of initiatives have been taken in order to raise quality. For example, both in the Swedish Graduate School and at UiA in Norway we have introduced what is called ninety percent seminars. This means that when the student and supervisors agree that there is a manuscript of about 90 % of the final thesis a seminar is organised. To this seminar an international scholar, who is expert in the area of study, is invited. He or she reads in advance the 90 % finished manuscript and gives constructive and creative feedback during the seminar, which is organised as a disputation. The intention of the seminar is to inspire the doctoral student to raise quality in the final phase of writing and to get fresh ideas how to improve the dissertation and to be aware of possible criticism before it is too late. The seminars have proven to be of utmost value to both the doctoral students and the supervisors. International scholars have generously given from their expertise in these discussions.

International studies

Another feature of importance for quality is international collaboration and studies abroad. There is an expectation for the students to spend one semester at another university, thus learning about a different academic institution and meeting other mentors and supervisors. This has functioned in Sweden, where the programme is often taken over five years but has been difficult to realise in Norway within a three year programme. The students feel the time pressure too hard for going away for such a long period of time. As compensation we have invited many international scholars to give seminars at UiA, but this is of course not the same as spending time abroad. We are working on how to improve this feature of the education. Internationalisation is also a concern of the Norwegian and Swedish educational authorities (SOU 2004:27). The issue of internationalisation is one that will be handled after the evaluation of the programme.

Models for supervision

Supervision is a crucial part of the doctoral education. In order to ensure good and continuous quality in supervision we have decided at UiA to have at least two supervisors. Supervisors move, get sick or retire and it is important that the students are not left in an unstable situation. Joint supervision and other forms for organising supervision must be considered. At Luleå University of Technology a dynamic model of supervision with many levels have been used and proven successful (Grevholm, Persson & Wall, 2005). The model reflects an apprenticeship theory for the doctoral education, which seems to be embraced by many of the supervisors. International experiences from work with quality of supervision and design of programmes have been followed closely by the Nordic community (Lester & Lambdin, 2003; Schoenfeldt, 2003).

Public defence of the dissertation

A public defence of the dissertation and invited international opponents is typical of the Nordic doctoral education. It seems very important to have open discussions, where anyone can question and criticise the dissertations. Also the publication of theses, which makes them accessible in libraries to everyone, is valued in the democratic Nordic societies. The publishing of thesis is the normal situation in Finland and Sweden and often is the case also in Denmark and Norway. Nowadays in addition to the printed books with theses there is often also an electronic version on the internet.

A Nordic Journal for Mathematics Education

The close collaboration with the journal Nordic Studies in Mathematics Education, Nomad, is of great value to supervisors and doctoral students in the Nordic countries. This journal is the natural choice for the first publications of the students. But many of them prefer other international journals as ESM, JMTE, IJSME or FLM.

Crucial or critical issues for the mathematics education doctoral programme in Norway– issues to investigate in the evaluation of the programme

Supervision in a new research field

Trying to build up and expand a new research field is not an easy task. The most problematic issue has been that here have not been many experienced researchers, who can function as supervisors. In Sweden, for example, many mathematicians accepted to be supervisors when the national graduate school started. Some of them realised that they could only be of help for general matters in the education and someone else had to do the actual mathematics education supervision. But others actually thought that they had the expertise (being expert mathematicians but amateurs interested in teaching and learning of mathematics). Thus over the years there has been a number of situations, where the board of the graduate school had to assist in finding new supervisors, often by using an international scholar as additional supervisor. Also it happens that the student and supervisors are not getting along in a good way and a shift of supervisor has to be made. This is difficult when not many choices are available. Thus some supervisors have been used to an extreme extent over some years.

As the access to experienced supervisors was limited there was a need to build competence. This has been tried both in the Swedish Graduate School and in the Nordic Graduate School. The success was limited in the first case because of lack of interest within the group of supervisors. In the Nordic Graduate School it seems to work well. The education of new supervisors is crucial for future survival of the area and we are focussing on getting all the new doctors to participate, thus fostering the future generation of supervisors. The quality of supervision is critical for the outcomes and international contacts and links are of extreme value here.

A first national conference on supervision of doctoral students was held in Sweden in 2003 and some research has been carried out in this area (Strömberg, 1979; Strömberg Sölveborn, 1983; Lindén, 1998). The international community in mathematics education has also cared for the issue of supervision (Hart & Hitt, 1999; Leder, 1995).

Intersubject collaboration

Collaboration between researchers in mathematics, mathematics education and general education has been tried in all the Nordic countries with varied success. In the beginning of the Swedish Graduate School there seemed to be a mini Math War going on. Later this faded away, probably because the mathematicians realised that what was going on is not dangerous for them, rather contrary. This development is even visible in the evaluation of the graduate schools in Sweden (Persson, 2006).

Issues of format and language in theses

The format of the thesis – monograph or selection of papers with preamble ("kappa") has been much discussed in the graduate schools. The tradition from pedagogy is to write a monograph and from mathematics it is a selection of published papers with a preamble. As most of the students have been situated in mathematics departments they have been strongly influenced to write a selection of papers. From the 9 in Sweden finished so far there are only a few "strict" monographs. One author wrote the licentiate thesis as a monograph and the second part of the thesis as a selection of papers. Another discussion is how many of the papers must be published in journals before the dissertation. In mathematics there has been a development towards accepting theses where none of the papers are published. So there has also been shifting of traditions in mathematics education. One of the Swedish students had 6 published papers in the thesis, and others had only two or three non-published papers.

Another critical issue is the question of language for the dissertation - mother tongue or English? In Sweden there has been a public debate about scientific papers written by Swedes in bad English. They are claimed to make fools of themselves internationally. It is obvious that almost every non-native English speaking writer is much better in expressing fine nuances in the mother tongue than in English. But it is also clear that writing in English opens for international readers. And later on researchers must write papers in English anyway. Not using mother tongue leads to a poor scientific language in the local languages and publications that will not be read by teachers in school. There are many pros and cons to consider before the decision on language is taken. In the end it is up to the student and the supervisor and must be taken in each specific case taking care of the circumstances for each student. A student who has writing difficulties anyway will have still worse problems if the writing is in English. In the programme in UiA careful discussions are held between the doctoral student and the supervisors before the decision about choice of language is taken.

Financing during and after the dissertation

The sources for financing doctoral studies differ from one place to another. In Sweden and Norway the student must have guaranteed financing for the studies before he or she can be taken up in a doctoral programme. The state offers a number of doctoral positions and there can be positions inside specific research projects. The student is employed by the university for 3-4 years and has legal rights as an employee. The salary can be compared with that of a beginning teacher. After the dissertation the position is finished. There is a lack of post doctoral positions in didactics of mathematics and this creates problems for those who want to go on at once after the dissertation and qualify themselves to become a docent. In Sweden and Finland this is an academic title for which one must qualify through research and publications after the doctoral degree (the same as Habilitation in Germany). The normal rule of thumb is to publish as much as a second thesis. An application must be made to the faculty and the scientific work is evaluated by external international experts and a public popular scientific lecture is given and evaluated by a scholar in another research field. Based on these activities the decision is taken about receiving the docent title. In Sweden the main supervisor of a doctoral student must be at least on the level of docent.

In Norway an academic teacher can apply to be promoted to docent, based on the scientific production and experience. This position could be considered to be at the level of a professor in non-Nordic countries.

In the Nordic countries academic studies are free, no costs are paid by students but all is paid by tax-money. Thus the salary of a doctoral student can be used entirely for the private consumption.

Vulnerability of small research environments

Another critical issue is the fact that many research environments in mathematics education in the Nordic countries are small with only one or two faculty members and one or two students. It is difficult to solve the supervisor problem and to create a vivid and inspiring work situation in a community of researchers. One solution for this situation is collaboration between two or more institutions or to be part of a graduate school. The earlier evaluations indicate that graduate schools are efficient in offering what the student needs as a complement to a small environment (Persson, 2006).

Opportunities to finance collaboration in graduate schools or Nordic networks

Collaboration in networks of graduate schools is rewarding and helps to assure quality. But there must be financial resources for such work. In Finland the graduate school succeeded in getting a continuation but in Sweden so far this has been unsuccessful. It is critical to find opportunities to solve this problem. In Norway it remains to be seen if the application is successful. The research environments that have been built up during the time of the graduate schools can very easily be torn down again if there is no continuation of the collaboration.

Gender balance

The Nordic communities in didactics of mathematics seem to be in equal proportions of female and male students. But among supervisors there is an overweight of male academic teachers. Continued work is to be done in order to improve the lack balance among supervisors. The Nordic professors in didactics of mathematics were male dominated until 2003 when suddenly four female professors were appointed. Another additional female professor in 2007 almost creates gender balance in this small Nordic group.

The importance of knowledge of mathematics by doctoral students in the programme

The fact that the program is situated in a mathematics department indicates that mathematics plays an important role. A solid foundation of mathematics must be part of the bachelor and masters education that forego the doctoral education. In the Swedish Graduate School the emphasis on mathematics was still greater as doctoral courses in mathematics were a substantial part of the coursework.

The participants in the Norwegian doctoral programme

Doctoral students

Who are the doctoral students in the Norwegian programme? Most of them are Norwegian students with teacher experiences either on school of university level, often as teacher educators. Some have rather long teaching experience and thus are not so young any more. Few students come directly from the basic academic education. Many of the students have taken their masters degree at UiA where an interest for research has been created, and they later come back to continue the studies at doctoral level. Some students have academic positions as teacher educators and are encouraged by their institution to take a doctoral degree in order to fulfil their career at university. One of the students is a retired school consultant and has much experience from school development and curriculum development in Norway. He is excellent in writing about the Norwegian development of school mathematics over 50 years. Most students are in an age where family, housing and children are important questions. Thus they cannot easily move, go abroad or change their conditions. Long courses demanding presence at other places than the home university is problematic. These are all conditions that influence the opportunities in the doctoral programme and they must be taken into account in the planning of activities for doctoral students.

Teachers and supervisors

The supervisors in the doctoral programme have different backgrounds. The average age of the supervisors at UiA is 61 years and many have a long and varied experience as teachers, teacher educators and professors before they entered the programme. Only one supervisor is below 50 years of age, which is problematic for the future of the programme. The research education of the professors also varies from mathematics to history of mathematics and to mathematics education. All are active researchers in didactics of mathematics and have been so for a long time. A few of the professors have experience from building up programmes and management of doctoral education. Only a few had experience from supervision of doctoral students before the programme started in 2002 and not all had developed and taught doctoral courses.

The future of doctoral programmes in mathematics education in the Nordic countries

Do we have a critical mass of researchers in order to keep the activities alive? How do we ensure quality and endurance of programs? What opportunities are there to improve the programmes and in what ways?

Is there a need for more persons in the labour market with a doctoral degree in mathematics education? Do we need research on doctoral education in mathematics education? Will society continue to ask for research in mathematics education?

There are many questions to inquire into and try to answer about postgraduate education in the Nordic countries (Grevholm, 2007b). The cultural and social conditions are similar in the five countries and problems are often the same. Also solutions seem to be similar and the public debates have parallels.

In Norway the doctoral education was restructured in 2002 and the Ph D was introduced to replace earlier degrees. An intense debate is going on about how much resources should go into research and Norway is lagging behind the other Nordic countries so far. The number of new doctors is increasing though, but many seem to need far longer time than the planned three years. Many universities are worried about the prolonged study time and try to implement incentives to shorten the study time.

The government in Sweden has shown great concern about the research education. It was restructured in 1998 and a first evaluation of the results was published in 2007 (Högskoleverket, 2007). One outcome is that the students that graduate within a period of five years have increased from 16 to 28 % of the population. The number of degrees has increased with 50 % after the reform and stays at that level. In 1990 0.6 % of the working population (between 25 and 64 years of age) had a doctoral degree and that increased to 1.0 % in 2005. An investigation in 2002 took care of specific questions about the doctoral time and the time after graduation (SOU 2004:27). Doctoral education has expanded with 100 % between 1990 and 2000. In Sweden the number of doctoral students is about 13000 (fulltime equivalents). It would be astonishing if there were no problems in such a strongly expanding activity. A large generation of persons born in the 40ties is in the process of retiring and the new academics with a doctoral degree seem to have a prosperous labour market to enter into. As mathematics knowledge is seen as one of the tools a citizen in a modern society will need it seem probable that questions about teaching and learning mathematics to still larger groups of the population will be in focus. Most governments realise that we are moving into an international society, where the human capital resources in the form of education and competence are the means to survive and compete internationally through excellence and growth.

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ONE SUCCESSFUL ALTERNATIVE TO THE TRADITIONAL DOCTORAL PROGRAM

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Abstract

The traditional mode of completing a Master's program and teaching for several years before entering a doctoral program has not met the needs of the profession. The Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics, with funding from the National Science Foundation, created a multi-institutional, cohort model for doctoral students studying mathematics education with an emphasis on the issues and concerns of rural areas. West Virginia University, University of Kentucky, Ohio University, University of Louisville, and University of Tennessee collaborate in order to provide courses in mathematics, mathematics education, research and rural education. Distance education, summer institutes and an internship prepare students for research in the teaching and learning of mathematics in rural settings.

Introduction

The Carnegie Foundation devoted five years to the study of the future of doctoral education. Their work defined the purpose of doctoral education as preparing "stewards of the discipline." Stewardship embodies not only a knowledge base, but also a philosophy or set of principles. The holder of a doctorate should be capable of generating new knowledge, conserving the most salient existing knowledge, and transforming knowledge through representing and communicating those ideas clearly and effectively to the benefit of society (Golde & Walker, 2006).

Reys, Teuscher, Nevels and Glasgow (2007) acknowledged that the shortages of people with doctorates in mathematics education, first documented a decade ago (Reys, Glasgow, Ragan & Simms, 2001) is still hampering the discipline. Approximately half of those completing the doctorate return or remain in their current position. Positions in top institutions often take from two – three years to fill. Teacher preparation, quality research, and policy development suffer from this shortage.

The traditional mode of doctoral education follows the scientific mode where students complete a Bachelor's degree during which the best are guided into graduate work. After completing a Master's, a subset of the previously best are guided toward research and into a doctoral program. This model has created a highly talented supply of researchers and scientists. However, moving that model to education creates an inexplicable conundrum. Many doctoral programs in education require three more years of classroom teaching. This requirement makes sense in that preparing future teachers and doing research in teaching and learning needs to be grounded in the reality of the classroom. But, by the time potential students in education have met entrance requirements, they have accrued family and debt. Returning to graduate student poverty is not a realistic option. A full-time, multiple-year commitment is considered the gold standard for a quality program.

Both admission and residency requirements are impediments for many students. Admission requirements do vary and some programs admit those with little or no teaching experience. But, then upon completion of the degree, these new Ph.D.s often find their employment options limited in that some accreditation (NCATE) standards require those preparing teachers to have had classroom experience. Therefore, top tier programs turn away highly qualified candidates in order to be fully accredited. Residency requirements also vary from program to program in nature and administration. Some programs accommodate part time students who are holding full time jobs and the residency requirement is met by taking x number of credit hours within a specified period of time. This meets the letter of the requirement if not the spirit. The intent of the residency requirement is focused on spending a significant period of time working in a research oriented environment with researchers active in the field. Unfortunately, even for full time students this can degenerate into time spent assisting the institution in low cost teaching and tasks other than research.

Rural areas, with their geographic isolation, make pursuing an advanced degree even more problematic. With the nearest universities often four or more hours away by car, teachers are unlikely to be able to pursue advanced degrees even on a part time basis. Add to that the poverty and lower teacher salaries, continuing one's education is beyond the grasp of most rural teachers. These teachers have an untapped knowledge base of what it means to be a rural mathematics teacher, and represent an untapped partial solution to the shortage of mathematics educators nationwide. An alternative to the traditional doctoral program was and is needed to address both the need for more mathematics educators as well as the difficulties rural mathematics teachers face in trying to pursue an advanced degree. Although distance education has made inroads in these locations, the residency requirement of most doctoral programs puts a halt to many teachers' plans. Creativity, flexibility and technology can help reach this untapped pool of potential mathematics education leadership.

ACCLAIM

The Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM), with funding from the National Science Foundation, created a multi-institutional, cohort model for doctoral students studying mathematics education with an emphasis on the issues and concerns of rural areas. West Virginia University, University of Kentucky, Ohio University, University of Louisville, and University of Tennessee collaborate in order to provide courses in mathematics, mathematics education, research and rural sociology/education. The coursework and beyond-course experiences were designed to meet the requirements of each institution and to address the guidelines for doctoral programs in mathematics education as published by the Association of Mathematics Teacher Educators (2002).

The ACCLAIM program is non-residential with a large portion of the academic component delivered by distance education. This program offers highly capable and dedicated but geographically bound students the opportunity to pursue an advanced degree emphasizing the teaching and learning of mathematics in a rural setting, and it does so without requiring them to leave work and family for long periods. Students attend intensive five-week seminars each summer on a university campus. The rest of the coursework is completed via distance education in both synchronous and asynchronous modes. (See Appendix B)

Admission requires 15 semester hours of graduate mathematics, significant classroom experience, and a passion for rural education. The program of study includes 15 semester hours of graduate mathematics, 18 semesters hours of mathematics education courses, 9 semesters hours of rural sociology/education courses and 12 semester hours of research courses. The collaboration among the universities gives students access to a larger mathematics education faculty than would be available through any individual institution. The funding and the distance education model also allows the program to bring in noted scholars from across the country with expertise not found in any of the participating institutions. For example Dr. Paul Theobald, who holds an endowed chair in rural education in Buffalo, New York, has taught a course for each cohort.

Delivery of courses

The delivery of courses via a distance model was essential for ACCLAIM to reach the targeted rural population. A variety of software exists to support web-based instruction in both the asynchronous and synchronous modes. Blackboard and WebCT are two commercial class management systems. Moodle is open source software that provides the same kinds of tools: drop boxes for assignments, discussion boards, posting of announcements and assignments, grade books, etc. Centra, E-luminate and Adobe Connect are frequently used tools for synchronous communication. These tools allow for real-time discussions, shared applications, small group discussions, etc. Students can use icons to raise their hand, applaud, laugh, cry, and answer questions. The instructor can give students the "microphone," give them control of an application such as a spreadsheet or a dynamic geometry tool or send them to a particular website. Video is also possible through mini-cams.

ACCLAIM experimented with a variety of delivery modes but has been most successful with a combination of an asynchronous component managed by Blackboard or Moodle and a synchronous component mediated by Centra. The synchronous component is deemed absolutely necessary for a variety of reasons. To create a virtual classroom, discussion and interaction between students and instructor and between students is vital to promote deep thinking and the sharing of ideas in real time. Required meeting times help students involved in work and family to stay on task and on a reasonable timeline. For a cohort model to succeed all students must progress in relatively the same time frame.

Mathematics, mathematics education, research and rural courses have all been successfully, delivered via distance learning. Rarely, however, each content area has experienced a less than fully successful delivery. The content is not the deciding factor; the teacher is. Not all professors can adapt their teaching style and philosophy to the distance mode. Some thought they had to be available 24/7 and soon found themselves totally bogged down in responding to constant and repetitive emails. One responded by stopping all communication midway through the course which caused a mild revolt on the part of the students. This issue can be resolved by posting questions to a discussion board where all can see and setting consistent "email hours" analogous to office hours on site. Others used the synchronous time to assign a myriad of readings and assignments which were unreasonable in terms of time required to complete. Again a revolt on the part of students helped educate some but caused one or two to throw up their hands in despair. A quick discussion and comparison with the onsite syllabus usually created a reasonable compromise. A common complaint among successful faculty was the challenge of teaching from a stationery position—that is sitting in front of the computer. Some alleviated this with a wireless microphone during discussions but found it is still necessary to monitor the screen consistently to know who has their hand up and who may not be responding at the level desired.

The students accepted into this program were all highly talented, highly motivated students. Their persistence proved to be both a joy and a challenge. These students insisted upon learning, expected to work hard and expected the highest quality of teaching. They were not necessarily gentle in communicating when their expectations were not being met. As such, even mild revolts had to be addressed. The program did not cater to student whims but this dual responsibility—students and program administrators—helped to insure a quality program.

Assignments

All of the types of assignments are possible using technology strategically. Examples of "Investigate/Summarize/Discuss/Revise Tasks" include (Burke, 2007) ERMO Summaries. In this activity, students are assigned to read an article or set of articles and write an "Earn the Right to My Opinion" summary of the readings. Students read each other's ERMO summaries and write a critical reflection. More and richer feedback is possible using a discussion board format. Another type of assignment which parallels the traditional classroom is "Research Papers and Paper Conferences." Each student is asked to write a research paper for a course. On a specified date they post their research papers in a folder set aside for their group. Critiques are posted and a group discussion is held moderated by the professor.

One of the significant advantages of working with place-bound teachers in the online environment is access to the living laboratory of their classrooms. This enables students to validate and commiserate with those in like positions but also provides insight into classrooms at different levels and from different types of institutions. Secondary teachers gain a better appreciation of what it means to teach mathematics to second graders—they usually quite surprised by what 8 year olds can do. Community college teachers learn what it means to teach at a research active institution. The initial reaction is one of envy of a teaching load of 2 classes yet when the research responsibilities and expectations are communicated, most are appreciative of their 4-5 class teaching load.

Role Playing, Lesson Study and Curriculum review and development are just a few activities that can be mediated through technology. Sometimes "inquiry" takes on forms not captured by our previous categories of tasks. In the online environment, with its shared text spaces that students can contribute to asynchronously, Joint Productive Activity or Development Tasks progress sometimes more efficiently than when students attempt to meet face-to-face. These joint activities can build resources for the entire class to access. Professors set the task and its boundaries and contribute start-up resources. Students and the professor contribute to and monitor the development of the resources. Again as the teachers are thoroughly grounded in the classroom, these tasks become more real, more meaningful and more applicable. A by-product of this classroom involvement brings the students in those classrooms into the learning of their teacher. Many have no concept of higher education. Seeing learning being modeled and enjoyed by their teacher can be very motivating.

Many of our courses focus on mathematical topics. Problem solving with an exploration of some phenomena and then the posing of deep problems that challenge everyone in the class work well in both the synchronous and asynchronous modes. In groups students engage in dialogues and shared strategies. They look to each other for insights and eavesdrop frequently on the discussions of other groups. The professor can move from group to group monitoring the progress and the on-task behaviors necessary for learning. In addition, students may teach various segments of the courses, lead a discussion on an article, or demonstrate a selected piece of software. Mathematics can be done “on the fly” using one of various peripherals and student-created work can be shared either via document camera or through student work generated in real time.

Residency

The traditional residency requirement in doctoral work is designed to ensure a student of at least one year of in depth work in an academic setting with professors actively engaged in research, teaching and service. In lieu of a traditional residency requirement (usually technically defined as 2-3 semesters of full time enrollment), ACCLAIM has negotiated an innovative residency which includes two summers of 9 semester hours of coursework each at one of the participating institutions and the academic year in between (third year of the program). During this academic year, students are enrolled in 3 semester hours of internship each semester.

The internship is based on the *Guidelines for Doctoral Programs in Mathematics Education*, specifically guideline 6. “Mentored internships focused on acquiring expertise in collegiate teaching, supervising student teachers, designing and implementing a research study, designing and facilitating professional development activities for teachers, preparing grant proposals, and writing papers for publication; “(AMTE, 2002) Each student negotiates an individual contract outlining specific activities selected to add to their knowledge in the areas of K-16 teaching, supervising student teachers, designing and implementing a research study, designing and/or facilitating professional development activities, preparing grant proposals, and writing for publication. (See Appendix A.) For example with respect to K-16 teaching, a high school teacher would be charged with observing for several consecutive days in a primary classroom, in a middle school classroom, and perhaps, working with a college professor to re-design a syllabus. For professional development, if a student had never presented at a professional meeting, they would be charged with getting on the program at a local or state meeting. If they had presented at a state meeting, the charge might be getting on the program of an NCTM regional.

The students meet on-line every two weeks to discuss their activities and to share thoughts with respect to a paper addressing one of the areas of study. Blackboard is used for students to submit their completed activities and track their progress through their contract.

Summary

Staying on the job, while completing an advanced degree, allows the student's theoretical work to remain fully grounded in the reality of the classroom and does not further deplete the supply of mathematics teachers. Such alternatives can supplement existing programs and help alleviate a critical shortage. The power of technology should not be over estimated. Technology can provide access to help mitigate, but perhaps not solve, some of the problems in education. Geographically isolated, disabled, and place- bound populations can be reached with high quality educational opportunities. Shortages in critical areas can be alleviated. However, the power of tradition and inertia should not be underestimated. University policies and practices can create huge barriers particularly in working across institutional lines. Professors are often bound to tradition with good reason and reluctant to risk unproven methods and techniques. However, creativity, flexibility, and external funding can help. The ACCLAIM program is an existence proof. Duplication and replication are next steps.

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Appendix A:

INTERNSHIP WORKSHEET

Characteristics of ACCLAIM Internship

Individualized - Individual plans of study will be negotiated for each student by the end of the Summer Institute, 200x

Instructor of Record:

Mentored - various individuals will assist and support each internship including the ACCLAIM personnel and other professionals in the field.

Electronically supported - through Blackboard and Centra with bi-weekly discussion sessions and through web based log books

Clock Hours - approximately 90 clock hours is required for a 3 semester hour lab course

Objectives

During the two semesters of internship students will acquire expertise in:

k-16 mathematics teaching and the preparation of K-16 mathematics teachers,

supervising student teachers,

designing and implementing a research study,

designing and/or facilitating professional development activities

preparing grant proposals, and

writing papers for publication.

These objectives will be met through observation, reading, reflecting, conversation, practice, interviews, etc.

I. k-16 teaching

Observation in classrooms at levels, (elementary, middle school, high school, and/or college) with which you have no recent experience

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

b) Read and reflect on critical issues at various levels

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

c) Explore the political climate of mathematics education

Accreditation issues – K-12, Higher Ed
Certification issues – K-12
Impact of rural context in political decisions

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

d) Consider the role of technology in mathematics education at the various levels

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

II. Supervising student teachers,

Assist in the evaluation of a student teacher/intern
Mentor a student teacher/intern
Shadow a supervisor of student teachers
Develop/critique the syllabi for mathematics methods courses in the region.
Attend Association of Mathematics Teacher Educators Annual Meeting (required)

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

III. Designing and implementing a research study

Capturing potential research
b) Narrowing the questions
c) Developing logical argumentation/literature review
d) Conducting and sharing conversation(s) with researcher(s)
e) Attending/participating in an ACCLAIM Research Symposium

f) Attending/participating in the Research Pre-Session (NCTM)

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

IV. Designing and/or facilitating professional development activities

a) providing pd activities for colleagues
b) attending/participating in ACCLAIM TE/PD opportunities
c) evaluating on-going pd activities for a school or department

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

V. preparing grant proposals, and

a) being a part of a concept team in a proposal development
b) being a reviewer of proposals for a funding agency
c) writing and submitting a proposal

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

VI. writing papers for publication

answering a call for papers by a professional journal
writing an occasional paper for the Rural Mathematics Educator
being a reviewer for a professional journal

Recent Experience	Projected plans
1.	1.
2.	2.
3.	3.

Appendix B

Tentative Template for 2004 ACCLAIM Cohort

Summer #1 (2004) – Location: Athens, Ohio

		Content	Institution	Instructor
	3 hours Mathematics	Geometry	UK	Lee
	3 hours Education/Mathematics Education	Learning/ ssessment	WVU	Mayes
	3 hours Rural Sociology/rural education	Historical perspective	OU	Howley/Howley
Fall #1 (2004)	3 hours – Mathematics Education	Curriculum	OU	Schultz
	3 hours Mathematics	Linear Algebra	UK	Lee et al
Spring #1 (2005)	3 hours – Mathematics	Discrete Mathematics	UT	
	3 hours – Research	Quantitative	UK	Xin Ma

Summer #2 (2005) Begin residency Location: West Virginia

	3 hours Mathematics	Advanced Algebra	WVU	
	3 hours Education/Mathematics Education	Pedagogy	UT	Long
	3 hours Research	Qualitative	WVU	Webb-Dempsey
Fall #2 – (2005)	3 hours – Rural Sociology/rural education	Current Status	UK	Theobald

	3 hours – Internship/Mathematics Education	Technology	UT	Long
Spring #2 (2006)	3 hours - Mathematics	History of Math	UK	Lee
	3 hours – Internship/Research	Research	UL	Bush
Summer #3 (2006) Complete residency Location:		Louisville		
	3 hours Mathematics	El Math from Adv. Standpoint	WVU	Mays
	3 hours Education/Mathematics Education	Research Trends in MTE	UL	Bush
	3 hours – Rural Sociology/rural education	Implications for Practice	UK	DeYoung
Fall #3 (2006) -				
	3 hours – Mathematics	Adv. Calculus	OU	Connor
Spring #3 (2007)	3 hours – Research	Design	OU	Arlie
	3 hours – Mathematics Education	Adv. Studies in ME	UT	Taylor
Summer #4 - Write Comprehensive exams/Begin Dissertation				

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COMPLEXITY AND UNCERTAINTY AS DRIVERS FOR A PH.D. IN MATHEMATICS AND SCIENCE EDUCATION

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University of Wyoming, USA

Keywords: *Complexity, Cognitive Science, Interdisciplinary*

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Introduction

The Science and Mathematics Teaching Center (SMTC) at the University of Wyoming has been collaborating with the College of Education on revising the Ph.D. program for Mathematics Education and the Ph.D. program for Science Education. Currently the programs are under a college wide Ph.D. in Education which requires a significant number of general education courses (16 hours), an advance research core of courses (12 hours), and the standard dissertation hours (16 hours). These required 44 hours of courses leave too little room for innovative cognates in mathematics and apprenticeship experiences. We are striving to create a novel Ph.D. program that integrates concepts of complexity and uncertainty in mathematics and science, integrated science and mathematics cognates, and apprenticeship experiences in mathematics and mathematics education.

Complexity science, computational science, and cognitive science provide new paradigms for the study of mathematics education. First, complexity science can serve as a driver for both mathematics content and mathematics education. Two of the most pressing and complex problems of our time, energy resources and environmental issues, require citizens that can bring an integrated mathematics and science perspective to bear on the problems, as well social, political, and economic lenses. These issues will provide the context for the study of complexity and uncertainty in our program. Complexity theories can also be applied to the science of learning systems, accounting for the interactions of multiple agents, as opposed to the study of individual components of a system (Davis & Simmt, 2003).

The field must move beyond efforts to bridge individual learning and social learning, to trying to understand the emergent classroom community. Second, cognitive science provides theoretical underpinnings for the teaching and learning of mathematics. Findings in this field have a major influence on how the teaching and learning of mathematics and science are viewed. Third, computational science with its focus on modeling scientific phenomena, large data base analysis, and computational efficiency is driving efforts at integrating science and mathematics. The integration of science with mathematics through modeling should impact K-12 schools in significant ways, making this an important potential driver for the Ph.D. program.

Ph.D. programs need to develop future leaders with expertise in issues of complexity, uncertainty in modeling, and in integrated approaches to science and mathematics. At the University of Wyoming we are revising our Ph.D. program to include:

Cognates in key areas:

- Mathematics Education Cognate with an emphasis on complexity, uncertainty, and rural education;
- Mathematics Cognate with an emphasis on mathematical modeling, simulation, and applied algebra;
- Cognitive Science Cognate specializing in mathematics cognition, learning theory, and assessment.

Immersion in authentic mathematics education and teacher education experiences, including:

- Teaching apprenticeship in undergraduate mathematics and mathematics education;
- Outreach apprenticeship in professional development through the Science and Mathematics Teaching Center (SMTC);
- Mathematics education research experiences in the study of teaching and learning complexity and uncertainty and rural education;
- Mathematics research experiences in modeling and computational sciences, potentially in conjunction with the NCAR Super Computing facility coming to Wyoming.

In this paper we will discuss the components of such a program and why they are important.

National Trends

Reys, Teuscher, Nevels, and Glasgow (2007) researched current doctoral programs in mathematics education. They found that over half of the institutions in the United States require a student pursuing a secondary emphasis in mathematics education have a BS/BA in Mathematics or Mathematics Education and over half strongly encouraged applicants to have a master's degree in one of these areas. The institutions in the study reported that the strongest areas of emphasis in mathematics education doctoral programs are Research in Mathematics Education (98%), Research Methods (97%), Mathematics Content (90%), Learning Theories (83%), Teaching/Professional Development (83%), and Mathematics Curriculum (80%). The Association of Mathematics Teacher Educators (AMTE) and the National Council of Teachers of Mathematics (NCTM) established *Principles to Guide Doctoral Programs in Mathematics Education* (AMTE & NCTM, 2002) in which they supported the areas of emphasis identified above. They emphasized the need for the mathematics content to broaden and deepen the mathematical knowledge around the big ideas in the pre-K-14 mathematics curriculum and to examine how those ideas develop throughout the curriculum. They recommended the inclusion of seminars, clinical experiences, internships, assistantships, and independent study to support coursework. The revised Ph.D. program at the University of Wyoming will incorporate these recognized components and recommended approaches. We also wanted to build on the strengths of the University of Wyoming in the areas of energy and environment. The nexus between energy and environment is complex and uncertain, lending itself to study through computational science and mathematical modeling. Our goal is to integrate cognates between a Ph.D. in Science Education and a Ph.D. in Mathematics Education, using energy and environment issues as the context.

Energy and Environment Nexus Driver

By the year 2050 the ever increasing demands for natural resources, energy, and water will require two planet Earths to satisfy. Worldwide issues of natural resource depletion, energy consumption, CO_2 emissions, climate change, and water shortages will be the pressing scientific problems of the next generation. A driving force in research will be the resolution of the natural resources, energy, and environment crisis confronting the world. Research addressing this issue will be interdisciplinary, require the collection and analysis of large amounts of interrelated data using technology, and engage scientists and politicians in complex problems with both scientific and social consequences. The real-world energy-environment nexus should be a driving force in science and mathematics education across the K-16 continuum, so we have an educated democratic citizenry that can make informed decisions and one that has opportunities in science and mathematics related careers. We want to create innovative Ph.D.

programs in science education and mathematics education with an emphasis in energy and environmental education which:

1. develops teacher educators who are leaders in integrating the concepts of complexity and uncertainty in mathematics and science into the K-12 classroom;
2. develops researchers addressing the significant research question: What is the cognitive capacity of K-12 students to develop a conceptual understanding of issues of complexity and uncertainty in mathematics and science?

To accomplish this we will create an energy-environment cognate consisting of a collection of courses where mathematics education and science education graduate students apprentice with mathematicians and scientists to develop expertise in energy and environmental sciences.

The graduate students in this cognate will develop expertise in cutting-edge science addressing the energy-environmental nexus so they can study its integration into the classroom at the K-12 level. Problems in this area require an integrated science and mathematics approach supporting expertise in the collection and analysis of large data sets, modeling of those data sets to make predictions, and integrating resources from science and politics to determine policy decisions. The Haub School of Environment and Natural Resources (HENR – policy issues in environment), the School of Energy Resources (SER – research on energy issues and alternative energy resources), and the Program in Ecology (PiE - expertise on diverse ecological aspects of energy development) at the University of Wyoming will partner to provide mentoring and collaborative research opportunities in the interplay of energy and environment. Special seminars will bring together graduate students with faculty in education and the sciences to explore issues of how energy and environment research should and can impact K-12 classrooms.

Permeating the STEM classroom from K through 16 with the energy-environment nexus will require substantial change in the educational system. This change must include the creation of curricula appropriate for different grade levels, authentic assessment tasks that measure students' conceptual understanding, content-based professional development that enables teachers to enact the curricula in meaningful ways, and research into the cognitive development of children with respect to issues of complexity and uncertainty. The Ph.D. programs will develop future mathematics educators and science educators that can address these future needs.

Complexity and Uncertainty Drivers

The Energy and Environment Driver provides a context for the principal themes driving the Ph.D. program: complexity and uncertainty. Energy and environmental research represent two strengths at the University of Wyoming, so they are a natural choice as a context for our university. While the context may differ depending on a university's strengths, the themes of complexity and uncertainty can be universal drivers for Ph.D. programs in mathematics education and science education. These themes lead naturally to a focus on computational science issues such as large database analysis and modeling, as well as technology's impact in the area of data collection, visualization, and data analysis. The graduate students will research the developmental aspects of students exploring complexity, uncertainty, modeling and scale in mathematics and science. What are the developmental aspects of gathering information, representing and modeling that information, analysis using technological tools, and decision making when there is a level of uncertainty? What are the appropriate developmental levels and learning trajectory for complexity and uncertainty across K-16 grade levels?

COMPLEXITY: Why complexity as a driver? The energy-environment nexus is a non-linear complex adaptive system with a number of diverse and independent agents, including scientific, social, and political, that are constantly changing and interacting with each other. Environmental challenges such as climate change and the loss of biodiversity display non-linear response, long range correlation, and disequilibrium through significant fluctuations leading to extreme events (Canziani, 1999; Hallam & Funasaki, 1999; Hull & Falcucci, 1999; Jorgensen, 1999; Giampietro, Mayumi, & Pastore, 1999, Svirezhev, 1999). Complexity science provides a theoretical framework for studying such complex adaptive systems (Kelly, 1994; Waldrop, 1992, 1996; Wheatley, 1999). Complex systems are non-linear, meaning that a small perturbation may cause a large effect (butterfly effect), a proportional effect, or no effect at all. Complex systems are open in that they are far from equilibrium, but they change over time in ways that can influence future states and therefore may produce emergent phenomena. The Ph.D. program will endeavor to have students move from viewing the natural world through a machine-like reductionist perspective where a complex system is understood by taking it apart and examining the components, to a complexity science view where there are a number of diverse and independent components constantly changing and interacting with each other. Studying only the components of energy and environment in isolation produces an incomplete understanding of the whole. Complexity science suggests that the natural tendency in problem solving of breaking down the problem into parts and solving a simpler problem is not sufficient for complex problems. A better approach to solving complex problems is to implement multiple approaches and then gradually shift time and attention towards those things that seem to be working best.

Davis and Simmt (2003) of the University of Alberta have turned the lens of complexity onto learning in mathematics. This provides a dualistic use of the complexity science perspective; not only are mathematics and science driven by issues of complexity, so is the teaching and learning of science and mathematics. They are researching the application of principles of complexity to the teaching of mathematics. Complex systems are adaptive and emergent. They define how a complex system adapts in a Darwinian evolution manner, changing its own structure; and how it is emergent in that it is composed of and arises in the co-implicated activities of individual agents. The central thesis of their work is that mathematics classrooms are complex systems in and of themselves, systems which are adaptive and self-organizing. The contrast between current theories of knowing, such as constructivism, with complexity theories of knowing is striking. The constructivist epistemologies are focused on particular phenomena, such as an individual's or group's construction of knowledge. But complexity science

> *is concerned with a range of nested learning systems which includes the co-implicated processes of individual sense-making and collective knowledge-generation. We might say that complexity science is more a meta-discourse, useful for reading across theories that are concerned with different levels or aspects of complex nested learning systems* (Davis and Simmt, 2003).

This view implies that we must move beyond efforts to bridge the phenomena of individual and social learning; from constructivism to trying to understand the emergent classroom community. It is our goal to take a complexity science view of the learning and teaching of mathematics and science in our programs.

UNCERTAINITY: Why uncertainty as a driver? The energy-environment nexus, as in many other areas of mathematics and science, requires that decisions be made with an acceptable level of uncertainty. The theoretical framework of uncertainty analysis is relevant to the Ph.D. programs' desired outcome of students working with uncertainty in mathematics and science. **Uncertainty analysis** aims to quantify the overall uncertainty associated with the response as a result of uncertainties in the model (Sayers, Gouldby, Simm, Meadowcroft & Hall, 2002). Uncertainty is divided into natural (aleatory) variability which refers to the randomness observed in nature and knowledge (epistemic) uncertainty which refers to the state of knowledge of a physical system and the ability to measure and model it. In science knowledge is often captured through an imperfect model or theory, causing the boundary between natural and knowledge uncertainties to blur and change over time. Analysis of knowledge uncertainty has three key components: define what is uncertain in the modeling process (sources of uncertainty), define how to quantify output uncertainty consequent on the sources of uncertainty, and define how to condition the uncertainty estimate as data on model-predicted variables become available. Students studying the energy-environment nexus will examine large data sets, create a conceptual model and represent it quantitatively with a graph or equation, and finally implement a procedural model that will provide quantitative predictions (Abbot, 2002; Beven, 2001). The model may not be an accurate mathematical description of the physical processes, so it is subject to three different forms of knowledge uncertainty: process model uncertainty (all models are an abstraction of realty and so have inherent error), statistical inference uncertainty (error in estimating the population from a sample), and statistical model uncertainty (multiple models may fit the data equally well over the sample, so which is best for extrapolations/interpolations). Mathematical and statistical quantitative aspects of modeling large data sets will be an enduring understanding that is valued as a student outcome in the Ph.D. programs.

Analyzing complex problems in the energy-environment nexus requires modeling large sets of data. There are a number of large, natural sciences data bases available for students to analyze such as: World Data Center System: NOAA's National Geophysical Data Center, Boulder, CO; Water Events Worldwide: United Nations Educational Scientific and Cultural Organization; Global Change Master Directory: Goddard Space Flight Center; and Global Resource Information Database: United Nations Environment Program – Sioux Falls, SD. The analysis of the data will require both quantitative and qualitative methods, including developing mathematical models to use for studying trends and making predictions.

Integrated Science – Computational Science Driver

The types of complex environmental, energy development and related issues facing society today simply cannot be addressed by any one traditional discipline or approach, and they cannot be resolved by basic or applied science alone. In response to this and related problems identified by the National Science Board (NSB, 2000), the National Science Foundation (NSF) convened the NSF Advisory Committee for Environmental Research and Education. This committee's report entitled *"Complex Environmental Systems: Synthesis for Earth, Life, and Society in the 21st Century"*

(Pfirman, S. and the AC-ERE, 2003) presents a challenge "…to develop environmental synthesis to frame integrated interdisciplinary research questions and activities and to merge data, approaches, and ideas across spatial, temporal, and societal scales. An essential part of this process is the effective communication of scientific information, models, and conclusions to and among researchers, educators, students, resource and industrial managers, policy makers and the public."

These recent NSF reports directly relate to the real-world complexities and uncertainties associated with analysis and management of any energy-environment project or policy, which can be among the most complex and difficult issues facing society today. As such, these kinds of projects and policies demand an interdisciplinary approach, encompassing the traditional disciplines of the physical, natural and social sciences; mathematics and statistics; law and politics (and more). This Ph.D. program will integrate graduate students into an interdisciplinary mix already in use at the University of Wyoming to conduct cooperative course delivery and research in the Haub School of Environment and Natural Resources, the new School of Energy Resources and the Program in Ecology.

The Haub School of Environment and Natural Resources (ENR) strives to prepare students to address societal complexity and uncertainty associated with estimating environmental responses to energy development scenarios, ENR coursework and research strategies use a "problem-based learning" approach, wherein student teams, guided by faculty mentors, attack a highly complex and seemingly intractable real-world project or policy problem and complete a major integrated assessment of the project or policy consequences. A graduate capstone experience and research opportunities will bring students and faculty from disparate disciplines together, serving as culminating experience for students in their preparation as practitioners and educators.

Several of the modern integrating approaches to deal with the complexities and uncertainties of major energy-environment projects and policies that are used in these courses and research projects include the following: conceptual modeling of complex science and management options for understanding the key drivers of environmental responses to energy development alternatives (Henderson and O'Neil, 2004); risk analysis models for estimating rate functions within action-response networks in energy-environment systems provided by the U.S. Environmental Protection Agency in 1998; adaptive management strategies for handling major uncertainties inherent in energy-environment project tradeoffs, including staged development (e.g., partial oil field development), monitoring of environmental and economic responses, and altered next-stage development strategies; and collaborative education and involvement strategies for engaging decision-makers, stakeholders and the public in project and policy related decision making from the Council on Environmental Quality in 2006.

Cognition Driver

The research program for the Ph.D. will focus on cognitive science related to the learning and teaching of complexity and uncertainty in STEM disciplines. Cognitive science is the interdisciplinary study of mind and intelligence, embracing philosophy, psychology, artificial intelligence, neuroscience, linguistics, and anthropology. The theoretical framework for cognitive science includes:

- computational models analogous to mental operations complementing psychological experiments on deductive reasoning, concept formation, mental imagery, and analogical problem solving
- linguistic approach to identify grammatical principles that provide the basic structure of human language
- neuroscience focus on the nature of the brain and what regions are involved in mental imagery and word interpretation
- cognitive anthropology using ethnographic methods to explore culture influences in cognition
- The expert-novice cognitive research is influential in cognitive science

We want to develop graduate students with the capacity to be experts in cognitive science in the area of STEM disciplines. The program of study on complexity and uncertainty will incorporate cognitive and affective analysis of how students develop such reasoning across STEM disciplines and across the divide of high school and college. A focus on how student cognitive misconceptions of complexity and uncertainty develop and methods of addressing those issues will be studied. Current cognitive science theoretical approaches about how the mind works will be incorporated into the study, including mental representations interpreted as formal logic, rule-based systems, concept schema and scripts, analogies in problem solving, and visual and spatial imagery.

Graduate students will engage in research on children's cognitive development in the area of complexity and uncertainty in science and mathematics across the K-12 grade continuum. A component of the field internship will be the engagement of a cohort of research scholars in a common research agenda focused on children's cognitive development in this area. This development will encompass students' ability to model complex problems, critical thinking, reasoning, communication, and problem solving processes. Cognitive science research will be brought to bear on what energy and environment topics are appropriate on varying grade levels and on the learning trajectory for computational science and mathematical reasoning that support this science. A number of questions related to the complexity and uncertainty concepts will be of interest. What impact will integrating issues of complexity and uncertainty into the science and mathematics classroom have upon student's ability to critically reason about and solve complex problems? What is the level of cognitive processing that can be elicited across the K-12 science and mathematics curriculum by engaging students in large database research and technology-based data gathering? How do we promote conceptual understanding of science and mathematics through the study of complexity and uncertainty? What is the affective impact of engaging students in the real-world problems of complexity and uncertainty? What is the impact on student achievement gaps in science and mathematics for underserved populations in an integrated science and mathematics approach addressing complexity and uncertainty?

The focus on complexity and uncertainty in mathematics and science education carries with it questions of developmental and conceptual ability of students across the K-12 continuum. Graduate students will need to be versed in cognitive science in order to study this question. Efforts by psychologists to understand the acquisition of scientific knowledge and knowledge about scientific method, though reflecting variety in theoretical orientation, have illuminated important factors in the development of scientific understanding. One such factor is the role of prior knowledge of the domain which has been shown to figure importantly in the formulation of questions and hypotheses (Klahr, Fay & Dunbar, 1993; Penner & Klahr, 1996; Schauble, 1990; 1996). Another is the ability to distinguish between, and to coordinate, theory and evidence (e.g., Klahr & Dunbar, 1988; Kuhn, Amsel & O'Loughlin, 1988). Carey & Smith (1993) have noted that many students do not recognize that science is fundamentally a theory-building endeavor. Another factor that may influence the development of scientific reasoning is awareness of one's own thinking; recent studies in children's "theory of mind" have suggested important developmental changes that may bear on this element (e.g., Chandler, Hallett & Sokol, 2002). Several lines of research have converged on the characterization of children as moving from a view assuming straightforward, sensory-based knowledge in which truth is an easily obtained objective to a view in which science is admitted to involve active interpretations of deliberately staged experiments, mental manipulations, and theories (i.e., frameworks for knowledge that may yet contain uncertainty) (Carey & Smith, 1993; Grosslight, Unger, Jay & Smith; 1991). Thus, children's understanding of models as a scientific tool undergoes significant change. These known factors in the development of scientific thinking will be taken into account (maybe even treated as aspects of manipulations and/or measured outcomes) in activities fostered by the proposed program.

Apprenticeship Drivers

The graduate students will develop personal and professional skills by engaging in a professional apprenticeship model. They will work as a community of scholars on issues of complexity and uncertainty in mathematics and science education. As members of the community they will attend seminar series focusing on STEM research and education issues. They will complete a research project and present the results at a regional or national meeting. They will submit a state and national grant supporting the dissemination of their research and broadening its impact. The grants will provide for continued research on complexity and uncertainty in science K-12 classrooms, as well as the development of curricula. The cohort will publish papers in STEM education in collaboration with faculty in science and education.

STEM RESEARCH PROJECTS (at pre-dissertation stage)**:** The doctoral cohort will engage in integrated mathematics and science research internships with University of Wyoming mathematicians and scientists in the areas of energy, environment, and computational science at the pre-dissertation stage. They will participate in the numerous seminars offered through HENR, the PiE, and SER, which invite national and international speakers to the University of Wyoming. In collaboration with HENR and SER, the graduate students will participate in a research and policy project on energy and environment. They will have the opportunity to interact with the internationally renowned board of advisors for the HENR. They will also have opportunities to collaborate with mathematicians and scientists working on modeling projects with the National Center for Atmospheric Research (NCAR), including use of the new super computer to be built in Wyoming.

STEM EDUCATION – Preservice Teacher Apprenticeship: Graduate students will partner with College of Education faculty to integrate complexity and uncertainty in mathematics and science into the professional development of K-12 pre-service teachers. With recent education reforms focused on a socially-relevant science curriculum and incorporation of exciting discoveries and applications, the role of scientists in teacher professional development has become paramount (Drayton & Falk, 2006). In pre-service science teacher development programs it is believed that integrating scientific experiences benefit the field "by exposing them [students] to the leading-edge techniques/technologies." (Bloch, 1990, p. 841).

Most programs involving mathematicians or scientists and teachers working in collaboration are limited to sporadic experiences for both groups, with management and control mostly executed by the invited higher education faculty member. There have been different ways in which scientists have been involved in science teacher education programs. For instance, a scientist and science teacher team teach a lesson (Anderson, 1993; Wier, 1991), or participate in summer internships that include meetings during the following year. This proposal is oriented towards establishing a new model in mathematician-scientist-science teacher collaboration. In our program, both pre-service and in-service teachers are engaged in addressing issues related to energy and environment. Experts in the field and researchers (i.e., graduate students, scientists) assist pre-service teachers during their junior and senior years. This is a crucial stage in the mathematics and science teaching certification program because it is when pre-service teachers incorporate their content knowledge into the planning, implementation and assessment of teaching and learning before their residency semesters and full-time careers. This partnership effort is also significant because mathematics and science educators, as recommended by the National Research Council's standards, are to seek connections with other groups of practitioners within the local, national and international community. Therefore, there is evidence that highlights (1) the merit of this collaboration, especially from the standpoint of the value of pre-service teachers' involvement in research experiences, and (2) mathematics and science teachers gain great understanding of the scientific enterprise and its features (i.e., uncertainty) in connection to science teaching (Cunningham & Helms, 1998; Helmer, 1997).

The scientist (graduate student)-pre-service science teacher collaborative work in this proposal is focused on features such as uncertainty and complexity of the scientific knowledge as central characteristics of the scientific practice. This is a valid framework that researchers (Bowen, 2004; Roth, 1995; Varelas, House, & Wenzel, 2005) have used to understand and associate the practice of science in educational settings. The finished science products, as reported in science textbooks, are not the only pictures we want our pre-service science teachers to portray and show in their classrooms. They need to tell their student not only 'where we are not (knowledge of science) but also how we get there (knowledge about science)" (Wandersee, 1992, p. 428). Science is a human endeavor and builds a type of knowledge that is durable but tentative. These are features that translate into attitudes educators observe in their classrooms; human curiosity oftentimes has resulted in scientific breakthroughs as scientists wonder about reality or practice their problem-solving skills to tackle phenomena in nature. This approach to scientist-science teacher connection highlights the goal of our STEM teacher education by "helping science teachers challenge and refine their ideas about teaching and learning science and learn how to learn from experience" (Bryan & Abell, p. 137).

STEM EDUCATION – Outreach Teacher Apprenticeship: The graduate students will also work with teams of K-12 teachers, scientists and mathematicians, and science and mathematics educators in an outreach internship lead by the Science and Mathematics Teaching Center (SMTC) at UW. The internship will place them in a leadership role in providing professional development for teachers in the field. The professional development theme will be complexity and uncertainty issues in energy, environment, and computational science. Partner K-12 schools in Wyoming that are teaching environmental and energy issues in the classroom (Journey School, Jackson; Star Lane Academy, Casper) will serve as partners in research on student development of understanding complexity and uncertainty in science and mathematics.

Recruiting, Retention, and Diversity

If the graduate students are to become leaders in mathematics and science education around the country, it is important that they be broadly representative of diverse ethnic, racial, and cultural backgrounds, gender, non-traditional students and international students. A comprehensive recruitment plan will be developed with special emphasis on locating prospective students from these underrepresented groups. The President's Advisory Council on Minorities' and Women's Affairs was established at UW to provide funding for diversity-related activities and to support recruitment and retention efforts. Additional funding to support recruitment of these five graduate students will be sought from this source which is available throughout the year.

A team of University of Wyoming faculty who are Hispanic, Native American and African American from across campus have met and agreed to take a lead role in recruiting and mentoring underrepresented graduate students for the program. They include Professor of Educational Studies, Dr. Francisco Rios; Director of American Indian Studies, Dr. Judith Antell; Director of African American Studies, Dr. Gracie Lawson-Borders; Director of Chicano Studies, Dr. Ed Munoz; Director of Employment Practices and

Affirmative Action, Dr. Nell Russel; and Professor of Secondary Science Education, William Medina-Jerez.

Students will be recruited from within the University of Wyoming and from other universities, as well as potential students who are working in professions. Recruitment will start at UW with all of the science-related departments and College of Education to identify potential candidates among their students completing bachelors and masters degrees. In addition, presentations will be made to the McNair Scholars on campus (a graduate school preparation program at UW for low income and first generation college students) as well as students who have participated in the federally-funded TRiO Math Science Initiative at UW, the Summer Research Apprentice Program (SRAP) and the Minority Engineering Program. Partnership programs such as the one that UW has with Shanghai University to train undergraduates in the sciences will also be contacted. We will collaborate with the Graduate School and a variety of UW academic departments and programs in their outreach efforts including the Minority Student Recruitment program. Recruitment from other universities will focus on Minority-Serving Institutions (Historically Black Colleges and Universities, Tribal Colleges, and Hispanic-Serving Institutions); large public universities with good minority recruitment programs, such as UCLA, the University of Michigan, and the University of Texas at Austin; private colleges and universities that do an excellent job of enrolling a diverse student body, such as Harvard, Stanford, Wesleyan, and Yale Universities; majority institutions with large minority enrollments, such as Arizona State University; and intervention programs with good track records, such as the Meyerhoff Program at the University of Maryland (a program that focuses on highly able African-American students who aspire to become leading research scientists and engineers – some may be interested in a career in teaching in higher education). Faculty and staff will also contact science and education departments at universities that do not offer PhD programs in their specific areas and ask them to recommend good candidates.

Specific strategies for identifying and attracting members of underrepresented groups who are working in various science-, education-, environment- and energy-related professions include recruiting at various conferences, advertising in their publications and websites, and sending direct mail. Organizations will include the Society for the Advancement of Chicanos and Native Americans in Science, the American Indian Science and Engineering Society and its college chapter at UW, the Society of Hispanic Professional Engineers, the National Association for Multicultural Education, and the National Science Teachers Association and its affiliate

Association for Multicultural Science Education. The graduate program opportunity will also be advertised to teachers currently teaching in the sciences in Wyoming and the region.

A Summer Research Institute for Graduate Students will be held with the express goal of recruiting underrepresented students for the doctoral program. The institute will introduce prospective students to STEM and STEM education faculty, allowing them to explore common research interests and examine labs. The prospective students will also meet with faculty to discuss issues of support and mentoring at the University of Wyoming. The University has strong student support groups and student services. The graduate students will have mentors and advisors who will have been provided with special training. A special emphasis will be placed on involving the graduate students in research, professional societies and the intellectual life of the university.

The University of Wyoming will provide a rich experience for students from diverse backgrounds. The graduate students will work with highly diverse urban public schools in Denver as well as rural tribal schools in Wyoming as part of their program. UW has well-established relationships with schools in both areas that have been a part of the education program for all teacher-education candidates for many years. Diversity is supported and encouraged through the UW President's Advisory Council, the Ethnic Studies Program, Martin Luther King Days of Dialogue, the Women's Study Program, and the nationally renowned Shepard Symposium on Social Justice. In addition, the University of Wyoming has a new initiative, the Social Justice Research Center, which will provide graduate students additional mentoring and research support.

Summary

The proposed Ph.D. in Mathematics Education incorporates cognates and apprenticeships that will engage the students as practitioners in a community of STEM scientists, mathematicians, and educators. The primary drivers of complexity and uncertainty motivate an integrated science approach based in modeling real-world phenomena using mathematics and technology. Graduates of such a program are uniquely poised to address pressing needs in K-12 STEM education. There is a pressing need to move curricula from the current silo approach to teaching mathematics and science as a collection of isolated facts, to an integrated approach that coalesces STEM disciplines around real-world problems. There is a pressing need to provide preserve and inservice teachers with professional development that prepares them to teach mathematics and science through a problem/project based pedagogy that engages and motivates students by demonstrating the utility of science. There is a pressing need to develop teacher educators that are enculturated into the STEM communities way of knowing (what does it mean to DO science or mathematics) and reflect the central concepts of scientific inquiry and mathematical problem solving/proof in their practice. Finally, there is a pressing need to bring educational research in the area of cognition to the classroom in a way that impacts teacher's practice and student learning.

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DOCTORAL PROGRAMS IN SLOVAK REPUBLIC

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Key words: *Study plan, study and scientific part, common core of knowledge, thesis*

Slovakia is a young European country (establishment of the state in 1993) that is still in progress. During the 15 years of our history changes were introduced in all areas of the society including the education.

At the Faculty of Mathematics, Physics and Informatics of Comenius University there are 19 PhD study programs including *Theory of mathematics education* that was accredited by Accreditation Commission in 2006.

Graduates or students of the last study year of the Master, Engineer or Doctor (previous form of PhD study) university study in any study program can apply for PhD study at the Faculty (generally they are fresh from college, rarely persons from practise). In the application the following is given by the applicant: the study programme, topic of dissertation thesis, supervisor and form of PhD study (daily form, i.e. with stipend, or external form). The topic of dissertation thesis is chosen by the applicant from the list of dissertation theses topics for applicants for PhD study.

The entrance interview consists of two parts:

a) Written test examining the basic knowledge in the field.

b) Personal interview by the entrance committee. It is aimed at attestation of the applicant's preconditions for PhD study in the selected field, a more detail specification of the supervisor's proposal and the topic of dissertation thesis and discussion on applicant's conception and plans and his/her scientific program.

The assumption for the nearest future is to admit 2 till 5 applicants of doctoral study per year (it depends on financial resources and on personnel capacity at Faculty).

Duration of daily post-graduated study in Slovakia is at least three years (humanitarian and social science's educational programs) and at most four academic years (natural science's educational programs). Duration is at most 5 years in external form.

The PhD study at Faculty of Mathematics, Physics and Informatics consists of the *study part* and *scientific part*. Educational programs of *Theory of mathematics education* contain study and scientific parts in ratio 1:2. The study plan of each PhD student is compiled by the supervisor that manage himself periodicity of meeting with PhD student and working-out of his/her thesis. The choice of research problem depends on agreement between supervisor and PhD student. In study program *Theory of mathematics education* we prefer research problems that could be useful for secondary teaching (analysis of different subjects in secondary teaching, implementation of ICT in teaching/learning process, motivation in educational process, way of increasing effectiveness of teaching/learning process...). In Slovak Republic the supervisors have to be appointed professors or docents.

The *study part* of the PhD study consists especially of the lectures, seminars and of individual studies of the literature, which is related to PhD thesis. Lectures and seminars are usually ended with exam. Individual study of scientific literature can be divided into the phases and is ended by tutor who gives the students the appointed amount of credits. The study part is focused on a survey of mathematical and pedagogical-psychological disciplines and on intimate knowledge of areas related to subject of doctoral thesis that the PhD student has to acquire. After studies the student should be able to enlarge and deepen knowledge acquired from mathematical disciplines of wider fundamentals of the study program – Mathematics Education, pedagogic-psychological disciplines as well as the methods of quantitative analysis (special statistical methods). He/she obtains new and deeper knowledge concerning utilization of ICT and new trends in the teaching/learning process and he/she learns to follow them. He/she adopts methodology of work with scientific library and periodical literature and by utilizing Internet sources. He/she will manage to present the results of his/her work at domestic as well as international academic conferences.

The current subjects in study parts are Theory of Didactical Situations, Didactical software in Teaching of Mathematics, Epistemology and Cognitive Psychology, Multidimensional Relations of Didactics of Mathematics, Didactical Engineering, The experiment in Didactics of Mathematics. There is a core for coursework studies, but particular methods (like statistic analysis with using of CHIC) and theories (like Theory of Didactic Situations) are privileged.

The *scientific part* of the PhD study consists of individual or teamwork of the PhD student, which is bound to the theme of the dissertation. In the scientific part of study the students are led to look up and evaluate scientific information, to adopt the standard methods and forms of scientific research process, to interpret and present results of their work at national and international conferences. After the studies the student should be able to apply theoretical skills to problem solving in social practice, to communicate with experts, to specify and analyze their school praxis problems and suggest them model solutions and to help with their implementation. He/she should be able to formulate mathematical problems, projects and other forms of activation tasks for students and contribute to the development of scientific discipline related with mathematics education, but also within the school subjects' relations.

In addition to theoretical knowledge and practical skills and abilities the PhD student should be able to lead on a professional level the teaching/learning process in first level university education (Bachelor's), to participate in organizing student's scientific work and students' symposiums (including international ones), that are touched to the branch problematics, to organize scientific research events, including events with international participation, to involve and lead students of lower university level to the actual scientific problems in the branch and effectively present his/her results using modern ICT tools.

To all forms of doctoral study the credit system is applied. One credit is a base value of student work, and in PhD study it was defined equivalently as in bachelor's, master's and doctoral studies. Standard activities during academic year represent 60 credits. Assessment of PhD students contains study part (lectures, seminars...), creative activity in scientific field (publications, scientific work...), leading of the teaching/learning process at university and working-out of PhD thesis. To finish the PhD study successfully both in internal and external form one has to get at least 180 credits including credits of his/her doctoral thesis. PhD student can ask for the authorization process of thesis' defence when his/her supervisor recommends thesis' acceptance and student has get 150 credits during his/her studies. Requirements for the thesis include high level analysis and synthesis of knowledge and adequate overview of scientific literature (minimally 100 and maximally 160–220 pages of thesis). The constitution of the committee for the defence of doctoral thesis is determined by a Common specialization committee (nationwide committee) according to its chairman's suggestion.

The graduates of specialization *Theory of Mathematics education* are qualified for the following positions: research – pedagogic position at University, research position at Slovak Academy of Science and at research institutions, leader of a team concerned with applications (e.g. Methodical centre) and manager of Education department. Participation of PhD students in the life of the Faculty after their study is low (4 years ago Dean of the Faculty ensured each doctoral student to stay at Faculty in position of assistant lecturer, but in this time it is hard because of low financial resources). Candidates leave the school and often go abroad or in commercial sphere.

Accredited PhD study program *Theory of Mathematics education* in Slovakia is very young. Therefore the position of PhD students in labour legislation is often changing and some particular methods and theories prevail in common core of knowledge. Also there are differences between mathematics and mathematics education thesis according to mathematicians because they refuse the existence of scientific part in mathematics education. For that reason I consider as the most important the question of common core of knowledge that could help us to decrease differences between countries in PhD students´ competences, to compare the range and depth of mathematics content required and manner in which research competence is acquired and so to improve international cooperation.

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DOCTORAL PROGRAMS IN MATHEMATICS EDUCATION—CHALLENGES AND A VISION

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Background

Doctoral programs in mathematics education vary greatly within and across countries. Some doctoral programs require K-12 teaching experience prior to admission. Others require collegiate teaching experience. Still others require no prior teaching experience. Some institutions require full-time residence for multiple years in order to complete a doctorate, other programs can be done on a part-time basis and a doctorate be completed while working full-time in another position. Still others can be done primarily via distance learning. Programs also vary greatly in the range and depth of mathematics content required, as well as the manner in which research competence is acquired. Institutions vary greatly in the number of faculty members as well as the number of graduate students. Some institutions have only 1 or 2 faculty members in mathematics education, whereas other institutions may have more than 10 faculty members. Some institutions graduate several new doctorates every year, whereas other programs graduate one doctorate in mathematics education every couple of years. Many different variations in doctoral preparation have been reported (Reys & Kilpatrick, 2001; Reys & Dossey, 2008).

Some people view this diversity in doctoral programs as a strength, others as an area of concern. It certainly raises at least one important question: Is there a central core of knowledge/experiences that doctorates in mathematics education possess? An equally important question is: Should there be a common core of knowledge for graduates with doctorates in mathematics education? That is, when someone says they have a doctorate in mathematics education, what is reasonable to assume about the knowledge they possess with respect to mathematics education?

If the answer to this question "Is there a central core of knowledge that doctorates in mathematics education possess?" is Yes, then several natural questions follow, including:

What should constitute this common core of knowledge?
Who should decide what constitutes this common core?
How should it be delivered?
How should competence in mathematics education be assessed?

Should there be an accreditation of doctoral programs in mathematics education?

One could argue that answers to these questions would provide useful guidance to doctoral granting institution. Others may argue that such information would be too prescriptive, and therefore run the risk of curtailing creativity and uniqueness currently associated with doctoral programs in mathematics education.

A vision for the future

A vision for the future is that doctoral programs in mathematics education become more convergent. Does this mean that all doctoral programs in mathematics education would be alike? No, definitely not. Such convergence does not exclude interdisciplinary experiences, but it would insure that doctorates in mathematics education would share a common core of knowledge. Unless a common core of knowledge exists, it is hard to justify mathematics education as a discipline of study.

The Association of Mathematics Teacher Educators developed a document entitled *Principles to Guide the Design and Implementation of Doctoral Programs in Mathematics Education* that included the identification of core knowledge areas. Their core knowledge elements included, mathematics content; learning theory; mathematics curriculum, research, technology, assessment, and history of mathematics education. Policy and diversity have been other topics that have been recommended for inclusion in the core of knowledge of doctorates in mathematics education. While a list of 'core knowledge' may never be universally supported, it at least provides some talking points for those who have responsibility to develop and shape doctoral programs in mathematics education. If there is agreement that some refinement of this type of effort would be of value internationally, then perhaps some plans could be made to move at ICME-12 in that direction.

Ideally a core of knowledge will prepare doctoral students for their career as mathematics educators. This goal is challenging when the range of diverse career directions are considered. For example, while the majority of doctoral graduates in mathematics education take positions in higher education, other graduates take positions as K-12 classroom teachers, and mathematics supervisors. Still other graduates are employed by test development companies and textbook publishers. Even those employed in higher education assume a range of teaching responsibilities, that may include teaching mathematics content courses anywhere from undergraduate to graduate courses in mathematics, or teaching undergraduate and graduate courses in mathematics education. This wide range of career options underscores the difficulty in designing doctoral programs in mathematics education that adequately prepares everyone for their potential employment. Clearly designing a common core of knowledge for such diverse careers represents a significant and continuing challenge.

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"HISTORY AND MATHEMATICS EDUCATION, HISTORY AND PHYSICS EDUCATION, HISTORY AND CHEMISTRY EDUCATION": A PARTICULAR INTERNATIONAL DOCTORATE

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The Research Doctorate (PhD) in "History and Didactics of Mathematics, of Physics, and of Chemistry", at the University of Palermo, is offered in collaboration. The colleggio is composed of members of the following departments, who gather on-line. This international doctorate is offered between the following academic centres:

- CIRE (Inter-departmental Educational Research Centres, University of Palermo)
- Department of Mathematics and Mathematics Applications, University of Palermo
- DIFTER (Department of Physics and Technologies), University of Palermo
- Department of Inorganic and Analytical Chemistry, University of Palermo
- Department of Mathematics (University of Bologna)
- Department of Physics (University of Bologna)
- Department of Chemistry (University of Bologna)
- Department of Mathematics (University of Catania)
- Department of Mathematics (University of Pavia)
- Department of Algebra, Geometry and Mathematics Education (University of Bratislava, Slovakia)
- Department of Mathematics (University of Nitra, Slovakia)
- Department of Physical Sciences (University of Napoli, "Federico II")
- Department of innovation and Didactic Training (University of Alicante, Spain)
- Department of Education (University of Cyprus)

The program is offered as a full time one. The *dottorandis*, for which candidates receive a 50 % scholarship, precludes full time employment. There is generally a single supervisor; there can be two supervisors depending on the nature of the thesis and the expertise of the supervisor.

The involved discipline sectors[1]

1. MAT/04 (Complementary Mathematics). This sector includes research competence related to foundations, history and didactics of mathematics, and also concerns the development of innovative teaching methods and technologies, as well as aspects of mathematics (complementary mathematics and elementary mathematics from a higher point of view) necessary for their treatment.[2].

2. FIS/08 (Didactics and History of Physics). It includes the expertise necessary to study the history of physics, starting from the origins of physical ideas and those necessary to study the development of didactic methodologies. The expertises of this sector also concern historical, epistemological and didactical problems related to the foundations of classical and modern physics.

3. History and Chemistry Education. It includes the expertise necessary to study the history of chemistry, starting from the origins of chemistry ideas and those necessary to study the development of didactic methodologies.

The last 40 years of research in the field of history of Mathematics, of Physics and of Chemistry have contributed to deepen the bond between historical-epistemological paths of mathematical concepts that are correlated with several experimental works on learning / teaching situations.

Courses, Program, Seminars, Workshops

	COURSES	SEMINARS	WORKSHOPS	VISITS ABROAD
	Preliminary description of the courses' programs and indication of the duration	Thematic indications, titles and anticipated or predictable times		
I YEAR	**1) History and Epistemology of Mathematics and History and**	1) "International research in Didactics of Mathematics" (10	At least one per year regarding one of the themes treated in the	1 month (at least)

[1] The disciplinary sectors are those also recognized by the Italian university system.
For example: "Matematiche Complementari" (Code MAT04) is a disciplinary sector for "Mathematics Education, History of Mathematics, Fundamenta of Mathematics, Mathematics Elementary"; Code FIS08, "Physics Education, History of Physics" is another sector, etc.

[2] The Italian and German tradition of the "Elementary Mathematics from a higher point of view" go back to the end of the `800. There also exist the "Encyclopedias of the Elementary Mathematics from a higher superior point of view" from the beginnings of the '900.

	Epistemology of Physics or History and Epistemology of Chemistry; **2) Didactics of Mathematics end Didactics of Physics or Didactics of Chemistry;** **3) History of Sciences**.	Days) or "The International research in Didactics of Physics" or "The International research in Didactics of Chemistry" . 2) Monographic courses of History of Mathematics (10 Days) or Monographic courses of History of Physics (10 Days) or Monographic courses of History of the Chemistry (10 Days). 3) Monographic courses of Epistemology (10 days).	courses.	
II YEAR	**1) Computer technologies for scientific communication;** **2) Philosophy of Sciences.**	1) Cognitive Sciences also from the point of view of Neurophysiology (10 Days) 2) Tools and methods for communication: the role of technologies. (10 Days)	At least one per year regarding one of the themes treated in the courses.	1 month (at least)
III YEAR	Layout of the Doctorate Thesis.	Seminars defined according to the subjects of the thesis: Cognitive Psychology, Experimental Pedagogy, Neurosciences etc… (10 days).	At least one per year regarding one of the themes treated in the courses.	1 month (at least)

The Theory of Didactic Situations, the study of didactic obstacles, of epistemological obstacles, of misconceptions, etc… regarding the single scientific concepts have allowed the understanding of well focused communicative tools that also represent today exportable results outside the scholastic world; for example, scientific popularization in general and museums' communicative activity, etc… Disciplinary communication represents today a basic element to be able to go beyond the boundaries of Science on the one side and to deepen the epistemological contributions on the other.

All the students follow the courses of History and Didactics of the Mathematics, History and Didactics of the Physics and History of the Chemistry. Actually, up today no student of Chemistry has participated, and it is for this that courses of Didactics of the Chemistry are not activated, while the history of the chemistry is an obligatory course for everybody.

These courses concern the first year. The second year is more devoted to thematic seminars, concerning the subject thesis of the students. The third year is devoted to the thesis.

Expected graduate destinations

PhD graduates in "History and Didactics of Mathematics, of Physics and of Chemistry" are prepared for the following occupations:

a) preparation of future's inspectors[3] in the educational sector that deal with teaching / learning in training agencies[4],

b) preparation of researchers in the History and Didactics of Mathematics, of Physics and of Chemistry.

Such sectors include disciplines that are taught in Masters' Degree in Education both for Primary and Secondary School.

Quality parameters

For a PhD student, to learn to write a research paper, to know how to answer to the referees and to prepare reviews to scientific papers of others is very important.

The Italian Ministry of the University asks, for every year of the PhD course, the scientific production of the single *dottorandis*.

Quality parameters are defined by:

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[3] In Italy there are inspectors of disciplines: inspector of mathematics and physics, etc. Currently the preparation of the inspectors is based on the epistemology and didactics of the dicipline.

[4] For example, regional Institutes of educational research. In Italy there are institutes that have national and international contacts and they deal with the formation of the teachers in service.

1) PhD students' publications of papers in national and international journals;

2) Reports of experts involved in the project activities;

3) PhD student's lectures in National Conferences;

4) PhD student's lectures in International Conferences.

At the end of every year, PhD students and teachers who took part in the activities have to prepare an evaluation report.

The evaluation parameters regard the effectiveness of the seminars, the transferability of the experience to products of research, and the involvement of teachers and students.

The teaching staff establishes from year to year didactic activities according to the financial situation. We provide: on-line didactic activity, construction of a common web site, video conference meetings, on-line forums between students and teachers, publication of an on-line journal managed by the PhD students with scientific supervision by at least 2 members of the doctoral board.

The consortium's web site is established on the server of the Department of Mathematics, University of Palermo, managed by the GRIM (Research group in the teaching of Mathematics).

http://math.unipa.it/~grim/dott_HD_MphCh/dott_HD_index.htm

The meetings can be held in one of the University centres of the Consortium and also in video conferences. We also provide on-line meetings with the college staff. Every meeting is then certified through signatures. For on-line meetings we draft minutes that are sent to all the other members of the consortium.

Doctoral thesis

Every PhD candidate presents a thesis that must be equipped with a report of the thesis advisor (assumption of thesis responsibility) and then certified at majority by the teaching staff.

The candidates, by mutual consent with his thesis advisor, present their thesis to the teaching staff. The staff appoints a board of referees (at least 5) selected among members of the teaching staff but also outside according to the subject. The referees draft a maximum 3 page report with deadline fixed by the teaching staff. If three referees out of 5 express a completely negative judgement, the thesis cannot be discussed for the final examination. If the judgment is positive with only some remarks, the candidate will have to modify parts to address the referees' observations. After these possible further corrections the thesis can be approved for discussion. There is no established limit regarding the number of pages of the thesis. The size of the thesis depends on the introduced hypotheses, on the methodology and on the way according to which the conclusions are deduced.

The staff appoints a board of at least 6 members (at most 2 not belonging to the staff) that will examine the final editing and judge the discussion of the thesis.

The students' origin

Each student can choose one specialization of the PhD course. After the courses of epistemology, methodology of research in didactics and history of the sciences (1° year) he/she directs his/her study toward the didactics of the discipline or the history of this (2° and 3° year). Naturally the students, since the first year of the study course, already have an oriented research project that they could also change in timetable.

The students can be graduated in Mathematics, Physics or other scientific disciplines.

We already are at the third year and, at the end of this year, we will have the first PhDs. The following chart synthesizes the actual situation.

Students XX Cycle 2005-2008	**Students XXI Cycle 2006-2009**	**Students XXII Cycle 2007-2010**
1 History of Mathematics 2 Didactics of Mathematics 3 Didactics of Physics	3 Didactics of Mathematics	3 Didactics of Mathematics 2 Didactics of Physics
3 Graduated in Mathematics 2 with Master Degree in Education for secondary school (age 11-18) in Mathematics and Physics (2 years post graduate school)	1 Graduated in Mathematics 1 Graduated in Physics 1 Graduated in Economy 3 with Master Degree in Education for secondary school (age 11-18) in Mathematics and Physics (2 years post graduate school)	1 Graduated in Mathematics 3 Graduated in Physics 1 Graduated in Engineering 2 with Master Degree in Education for secondary school (age 11-18) in Mathematics and Physics (2 years post graduate school)

Stable connections with other doctorates of the consortium

For the preparation of the PhD students the exchange with other experiences is very important. Besides the stages in other university structures, some stable connections can be servants. This happened with the doctorate in Mathematics Education of Slovakia.

Since 1999 there are connections with the University "Comenius" of Bratislava (Slovak Republic). Exchanges of students, in average a month long, have taken place. Theses have been presented and discussed in English until 2005 in Bratislava and those of Italian students are at the following web addresses:

http://math.unipa.it/~grim/tesi_it.htm

PhD students' research activities have also been documented by mathematics education research journals managed by the respective research groups:

1. "Quaderni di Ricerca in Didattica (Mathematics Section)":
   http://math.unipa.it/~grim/menuquad.htm
2. "Acta Didactica Universitatis Comeniae – Mathematics" :
   http://www.ddm.fmph.uniba.sk/ADUC/index.html

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DOCTORAL PROGRAMS AT SOME AUSTRALIAN UNIVERSITIES

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Monash University, Australia

The following describes some aspects of doctoral programs at two Australian universities with which I am familiar. There are no specific requirements related to mathematics education doctorates that are different from the overall requirements. The following describes the goals of the programs, the requirements for entry, the supervision processes, the requirements for the thesis, alternates modes of completing the doctorate, and the examination process.

Goals

The comparative goals of the doctoral programs are described as:

- candidates for doctoral programmes will make a significant contribution to knowledge and demonstrate the capacity to carry out independent research (Monash);
- graduates who demonstrate academic leadership, increasing independence, creativity and innovation in their research work. In addition, professional doctoral studies provide advanced training designed to enhance professional knowledge in a specialist area, and encourage the acquisition of a wide range of advanced transferable skills. A thorough grasp of the appropriate methodological techniques and an awareness of their limitations must be demonstrated, together with an ability to communicate research findings effectively in the professional arena and in an international context (Melbourne).

On one hand, the stated emphasis on knowledge creation, and on the other hand, on research training.

Entry

For Monash University, the entry requirements are that students meet the University's minimum academic and English language proficiency entry requirements. The website lists the minimum academic requirements for entry into a higher degree by research as:

- a bachelors degree in a related discipline from Monash University requiring at least four years of full-time study, which normally includes a research component in the fourth year, leading to an honours degree at H1 or H2A level, *or*

- a course leading to a masters preliminary qualification at a level rated by the relevant department, faculty and committee as equivalent to an honours class I or IIA degree, *or*
- a masters degree in a related discipline, including a significant research component, at least equivalent to an Australian honours degree (a full-time year of research and an examined thesis). It is normally expected that a minimum grade of H2A has been obtained for the research thesis or project.

Admission into professional doctorate programmes may also involve a formal interview, and applicant must have at least three years of professional experience.

At Melbourne University, the entry requirements are that:

- Applicants are normally required to have completed at least a four-year honours degree at H2A standard from an Australian university, or a qualification or combination of qualifications considered by the RHD Committee to be equivalent. For particular disciplines applicants are also required to complete, at an appropriate level, a Graduate Management Admissions Test (GMAT) or a Graduate Record Entry (GRE) test.
- The completed degree must be in an area that is relevant to the intended PhD, including sufficient specialisation such that the applicant will have already developed an understanding and appreciation of a body of knowledge relevant to the intended PhD.
- Applicants are normally required to have completed a research project/component that accounts for at least 25% of their year's work at 4th year or at Masters level. Graduates of certain professional degrees at the University of Melbourne, including MBBS, BVetSci, LLB, BPhysio and BEng are deemed to have met this requirement. (Other evidence of research ability may include producing a sustained policy document, conference presentations, articles in professional journals, etc).
- Applicants must also meet the University's English proficiency standards.

As you can see there are minimal discipline specific requirements, but the overall requirements are substantial.

Supervision

At Melbourne University, key responsibilities of supervisors include:

- Facilitating the timely completion of graduate research

- Monitoring the quality of research in progress
- Knowing the relevant policy frameworks and requirements for graduate researchers
- Assisting graduates to develop transferable skills and prepare for their careers

Supervisors are members of the academic staff who have relevant research and supervisory experience and a continuing active participation in research. All supervisors must be appropriately qualified and the normal expectation is that supervisors will have a PhD.

There is a two-stage process for inducting new academic staff and new supervisors. All academic staff new to the University of Melbourne must attend a one-day orientation programme which includes a one hour session on the policy and procedures of postgraduate supervision. In addition, staff without recent postgraduate supervisory experience are required to attend a half-day workshop on postgraduate supervision. Attendance at these sessions is verified and recorded on staff HR records.

At Monash University, there are three levels of supervision: sole (100%); main (75%)/associate (25%); joint (two supervisors at 50% each). To be a sole or main supervisor of doctoral students a staff member needs to have supervised four students to completion, supervised students to completion at another university, or completed supervisor training at Monash University.

To be an Associate supervisor of doctoral students a staff member needs to have a Masters degree, but may not currently be a doctoral student. To be a supervisor of Masters or Bachelors of Education (Honours) students, in any capacity, a staff member needs to have a Masters degree.

The primary supervisor must be a member of the university's academic staff who has appropriate research experience and a continuing active involvement in research. Honorary members of staff, emeritus and adjunct professors may also be appointed as main supervisors, provided that they are undertaking teaching and research responsibilities expected of a member of the university's academic staff.

In both cases, the doctorate is predominantly an individually supervised study, like an apprenticeship, and while guidance is available, the responsibility for the quality of the doctorate is very much with the supervisor.

Thesis requirements

At the University of Melbourne, the length of the thesis varies with each discipline, with 80,000 words being the norm. The thesis should not exceed

100,000 words (or equivalent) without special approval from the Research Higher Degrees Committee.

At Monash University, the length of the thesis may vary across disciplines but will normally be 80,000 words, and will not exceed 100,000 words. (In relation to the EdD: doctoral-level units represent 25% of the work requirements and a thesis component of up to 75,000 words.)

In both cases, the length of the theses is substantial.

Alternatives

In response to changes to postgraduate education brought about by the electronic revolution, and the necessity to cater for students from a diversity of cultural and educational backgrounds, Monash has introduced flexibility in doctoral programmes, including candidature by off-campus or external mode. The prominence of the traditional PhD by thesis is inferred, but other options include:

PhD based upon published or unpublished papers;

PhD with coursework component - within the traditional framework, a PhD with formal coursework seeks to widen a candidate's knowledge base, to place the specialised research project within a broader context, and to enhance the candidate's research skills generally. It is possible, in some academic units, to take up to the equivalent of 12 months of full-time PhD-level coursework as an element of the PhD research programme. In these circumstances, a slightly shorter thesis may be submitted;

PhD in speciality of Visual Arts - The Faculty of Art and Design offers a PhD programme where the core of assessable work is an exhibition (or equivalent). Documentation supports and comments upon the work and seeks to explain its contribution to human culture, endeavour and knowledge. A three-unit coursework component provides a theoretical framework;

PhD in speciality of Music Composition - The Faculty of Arts offers a PhD programme in which the work submitted for examination consists of a composition portfolio, a critical commentary of between 20,000 to 25,000 words, and concert programme notes of the candidate's musical work. At least 50% of the music submitted for examination must have been performed publicly;

PhD in speciality of Creative Writing - The Faculty of Arts offers a PhD programme in which the assessable work is a piece of the candidate's own creative writing, together with an exegesis which places the work in context. The program is by 100% research;

Joint masters / PhD programmes - The Faculty of Education offers a joint programme in two discipline areas: the MPsych (Counselling)/PhD and the MPsych (Education and Development)/ PhD whereby two degrees may be undertaken during a four year postgraduate research programme.

Professional doctorates are also available, which combine research, coursework and in some cases professional work/industry experience. Predominantly research-based, professional doctorates focus on the improvement of professional practice: Doctor of Education (EdD); Doctor of Psychology (DPsych); Doctor of Public Health (DPH); Doctor of Business Administration (DBA); Doctor of Juridical Science (SJD); and Doctor of Information Technology (DIT).

Progress of the candidature

At Melbourne University students have an advisory committee that meets regularly throughout the candidature. At Monash University, there is a single formal process for confirmation of candidature.

The examination process

At Monash, the formative evaluation of progress is via the confirmation of candidature which takes place after the first year of enrolment. Progress reports are conducted on a yearly basis.

At the conclusion of the candidature, the head of the academic unit first consults with the supervisor with regard to the names of possible examiners, and supervisors should ensure that candidates are consulted. Two examiners, external to the University, are nominated. Candidates are advised of the names of the examiners. Examiners need to complete the examination within eight weeks.

There is a similar process at the University of Melbourne. Confirmation of candidature for doctoral students takes place after the first year of enrolment. All doctoral students complete annual progress reports.

A distinction is made between the examination of traditional PhDs and those incorporating art works: where a thesis consists of creative works and a dissertation, and where the creative work component includes performance or exhibition of visual art works, three examiners may be nominated from within Australia, and at least one of the three examiners appointed must be from interstate.

There is now some discussion on whether the processes for assessing creative works might well apply to practically oriented study.

Summary

The two universities are young in comparison to other major world universities. The two programs are distinctive in emphasis. There is not a strong culture of programmatic research, and where there are strong groups of mathematics education doctoral candidatures, this is usually the result of an active supervisor rather than a coherent program.

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