"Quaderni di Ricerca in Didattica", n14, 2004.
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Fractals as didactic material

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Abstract
This workshop explores the possibility to purpose some of the virtual objects on the web to the students in order to be used according the well known methods of mathematical modelling of real problems. In the first part the method of mathematical modelling of real problems will be briefly explained referring to an example. In the second part we will discuss, the possibility of using the same method to interpret and understand some of the objects on the web. In particular, we will refer to the study of fractals

Résume
Ce workshop explore l'hypothèse que certains des objets virtuels présents sur le web peuvent être proposés à l'étude des étudiants avec les méthodes bien connues de la modélisation mathématique des problèmes de réalité. Dans la première partie, on exposera brièvement la méthode de la modélisation mathématique de problèmes réels en faisant référence à un exemple. Dans la seconde partie on discutera, avec également des travaux de groupe, l'hypothèse que cette méthode puisse être utilisée pour interpréter et comprendre certains des objets présents sur le web. On se référera en particulier à l'étude des objets fractals.

1. Introduction

The Web offers a great opportunity to find didactical materials helpful for developing students’ mathematical activities. Many universities, centres of educational research and school institutes, publish a practically unlimited variety of didactical “tools” on the Web, often supported by exciting animations, which can be downloaded directly by students.

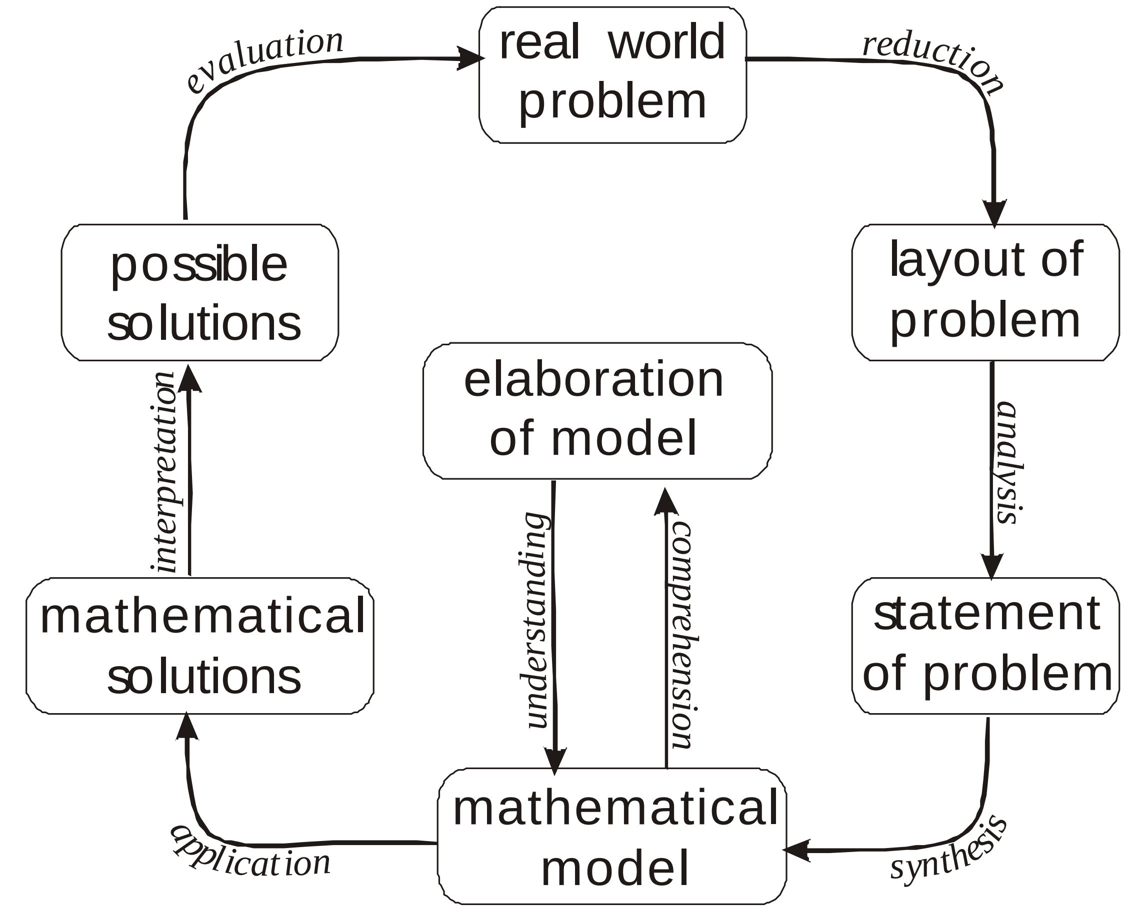


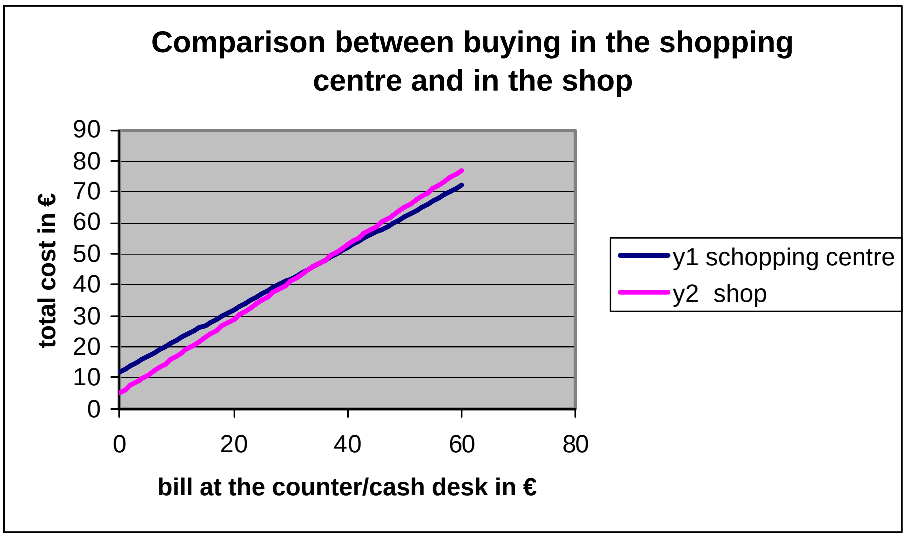
fig. 1

Schools can buy structured softwares which create sophisticated environments for learning mathematics, even in a cooperative way. In a computer laboratory interesting applications can also be realized directly by teachers and students using easy didactical languages. Especially in mathematics the didactic materials are becoming “virtual” ones. How can such new objects be interpreted from a didactical point of view? How can they be used in the teaching learning processes? The authors propose to use, of course with the necessary caution, the new “virtual” didactical objects in the same way in which the real ones should be used. In fact these new objects can be employed with the same principles, objectives and methods discussed many times in CIEAEM for constructing and developing mathematical concepts: the mathematical modelling of real problems (fig 1)

2. An example of developing mathematical concepts: the mathematical modelling of real problems.

Let's look over the operations of the map within a particular case in order to verify later if the same operations can be employed within' the study of fractal object

Real life problem	A shopping centre has opened in our area
reduction	*The real situation generates a problematical universe that the teacher has the responsibility of chanelling towards the individuation of a problem that falls within the objectives of the teaching programme: this inevitably leads to the investigation of one particular aspect of the problem*
Layout of problem	“Is it cheaper to buy in a small shop in the neighbourhood or in a far bigger store further away?” (algebric models)
	“What could the destiny of small shops be ?” (statistic models not examined in this example)
analysis	*The students, alternating group work and individual homework, are led to select suitable aspects among many possibilities, for instance the following:* • *the distance from home to the big store* • *the bill paid at the big store* • *the time spent shopping in the big store* • *the time spent shopping in the small shop* • *the value of the customer’s time* • *the travelling expenses (only for the big store)* • *percentage difference in prices*
Statement of problem	A family has to choose between a big shopping centre and the small shops in the neighbourhood for the weekly shopping. Knowing: • travelling expenses (only for the big store): 2 euro • time required for shopping in the big centre: 2 hours • time required for shopping in the small shops: 1 hour • value of the customer’s time: 5 euro • percentage difference in the prices: 20% Find the <u>minimum cost</u> to choose the big store
Synthesis	*The students using their previous knowledge has to transform the statement*
Mathematical model	$y1 = x + 12$ $y2 = 1,2x + 5$ with x => bill paid at the big store y1 => cost of shopping in the big store y2 => cost of shopping in the small shops
Comprehension and understanding of mathematical model	*The students are engaged study or “discovery” methods of solution of this kind of linear equation*
Mathematical solutions	X=35 Y=47

interpretation	
Possible solution	Discussing the graph, the student are involved in a wider interpretation which could be very helpful to introduce the study of inequalities
evaluation / **Real world proplem**	The student, at the end, should evaluate the effectiveness of the solution, eventually with practical experiments

A good reality problem allows us to refer back to the real situation which generated it with new and stronger instruments of analysis.

Real life problem	Is it now possible apply the model to everybody?
reduction	*The students changing the parameters of the problem solve many numeric systems. For many student could be a moment of drill and practice exercise; but the best of them develop the need of a generalization, and to focus the limits also.*
Layout of problem	Is it possible to solve the problem for everybody without handling the solution of a system of equation in each case?
analysis	*The students, alternating group work and individual homework, are led to generalize the problem*
Statement of problem	Many persons living in the countryside have to decide whether it is more convenient to go shopping in a big store or in local shops. To find the minimum bill to justify the choice of the big shopping centre they appreciate the following variables: - travelling expenses **g** € - value of the customer's time **c** €/hour - difference of time spent shopping **t** hours - percentage difference in the prices **p** %

The reader can image the prosecution. It is important to observe that the solution of a reality problem can introduce to new problem. For instance: *"The sales manager of the shopping centre is very interested in our formula. How could we show him the influence of the percentage difference in the prices on the value of minimum bill?"*. It is important observe that, working with the same method, reality problems became more and more abstract and mathematics more formalized.

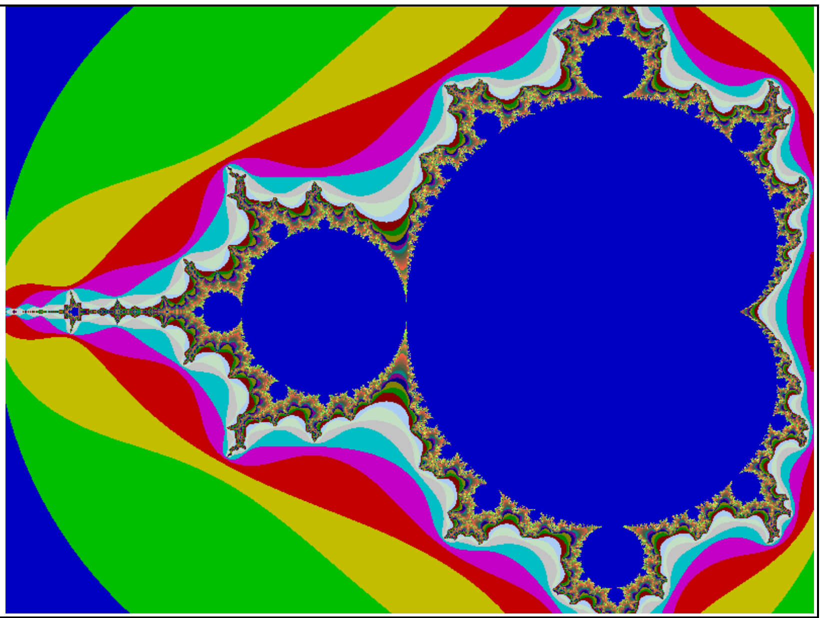
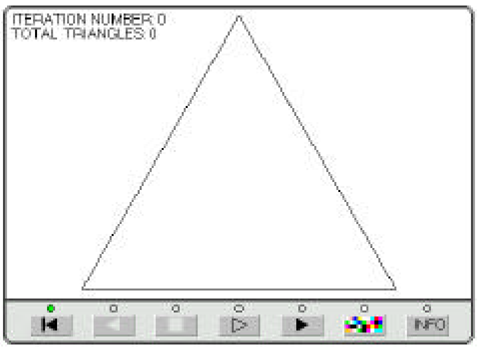
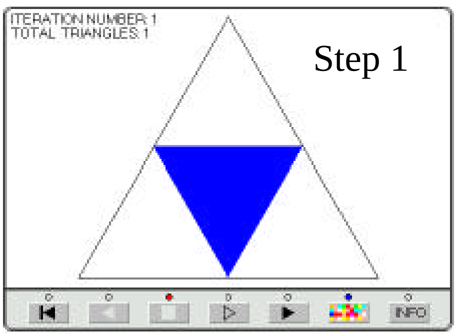
3. An hypothesis for the study of fractals in the schools.

In order to teach the properties of electric current at school, it is possible to abandon the deductive approaches that presume the knowledge of electrons, electric fields, electrostatics and electric machines. But, since electricity is part of our everyday life it may be appropriate to study it on the basis of partial hypotheses, linked to everyday-use measuring instruments. Starting from these partial hypotheses will be possible to extend and to deepen the understanding of all electrical phenomena. In the same way fractals are widely used by media. On the web there are many instruments that allow study their properties. In our hypothesis we will study some of the properties of fractal objects as starting point to extend and deepen the understanding of fractal generation and to study many mathematical topics. Let's now apply this general model at the study of fractals: in a computer laboratory is installed the software Fractint which can be download freely from the Web.

<table>
<tr><td>Real life problem</td><td>The Fractint software, downloadable from the Web

(http://spanky.triumf.ca/www/fractint/getting.html)

generates fractal objects and allows the opportunity of exploring them.

Shall we investigate the Mandelbrot fractal</td><td colspan="2"></td></tr>
<tr><td>reduction</td><td colspan="3">Students can "zoom" to explore deeper a fractal and describe its more evident characteristic. We expect they are likely to describe it as: "fractals presents the some shape within themselves in a smaller and smaller scale". The teacher, like in the previous case, focus the attention of students on a more simplified situation. Cinzia Lanius (http://math.rice.edu/~lanius/fractals/sierjava.html) gives a nice Java application, which reproduces the Sierpinski triangle</td></tr>
<tr><td colspan="2">Layout of problem</td><td colspan="2">The software proposed by C. Lanius reproduces on an ever-decreasing scale the shape of the triangle</td></tr>
<tr><td>analysis</td><td colspan="2"></td><td></td></tr>
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	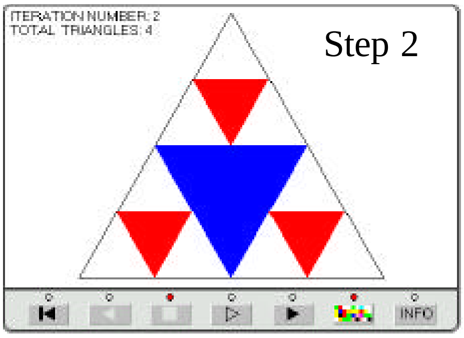	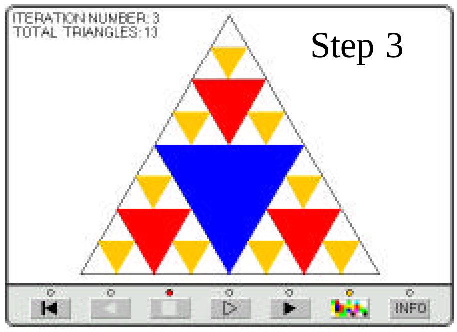	
Statements of problem	1. In the first step which fraction of the triangle is not coloured? 2. In the second step which fraction of the triangle is not coloured? 3. Have you found a rule? Are you able to predict the fraction of triangle that will not be coloured in the fourth step? 4. Try to write a formula that allows the calculation of the fraction of triangle that isn't coloured 5. Compare the figures on step 2 and 3……………………………. 6. Describe a procedure to construct the figures		
Synthesis	*The students, according to their school grade, develop experiences about the problem of representing in and codified mathematical language some properties emerging from the variety of a complex situation.*		
Mathematical models	$\mathbf{S_n = (3/4)^n}$ $\mathbf{a_0 = 1}$ $\mathbf{a_n = (3/4)\, a_{n-1}}$	similarity self-similarity fractal dimension	algorithm iterative algorithm recursive algorithm
Comprehension and understanding of mathematical model	*Students study the assigned sequence eventually linking them to recursive algorithm. (Employing Logo language (there are many free version on the web) they can write a recursive program, which can realise the triangle of Sierpinski and solve the problem n.6*		
Mathematical solution	to sierp :L triangle :L repeat 3[fd :L/2 lt 120 if :L>10 [sierp :L/2] rt 120 fd :L/2 rt 120] end	to triangle :L fd :L rt 120 fd :L rt 120 fd :L rt 120 riempi end	to riempi pu rt 30 fd 3 fill bk 3 lt 30 pd end
Interpretazion	*Students, even very young, (of course without deepen what happens on computer's memory stack) can access to recursive algorithm in Logo language in order to comprehend, at least essentially, recursive aspects of self-similarity*		
Possible solution	Finally students will have at their disposal a solution useful to interpret Mandelbrot fractal through two new concept: self-similarity and recursivity		
Evaluation	Can be useful to show to the students the similarity between the model which describe Mandelbrot (in Fractint software) fractal and the Sierpinski triangle (even without deepen complex number calculus)		**z(0) = c = pixel;** **z(n+1) = z(n)^2 + c.**
Real world proplem	Students got some mathematical concept useful to comprehend and interpret the Mandelbrot fractal generated by Fractin software Fully knowing, they can also modify some parameters.		

A similar path can be realised (http://math.rice.edu/~lanius/frac/koch.html) through the Kock snowflake in order to strengthen and extend the acquired tools. In this case too, it's possible to start a new research path within a more formal level.

4) Conclusions

The new technological context offers new opportunities to construct, elaborate and interpret sophisticated models with easy mathematical instruments. It permits the teachers to widely use the methods of modelling real problems. Generally the technological instruments are used to elaborate and develop mathematical models at different school levels; but, in some circumstances, a technological "tool" should also be considered as a "real" object from which the process of modelling can begin. From this point of view, fractals are one of the "didactic material" most published on the Web. Fractals need interdisciplinary approaches, they can be studied at different school levels to develop many mathematical topics: geometry of nature, self-similarity, logarithm, complex functions, recursive functions. For instance, students could discover the idea of self-similarity by directly exploring some fractals published on the Web, and, by schematising them, they could arrive at the development of some simple mathematical models of self-similarity like the curve of Koch. There are many reasons for using fractals as didactic material from elementary to secondary schools:

- the aesthetical aspects which also involve the students emotional intelligence
- the actuality of fractal models which are used in many applications, from medicine to cinematography
- a wide bibliography published on the web especially with didactical purposes
- the aid of computers allows the student to handle curves and concepts previously reserved to mathematicians and allows the teacher a lower use of technicalities in the program
- the nature of fractals emphasizes the perception that even in mathematics we "invent" rather than "discover"

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didactical proposals

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