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Volume Conception in late primary school children in Cyprus

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Abstract

This article concerns a research contacted in Cypriot schools to evaluate children's (age 10-12 years) learning on volume. Children's response to volume measurement tasks ranged from consideration of only visible aspects of the rectangular constructions, to realisation of the structural organisation of unit cubes in terms of columns and layers to the use of the formula V=LxBxH. Answers to volume conservation tasks ranged from indication full competence to conserve occupied and displaced volume to no conservation at all. It was concluded that there is strong association between understanding of conservation and measurement and that there are specific skills necessary for children to develop before we can expect meaningful use of the formula V=LxBxH.

Key words: Volume, conservation, measurement, primary, Cyprus, SOLO-Taxonomy, formula.

Resumé

Cet article concerne une recherche conduite aux écoles de Chypre pour évaluer l'apprentissage des enfants de 10 à 12 ans sur le volume. Les réactions des enfants aux taches des mesures du volume allaient de la considération des aspects visibles des constructions rectangulaires, a la réalisation de l'organisation structurale des cubes unitaires concernant des colonnes et des couches a l'utilisation de la formule V= LxBxH. Les réponses aux taches de la conservation allaient de l'indication de toute compétence à conserver, à une absence complète de conservation. Nous avons conclu qu'il y a une connections forte entre la compréhension de la préservation et de la mesure et qu'il y a des habiletés spécifiques que les enfants doivent développer avant que nous nous attendions a l'utilisation significative de la formule V= LxBxH.

Mots Clé: Volume, Conservation, mesure, primaire, Chypre, SOLO-Taxonomie, formule.

Riassunto

Questo articolo tratta di una ricerca condotta nelle scuole di Cipro per valutare l'apprendimento degli allievi di 10-12 anni sul volume. Le reazioni degli allievi sui compiti riguardanti la misura del volume vanno dalla considerazione degli aspetti visibili delle costruzioni rettangolari, alla realizzazione dell'organizzazione strutturale dei cubi unitari riguardanti le colonne e gli strati per l'utilizzazione della formula V= LxBxH. Le risposte sulla conservazione vanno dall'indicazione di conservare ogni competenza, ad un'assenza completa della conservazione. Abbiamo concluso che vi è una connessione forte tra la comprensione della preservazione e della misura e che vi sono delle abilità specifiche che gli allievi devono sviluppare prima che vi sia una utilizzazione significativa della formula V= LxBxH.

Parole Chiave: Volume, Conservazione, misurazione, primaria, Cipro, SOLO-Tassonomia, formula.

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VOLUME CONCEPTION IN LATE PRIMARY SCHOOL CHILDREN IN CYPRUS

INTRODUCTION

This is an evaluation of Cypriot children's learning of volume. Different aspects of conservation and measurement of volume are examined in children attending the two final grades of primary school in Cyprus. The concept of volume was chosen to be examined for several reasons. First, past research (Elkind, 1961; Towler and Wheatley, 1971; Enochs and Gabel, 1984; Campbell, Watson and Collis, 1992) has shown that children and adults often have difficulty in fully grasping the concept of volume. Such difficulties are most noticeable usually in the final grades of primary school, during the transition period from primary to secondary education when more abstract methods for measuring volume are introduced.

Second, the concept of volume is systematically encountered throughout the range of school disciplines at increasing levels of sophistication and complexity from early primary to secondary education. It is, therefore, important to examine children's understanding and relative mastery of the concept particularly in the final grades of primary school prior to entry to secondary education.

Lastly, volume seems most appropriate for examination in the area of mathematics because volume tasks in the age range examined can elicit clearly defined quantitative responses which require a variety of mathematics knowledge and skills as well as a clear understanding of certain language terms. Children in the final grades of primary school have already achieved a reasonable mastery of both language and mathematics systems rendering them able to explain their responses to volume tasks either verbally or through formal mathematical expressions.

The arguments in this paper stand on the outcome of a research study which was contacted with primary school children in Cyprus. Children' s responses to volume tasks seemed to follow the SOLO Taxonomy model which served as the main theoretical background. The methodology for the research combined qualitative as well as quantitative methods. The results were categorised according to the SOLO levels and the conclusions drawn concern children's understanding of the concept under investigation and implications to teaching practices.

1. THEORETICAL BACKGROUND

The material included in the Cypriot primary school curriculum and the teaching methods adopted draw heavily on the Piagetian model of intellectual development.

Piagetian theory argues for the existence of a transitional stage (sometime between late childhood and early adolescence) leading to the stage of formal operations when the child develops the abilities of reasoning, analysing, composing and drawing conclusions, thus becoming capable of volume conservation and can readily use the multiplication formula to calculate the volume of rectangular solids.

Piaget (1960) researched different aspects of conservation. Namely conservation of substance, quantity, weight and volume. To Piaget the concept of conservation demands the development of the schemas of multiple relationality and atomism. The first refers to the realisation that in an object under transformation if one dimension increases another decreases. The second one refers to the understanding that matter is composed of tiny units which interchange their location when the whole undergoes a transformation. Furthermore, the child must have developed the concepts of density and compression before he/she is able to conserve volume. Therefore, according to Piaget there is an order in which each kind of conservation is mastered: conservation of quantity and substance are mastered first, conservation of weight follows and conservation of volume is not mastered until about eleven to twelve years.

Two main aspects of volume conservation were distinguished by Piaget. Conservation of interior volume defined by the boundary surface of a block and conservation of occupied volume defined in relation to the object's surroundings in space.

Children's understanding of volume is described (Piaget, 1960) as a developmental process that goes through distinct stages: At the beginning the child is unable for any type of conservation (interior and occupied volume). There is a stage where he/she thinks in terms of one and later in terms of two dimensions, but is not yet able to generalise conservation into all transformations. At a third stage the child conserves interior but not occupied volume. It is not until eleven to twelve years of age when the child becomes capable of realising volume in relation to the surrounding spatial medium and is able to associate volume with the three dimensions.

Further research with Piagetian experiments lead Lunzer (1960) to the identification of a third aspect of conservation, namely conservation of displacement volume. This was defined as *"the equivalence of the quantities of water displaced by equal but dissimilar volume"* (Lunzer, 1969, p.200).

Research conducted in the 1960's (Lunzer, 1960; Lovell and Ogilvie, 1961) and the 1970's the Schools Council project in the UK, 1968-1974 (Hughes, 1979) confirmed children's progressive understanding of the different aspects of conservation of volume proposed by Piaget by demonstrating that younger children base their judgement of equality of volumes on perceptions rather than reasons.

Criticisms of the Piagetian stage theory stemming from its failure to explain the problem of decalage lead to a reconsideration of both the notion of intelligence and the primacy of the underlying cognitive structure in more recent theoretical models (Fischer and Silver, 1985; Demetriou and Efklides, 1985). Post-Piagetian research evidence identified a multiplicity of intelligences which in turn created a shift in emphasis from the study of the overall underlying intellectual structure to the importance of discovering the underlying structure in the specific subject matter within a discipline such as the concept of volume in mathematics.

One such theory which was developed to cope with the problem of decalage is the SOLO Taxonomy originally developed by Biggs and Collis (1982) and later modified and further developed by Collis and Watson (1991) and Biggs and Collis (1991). By separating the underlying hypothetical cognitive structure from the observed level of response (to a variety of content specific materials within the school curriculum), the theory allows for external factors such as language, specific learning experiences, motivation as well as underlying cognitive factors to mediate so as to produce the observed levels of response.

The SOLO Taxonomy theory proposes a structural model consisting of five broad stages: (i) Sensorimotor (from birth); (ii) Ikonic (from 18 months); (iii) Concrete symbolic (from 6 years); (iv) Formal (from 16 years); (v) Post-formal (from 18 years). Each stage has its own unique mode of functioning and its first appearance depends on both maturational factors and mastery of each previous mode in succession. According to Biggs and Collis (1982) the same learning cycle is repeated within each mode and can be identified by five levels of response ranging from an initial simple response to increasingly more complex ones. These levels of responses are: (a) Prestructural;(b) Unistructural; (c) Multistructural; (d) Relational and (e) Extended abstract responses.

During transition from one mode to the next, the most complex response (relational) achieved upon completion of the previous learning cycle (unistructural-multistructural-relational) becomes the unistructural response of the new learning cycle in the next mode of functioning.

In an attempt to examine the nature of the development within one mode (*intra-modal* development), Campbell, Watson and Collis (1992) conducted two studies in which they spanned the developmental sequence for the concept of volume measurement from the beginning of the concrete symbolic mode to abilities immediately prior to entry to the formal mode. The analysis of the results identified two learning cycles (unistructural-multistructural-relational) with the relational response of the first (upon completion of primary school) becoming the unistructural response of the second (beginning of secondary education).

It was concluded that developmental sequences of a specific concept are hierarchical and can be analysed as a sequence of very specific smaller skills. Any such sequences could be described by a unistructural-multistructural-relational learning cycle whereby the individual skills are used separately (unistructural response); many such skills are used in sequence (multistructural response); individual skills are co-ordinated (relational response). The number of such learning cycles within a single mode, is therefore, dependent upon the specific concept under investigation, the methods used and the breadth of the study.

These conclusions are reminiscent of the earlier Piagetian accounts of the developmental sequence observed regarding conception and measurement of volume. The young child starts by focusing on part of the figure only-its external aspects- and moves step by step to an appreciation of its internal structure.

Similar evidence was produced by Battista and Clements (1996) in their study of cognitive operations such as co-ordination, integration and structuring that appear to be required for students to conceptualise and enumerate cubes in three dimensional rectangular arrays. A developmental sequence was identified whereby at the initial phase students focused on the external aspects of the array and perceived it as an uncoordinated set of faces. At later phases as they reflected on experience of counting or building cube configurations, students gradually become capable of co-ordinating the separate views of the arrays and they integrated them to construct one coherent and global model of the array.

Overall structuralist research has demonstrated that the development of volume understanding follows a specific step by step sequence where children move from an appreciation of the external visible aspects of the object to its internal structural organisation in terms of units of measurement. An appreciation of different aspects of volume conservation such as conservation of interior, occupied and displaced volume, has been shown to correspond to different levels of competence in volume measurement.

Other research approaches have produced evidence that the development of the concept of volume in children is dependent not only on maturational factors (their age) or gender-performance differences in volume tasks observed between boys and girls (Hobbs, 1973; Walkerdine, 1988) but also on the development of skills acquired through activity interplay with content specific learning tasks.

This also points to the assumption that content specific activities of increasing order of sophistication might facilitate mastery of the concept of volume. Furthermore simple learning tasks which require individual skills (unistructural responses) have to precede more complex tasks which require individual skills in sequence (multistructural responses) which will have to be followed by tasks which require co-ordination of individual skills (relational responses).

This paper draws attention to children's conception of volume within the context of the Cypriot Primary School curriculum.

During their six years of primary education children are gradually introduced to the units for the measurement of volume and capacity and the methods of calculation of the volume of a rectangular solid. With reference to the teaching material on volume there seems to be a lot of emphasis on the use of the multiplication formula (V=lxwxh) in the last two grades of Cypriot primary school. Furthermore, according to the national curriculum, mastery of the use of the formula ought to have been achieved prior to entry in secondary education.

The present study examines children's performance on different volume measurement tasks of varying level of complexity and in conjunction with their competence on aspects of volume conservation. The possible relationship between children's understanding of volume conservation and their competence in the use of the multiplication formula for the calculation of volume is also considered.

Conclusions are drawn with reference to teaching practices and learning materials in the light of children's readiness to perform on volume tasks of different levels of complexity.

2. METHODOLOGY

2.1. Sampling procedure

The sample was selected from three state Primary Schools in the free southern part of Cyprus. The first school is located in a suburb of Nicosia (school A), the second in the rural area of Paphos (school B) and the third in the urban area at the centre of Larnaca (school C). All the schools were attended by pupils representing a wide range of attainment.

The total sample consisted of 90 Primary School children from the three different Schools. Thirty children were selected from each school, half of them attending grade five (15 children) and the other half attending grade six (15 children). The children from each class were selected randomly from the register, where surnames were listed alphabetically. An approximately equal number of boys and girls was selected from each grade, in each school. Table I. shows the distribution of boys and girls in each grade for each school selected for the study.

TABLE I

Distribution of children in the sample by school, grade and gender

School	Grade five		Grade six		Total
	Boys	Girls	Boys	Girls	
A	9	6	7	8	30
B	7	8	7	8	30
C	7	8	7	8	30
Total	23	24	21	24	90

The testing of the sample was carried out in November of 1996. Prior to testing it was made certain by the class teachers that the material on volume for the academic year 1996-97 had not yet been taught. The children were, therefore, assumed to have been taught only the material on volume and capacity included in the curriculum for earlier grades. Grade five children had already been introduced to the concepts of volume and capacity as well as to the cubic centimetre but not to the multiplication formula. Grade six children were assumed to have additionally been introduced to the use of the multiplication formula for the calculation of volume of rectangular solids.

2.1. Selection of Testing Instruments

The tasks considered for the purposes of the present study included the original Piagetian (Piaget et. al, 1960) transformation task of *“building a house of the same room on an island of different size”* and different versions of this task designed by Lunzer (1960) and Lovel and Ogilvie (1961) using prestructured question to make results more objective and replicable.

Other tasks considered were similar to those used in the CSMS project 1975-1980 (Brown et al., 1984) to investigate levels of children's (11-16 years) understanding of volume measurement. These tasks which were presented in written form included pictured blocks separated into unit cubes.

Prior to construction of the final version of the test items used in the main study, two pilot studies were carried out. Combined versions of the original tests used in past research, mentioned above, were administered to 15 English and Greek 10 to 12 year old children. The conclusions drawn after the analysis of the children's responses were as follows:

In the first pilot study the unit cubes used in the practical material presented were cubic centimetres which were later replaced by cubes of one inch in the second pilot study as they were considered easier for the children to handle.

The term used to describe volume as “*amount of room taken up by the block*” and its Greek translation proved to be the most appropriate for it appeared to have been well understood by all the children.

In the written versions of the tests children seemed to have no problems in conceptualising the pictured blocks which were separated into unit cubes as three-dimensional solids. Most children used an identical method to calculate the number of cubes in the blocks presented either in pictured or physical form.

In the case of undivided blocks presented in physical form the children used unit cubes to calculate the volume of the block.

In the case of undivided blocks presented in pictured form the children either separated for themselves the surface of the block into unit squares and proceeded as they did with the blocks which were already separated or just multiplied the three dimensions given in the diagram but without providing a meaningful explanation for using the multiplication formula for the calculation of volume. It was, furthermore, observed that some children produced a wrong result possibly due to numerical incompetence. This last observation led us to include some items to test the numerical competence of the children tested.

Regarding the transformation tasks, most of the children tried to calculate the number of cubes that made up the old house. The methods employed to calculate volume were the same as those used in the measurement tasks (physical and pictured blocks divided into unit cubes). Additionally, children who conserved interior volume in these tasks also showed conservation of interior volume in the main conservation tasks.

Responses to the conservation task were identical in the two versions (interview-physical and written-pictured form) of the test. The explanations provided by the children in the interview version were the same as those provided in the written version. Most of the children seemed to recognise conservation of interior volume while fewer seemed to be able to conserve occupied and displaced volume as well.

2.2. Testing instruments and procedure used in the main study

The testing materials were presented in physical and pictured form. A structural interview was conducted with each individual child. All the interview sessions were tape-recorded and later transcribed. They were summarised and translated into English.

The instructions to the questions contained various expressions commonly used in the language (in textbooks, in classroom instruction and in casual conversation) of 10-12 year old children. This was done in order to ensure that the expressions that were to be used in the main study would accurately convey the meaning of the original questions from English into Greek. These expressions were considered to be familiar to Cypriot children so as not to significantly affect their performance on the tasks.

Children were asked to provide an oral explanation for their answers to the interview tasks and a written explanation (such as an equation or words) to support their numerical answers to the written tasks.

For written tasks, a group testing procedure was used whereby children were tested in small groups (5 children on average) and were required to provide a written explanation to support their answers. The instructions to each consecutive task were first read out to the group upon completion of the previous task by all the children.

Tasks could be grouped under three general headings concerning: Measurement of volume, Conception of volume and Numerical competence. The interview part of the test was administered first and included some measurement tasks and the transformation task. The written part of the test was administered after all interviews for each school were completed. It included the numerical competence task, some measurement tasks and the main conservation task. The sequence in which the whole test was administered is described below.

2.4. Description of the tasks

(A) Measurement of volume

Each child was required to calculate the volume of rectangular blocks in terms of unit cubes and provide a verbal or written explanation to support their answer.

(1) Rectangular constructions separated into unit cubes.

(1a) Physical versus picture construction:

At the beginning of the testing sequence each child was presented with an actual rectangular 3x4x5 block separated into unit cubes and was subsequently required to calculate the number of unit cubes in the construction.

At the end of the testing sequence, after all the tasks had been completed, each child was again presented with a picture representation of the original 3x4x5 rectangular construction separated by lines into unit cubes and was asked to calculate the number of cubes in the construction.

(1b) Pictured versus physical construction:

Each child was presented with a picture representation of a 3x3x4 rectangular block separated by lines into unit cubes and was asked to calculate the number of unit cubes in the block.

Following this task, children who did not obtain the correct answer were asked to physically construct the 3x3x4 pictured block using unit cubes and were then asked if they could calculate the number of unit cubes in the construction.

(2) Capacity of a physical rectangular container.

The materials used for this test item were comprised of a physical rectangular empty box measuring 3x3x5 and a pile of unit cubes. Each child was asked to find the number of cubes that could fit into the box.

(3) Volume of a physical rectangular solid block.
The materials used for this test item were a solid rectangular block measuring 2x3x4 and a pile of unit cubes. Each child was asked to find the number of unit cubes needed to construct the block and to explain what was represented by the number they found.

(B) Conservation of volume

(1) The transformation task:
This task involved a construction similar to that used by Piaget et al (1960). In a structured interview setting individual children were presented with a 3x4x3 cuboid separated into unit cubes which they were told was a block of flats built on an island. The task was introduced as follows:
*"**This is a house** (The interviewer shows the original construction with dimensions 3x4x3 inches). **The house is built on an island**. (The interviewer puts the construction on a white card base separated by lines into 3x4 square inches then onto a blue cardboard sheet representing the sea). **But the inhabitants of the house have to leave it. So they decide to build a new house on another island. This island here** (She shows another white card base separated by lines into 2x2 square placed inches on the blue cardboard). **They want their new house to have as much room as their old one**."*
Conservation (interior volume) was determined by the children's responses to the following question: ***What will the new house look like?***
In the case the child could not understand or could not foresee that the new block would be taller the interviewer provided a further explanation that each unit cube represented one room and that each inhabitant had a room in the old house and would have to have a room in the new house as well. After this explanation children were asked to calculate the height of the new house. Responses regarding the height of the new construction were taken to be indicative of a child's understanding of conservation of interior volume.

(2) The conservation task:
The conservation task was adopted with the aim of testing children's understanding of all the aspects of volume conservation. In the pilot studies the task was presented in two versions. One using physical items and one using picture items. Individual responses to items presented in different form were identical. It was, therefore, considered more appropriate to use only the written version of this task (with picture items) for the main study for two reasons: Firstly because children could be tested in small groups making the testing procedure less time consuming and secondly -and more importantly- the questions would be the same for all children making individual responses more objective and more easily comparable to past research.
Prior to the main task children are presented with a container full of water and a unit cube said to be made out of iron. Children are asked what would happen to the water when the cube is put into the container. This was considered necessary prior to the main task to make certain all children understood the concept of occupied volume.
The main task was introduced with a picture of a container half filled with water

and a 4x3x5 block which was said to be made of iron unit cubes. Children were asked what would happen to the water if the block is put in it and then to explain their answer.
Children were then told that the 4x3x5 block was dismantled and all the unit cubes were used to make a new 3x2x10 block. They were then asked what would happen to the water when the new block was immersed in it. (Conservation of displaced volume).

(C) Numerical competence.

To investigate whether possible problems with numerical multiplication could account for any failure of the children to use the multiplication formula for the calculation of volume of a rectangular solid, this written task was introduced.

It consisted of 24 numerical items arranged into four groups. The first group of items included six multiplications of two numbers, e.g. 3x3=--, the second group included six multiplications of three numbers, e.g. 3x3x4=--, the third group included six equations of multiplications in which one of the numbers was missing e.g. 4x9=3x-- and the fourth group included six division items, e.g. 60:5=--.

3. CODING OF RESPONSES TO MEASUREMENT TASKS

The methods used by children to measure volume appeared to follow the same pattern for different test items. Responses ranged from simple ones in which only visible aspects of the different constructions are taken into account, to more complex ones, which show understanding of the structural organisation of unit cubes in the constructions and of the use of the multiplication formula.

Methods are initially categorised into successful and unsuccessful. Within these broad categories, responses are listed with respect to order of complexity ranging from complex successful responses at the top to more simple unsuccessful responses at the bottom involving only visible aspects of the construction.

The coding of answers to volume measurement tasks and the subsequent interpretation of results is based on the SOLO taxonomy for it seems that the pattern of children's responses resembles very closely to the categorisation proposed by the SOLO taxonomy theory.

A primary objective in this study is to identify associations between answers to measurement tasks and understanding of different aspects of conservation of volume.

A further objective is to examine whether mastery of conservation means mastery of other related skills such as measurement and numerical competence.

1 SUCCESSFUL STRATEGIES

(Relational level of SOLO)

11 Multiplying lengths of dimensions. Use of the multiplication formula V=LxBxH. This corresponds to the relational level of the SOLO taxonomy response for children are no longer bound by the external aspects of each construction and can integrate the three Euclidean dimensions to calculate volume (Campbell, Watson & Collis, 1992, p.291).

(Multistructural level of SOLO)

121 Sequential addition or multiplication by number of layers.

122 Sequential addition or multiplication by number of rows or columns.

131 Counting visible plus invisible cubes in an organised manner which is structurally correct. For example, the child first counts number of cubes in a column then adds successively for each outside column and finally adds the number of cubes in each of the columns in the middle. It seems that children employing this strategy are still somewhat bound by the visible aspects of the structure for they count the cubes on the outside columns first before considering the columns in the middle.

Responses 121, 122, 131 correspond to the multistructural level of SOLO response for in these instances children use sequential processing of two dimensions. They process two dimensions at a time in terms of rows or columns or layers and then they use multiplication or sequential addition to calculate the number of rows or columns or layers. Thus whenever multiplication occurs it is only between two variables one of which can be a composite dimension say LxB and this product is successively added to itself or multiplied by the third dimension. That is (LxB)xH (Campbell, Watson & Collis, 1992, p.290).

2 UNSUCCESSFUL STRATEGIES

(Counting visible and invisible cubes)

(Unistructural level of SOLO - transitional)

241 Counting visible plus invisible cubes in an organised but structurally incorrect manner. For example, children first count some or all of the visible cubes (avoiding double counting cubes in the corners) and then they double that number in an attempt to account for the invisible cubes. It should be noted that in such responses there is no explicit reference to cubes in the middle of the structure.

242 Counting visible cubes only in an organised but structurally incorrect manner avoiding double counting cubes in the corners. For example, children count cubes on some or all of the faces of the construction.

Children who responded with strategies 241 and 242 show some signs of transition towards a multistructural level of response by trying to make some sense of the organisation of the unit cubes thus counting cubes instead of faces and avoid double counting cubes in the corners. They are however bound by perception of the external aspects of the cube arrangement and fail to make sense of its internal organisation in terms of columns and layers.

3 UNSUCCESSFUL STRATEGIES

(Counting area of outside surface)

(Unistructural level of SOLO)

351 Counting area, (that is, squares and not cubes as space filling) on some or all of both visible and invisible faces of the rectangular construction. For physical and picture constructions.

352 Counting area only (that is, squares and not cubes as space filling) on some or all of the visible faces of the rectangular construction. For pictured constructions only.

Responses 351 and 352 can be described as unistructural ones for children's attention focuses only on the visible aspect of the construction. That is children fail to realise cubes as space filling and can only consider them in terms of their faces/squares thus totally missing the structural organisation of the rectangular arrays.

4 OTHER

These strategies were not as clear cut as the ones listed above and could not be identified by a distinct category of response. Such responses included cases where a child may have started by employing a strategy and then changed half way through into another strategy. Nearly all the children who produced these responses did not show an appreciation of the structural organisation of the constructions presented and failed to measure volume correctly.

The strategies listed above proved sufficient to describe the great majority of responses to the measurement tasks. There have been some exceptions in the case of capacity measurement and measurement of volume of an undivided solid.

In the first case some children filled the empty box measuring cubes one by one. Such a strategy was listed as 4. Children who were successful in this task put unit cubes in single lines along the dimensions of the box and then multiplied using the formula or they first calculated the number of cubes in a layer using cubes one by one and then put cubes in a single corner column of the box to find the number of layers. Those who employed unsuccessful strategies built walls using unit cubes on the outside of the empty box and then counted either unit cubes or just unit squares.

In the case of the undivided block, children using successful strategies either put unit cubes in single columns along the three dimensions of the undivided block and then used the formula or found the number of cubes in a layer and then either added this number successively or multiplied by the number of layers. Children using unsuccessful strategies built walls around the block using unit cubes and then measured number of cubes or surface of squares.

4. RESPONSES TO MEASUREMENT TASKS

As one would expect there is an order of difficulty among the three measurement tasks compared. Capacity was calculated correctly by 82.2 % compared with 71.1% for volume of a block separated into unit cubes and 55.6 % for volume of an undivided solid. There is also a progressive shift in the strategies used as children moved from the easier task of measuring capacity to the more abstract task of measuring volume of an undivided solid. The number of children using the multiplication formula (code 11) progressively increased from the easier task to the more difficult one while the number of children that used a layer strategy in the capacity task also shifted to using a unsuccessful strategy involving measuring individual cubes or squares. These results are shown in Table II.

TABLE II

Frequency and percentage of methods used by children to measure volume

Method of calculation	Test Items: Capacity of physical rectangular container (3x3x5)		Volume of physical rectangular block separated into unit cubes (3x4x5)		Volume of physical rectangular solid block (2x3x4)	
	N	%	N	%	N	%
Successful	74	83.3	64	71.1	50	55.6
1	73	82.2	64	71.1	50	55.6
11	7	7.8	9	10	15	16.7
121	58	64.4	51	56.7	35	38.9
122	6	6.7	3	3.3		
131	2	2.2	1	1.1		
4	1	1.1				
Unsuccessful	16	17.5	26	28.9	40	44.4

2	1	1.1	6	6.7	2	2.2
241			1	1.1		
242	1	1.1	5	5.6	2	2.2
3	13	14.4	17	18.9	37	41.1
351			3	3.3	4	4.4
352	13	14.4	14	15.6	33	36.7
4	2	2.2	3	3.3	1	1.1
Total	90	100	90	100	90	100

4.1. Pictured representation versus Physical construction

To establish possible differences in responses between tasks presented in physical and pictured form we carried out a small experiment in a structural interview setting.

In the first phase all children were presented with a rectangular 3x3x4 pictured construction separated into unit cubes and were asked to calculate the number of unit cubes in the construction.

In the second phase all the children that were not successful in their attempt to calculate the volume of the pictured block were asked to physically construct the pictured block using unit cubes and were again asked to calculate the number of cubes in the block. The results of the comparison are presented in Table III below.

TABLE III

Performance before and after physically constructing a pictured block

	Test Items			
Method of calculation	Pictured (All children)		Physical (Unsuccessful children in pictured task)	
	N	%	N	%
Successful	57	63.3	8	8.9
1	57	63.3	8	8.9
11	8	8.9		
121	47	52.2	6	6.7
122	2	2.2		
131			2	2.2
Unsuccessful	33	36.7	25	27.8
2	7	7.8	5	5.6
241	3	3.3	1	1.1
242	4	4.4	4	4.4
3	23	25.6	16	17.8
351	14	15.6	5	5.6
352	9	10.0	11	12.2
4	3	3.3	4	4.4
Total	90	100	33	36.7

All the unsuccessful children in the pictured task managed to produce an identical physical block using unit cubes but not all of them managed to calculate the correct number of cubes in the physical block after they constructed it. It seems that children can produce accurate physical reconstructions of pictorial representations of rectangular separated blocks but that does not necessarily guarantee success in measuring their volume in terms of unit cubes.

The results in Table III show that the performance of only 8.9 % of the children who were unsuccessful with the pictured task improved after physically constructing the block.

It is also interesting to note that the children who improved their performance used layer or column strategies moving from a unistructural level of response in the pictured task to a multistructural response in the physical task. By physically constructing the rectangular block some children were led to realise the structural organisation of the block in terms of unit cubes arranged in layers or columns. Clearly, there is an indication that some amount of learning has taken place due to the form in which the tasks were presented.

4.2. Testing sequence and task form (Physical versus Pictured)

It was hypothesised that children's overall performance in measuring the volume of a pictured block separated by lines into unit cubes would improve after the whole testing sequence was completed. Children's performance on measuring the volume of a physical 3x4x4 rectangular block separated into unit cubes presented at the beginning of the testing sequence was compared with their performance on measuring the volume of the same block presented in pictured form at the end of the testing sequence (Table IV).

TABLE IV
Performance at the beginning (physical block) and at the end (pictured block) of the testing sequence.

	Test Items			
Method of calculation	Beginning of testing Physical		End of testing Pictured	
	N	%	N	%
Successful	64	71.1	65	72.2
1	64	71.1	65	72.2
11	9	10	20	22.2
121	51	56.7	44	48.9
122	3	3.3		
131	1	1.1	1	1.1
Unsuccessful	26	28.9	25	27.7
2	6	6.7	3	3.3
241	1	1.1		
242	5	5.6	3	3.3
3	17	18.9	19	21.1
351	3	3.3	10	11.1
352	14	15.6	9	10
4	3	3.3	3	3.3
Total	90	100	90	100

The results in TableIV do not show a considerable improvement in terms of more children employing successful methods to calculate the volume of the pictured block at the end of the testing sequence compared with their performance in the physical version of the same task at the beginning of the testing sequence. This, however, does not defy our hypothesis, on the contrary it supports it. While the performance of the sample on the pictured task was inferior to their performance on a similar physical task at the beginning of the testing sequence (see TableIII) now at the end of the testing sequence (see TableIV) children's overall performance on the physical task (beginning of testing sequence) seems to be at least no different from their performance on the pictured version of the same task (end of the testing sequence).

Furthermore results in TableIV show that a considerable amount of successful children that used a layer or column strategy (multistructural response) in the physical task shifted to the use of the multiplication formula (relational response) in the pictured task. This is an important qualitative change in the method used to calculate the volume of a pictured block and can only be ascribed to the learning that took place during the testing sequence.

5. RESPONSES TO CONSERVATION TASKS

Conservation of interior volume-The Transformation task

Conservers of interior volume were considered those children who responded that the new construction would have to be taller. Non-conservers of interior volume were considered those children who provided a response showing no realisation that the new house will have to be taller in order to be of the same room, even after the explanation (the new house will have to have the same number of rooms inside). These children did not seem to realise that when one dimension decreases the other will have to increase for the two houses to have an equal amount of rooms (cubes). The great majority of children in our sample showed understanding of conservation of interior volume (n=86, 95.6%) while only very few (n=4, 4.4%) did not.

The methods used by children to calculate the number of cubes in the original block ("old house") are directly comparable to those used by children to calculate volume in the measurement tasks.

The 4 non-conservers of interior volume did not show understanding of the structural organisation of the original construction in their attempts to calculate the volume of the "old house". Two of those four children responded that the new construction is impossible to build and did not attempt to measure the volume of the original block. The remaining two children produced a unistructural response (method 352) and failed to calculate the correct number of unit cubes in the original construction. They attempted to build the new construction "new house" using individual cubes but they insisted that the new construction would have to be of the same height as the original one.

To calculate the height of the new construction ("new house") after the number of cubes in the old house had been determined the majority of children (conservers of interior volume) used the following general strategies:

1. Multiplication (M):. Thought more in terms of the opposite of division. The height of the new construction was considered to be the number that had to be multiplied to the number of cubes in the bottom layer (4) in order to produce the total number of unit cubes in the "old house" (36).
2. Division (D): Children divided the number of cubes used in the "old house" (36) by the number of cubes in the bottom layer of the "new house" (4).
3. Rearrangement (R): Children using this method rearranged chanks of cubes of the "old house" on a 2x2 base and put them on top of each other until all the cubes were used. Some of these children did not provide a clear explanation as to how the volume of the old house had been calculated but the new construction -on the majority, if not in all such cases -was successfully constructed. Possibly due to mastery of the multiplication formula.
4. Building with individual cubes (B): Children built the new construction using as many individual cubes -one by one- as found to be contained in the original block.

Table V shows the methods used by those children considered as conservers of interior volume to calculate volume of the old house, the height of the new house and their final success or failure to complete the whole trasnsformation task

TABLEV

Performance on the transformation task

Methods of calculation of volume	Methods of calculation of new height					Completion of the Transformation task	
	R	D	M	B	Total	Successful	Unsuccessful

Successful	10	14	22	22	68		
1	5	14	22	22	63	63	
11	1	7	4		12	12	
121	4	6	18	20	48	48	
122				2	2	2	
131		1			1	1	
4	5				5	5	
Unsuccessful		1	1	16	18		
2					2		2
241							
242				2	2		2
3					14		14
351		1	1	2	4		4
352				10	10		10
4				2	2		2
Total	10	15	23	38	86 100%	68	18

As expected, all students who calculated the number of cubes in the original block using the multiplication formula did not have to built the new house to find its height. They instead used rearrangement, division or multiplication. The majority of students (28 students) who used a layer method to calculate the volume of the original block also used rearrangement, division or multiplication to calculate the volume of the new construction while a considerable number of those students (20 students) still had to build the actual construction to find its height.

On the other hand, students who were not successful in calculating the volume of the original block resorted almost solely to building the new block with individual cubes to find its height.

5.1. Conservation of displaced volume

Responses to the introductory task of placing a unit cube made of iron in a container full of water showed that all children understood occupied volume for all children responded that the water would overflow when the unit cube is placed into the container.

Answers to the main conservation task were taken as indicative of a child's competence to conserve occupied and displaced volume. Based on children's answers four categories of responses were identified:

Conservers (C): Those children who clearly stated that the water level will rise to the same level when blocks G and H are immersed into the container and supported their answer by providing an explicit reason such as that: "The water level will rise the same because the two blocks are made of equal numbers of cubes or have the same volume or that the water displaced will be exactly the same because the volume of the two blocks is the same."

*Non-Strong-Conservers (NSC):*Those children who stated that the water level will rise the same but failed to provide an adequate explanation for their view either because they were constrained by their ability to express themselves in written language or because they understood the truth of such a statement intuitively but not in formal mathematical terms.

These children are considered to be able to conserve interior volume but seem unable to provide a concrete explanation or show adequate understanding of all aspects of volume conservation.

Non-Strong-Conservers-Position (NSCP):Those children who failed to respond that the water will rise the same if the two blocks are successively immersed in water. Furthermore, those children are identified under the above category specifically because they additionally stated that the level of water will rise less when the second block is immersed into the container clearly because they are distracted by the positioning of the second taller block. They explicitly responded that the second block will not be totally immersed in the water.

Non-Conservers (NC): Those children who stated that the water will rise more when the second block is immersed in the water and they qualified their answer by stating that the second block is larger because this block is taller and therefore it's bigger. Clearly those children did not show any indication of understanding conservation volume in all its aspects. In some cases not even interior volume.

A primary objective of the present study is to observe any connections between the performance of the children to volume measurement and volume conservation tasks. Table IV describes the answers of the children to three different measurement tasks in terms of response categories in association with their responses to the conservation task.

TABLE VI
Performance on three measurement tasks and the conservation task

	Method of calculation	**Conservation of Displacement volume**				
Measurement Task		**C**	**NSC**	**NSCP**	**NC**	**Total**
	1	23	21	14	16	74
Capacity of	11	5	1	1		7
container	2				1	1
(3x3x5)	3		6	1	6	13
	4		2			2
Total		23	29	15	23	90
	1	23	18	12	11	64
Volume of	11	4	4	1		9
separated block	2		2	1	3	6
(3x4x5)	3		7	1	9	17
	4		2	1		3
Total		23	29	15	23	90
	1	23	10	10	7	50
Volume of solid	11	7	2	5	1	15
block	2		1		1	2
(2x3x4)	3		18	5	14	37
	4				1	1
Total		23	29	15	23	90
	1	22	17	13	12	64
Transformation	11	6	2	4		12
task- original	2		1		1	2
block (3x4x3)	3		7	2	7	16
	4	1	4		3	8
Total		23	29	15	23	90

The above results (TableVI) provide two clear cut observations specifically concerning the two extreme groups of conservers and non-conservers of displacement volume.

First, children identified as conservers of displacement volume are all successful at calculating volume in the different measurement tasks.

Second, the non-conservers although some of them (not the majority) are successful at calculating volume in the measurement tasks, they do not use the multiplication formula for the calculation of volume but resort rather to a layer or column strategy. Clearly, therefore, non-conservers at best produce a multistructural response in their attempts to calculate the volume of a rectangular solid but do not reach the relational response level which appears to be related to the use of the multiplication formula.

With reference to the remaining two response categories (NSC and NSCP) identified in the conservation of displacement volume task children's performance on this task does not seem to bear direct relationship to their performance on the different volume measurement tasks. These two groups are distinctly different from the group of non-conservers of displacement volume for the majority of students in both groups seem to be successful in calculating volume in the measurement tasks some of them using the multiplication formula.

6. OTHER FACTORS

6.1. Numerical competence

Items taping numerical competence of the children in our sample were included in the study to investigate whether children's proficiency with multiplication (axb=?, axbxc=?), simple multiplication equations (axb=cx?) and division (a/b=?) was significantly related to their performance and the methods used to calculate volume in the measurement tasks.

Specifically attention was drawn to possible relationships between numerical proficiency and the use of the multiplication formula for the calculation of volume.

Comparisons of mean performance on numerical competence items (obtained for each separate set of numerical items) between successful and unsuccessful students in the different volume measurement tasks prooved not statistically significant. This was the case for comparisons carried out for the whole sample and for students attending grade 5 and grade 6 examined separately.

Comparisons of the mean numerical performance of conservers and non-conservers of displacement volume also did not reveal any significant differences.

When the mean numerical proficiency performance of students who used the multiplication formula (code 11) for the calculation of volume in the measurement tasks was compared with the performance of successful students who used a layer method (code 121) some significant differences were identified (see Table VII). Students who used the multiplication formula (11) showed an almost perfect performance in all items taping numerical competence. This finding suggests that for students who are successful at calculating the volume of rectangular solids there is a strong association between numerical competence and the use of the multiplication formula. It seems that for students in our sample correct understanding of the structural complexity of the construction alone does not suffice to produce a relational response (use of the multiplication formula) in the calculation of volume of rectangular solids and that numerical proficiency with multiplication and division items is an important and possibly a necessary skill. Table VII presents the results of t-tests identifying the levels of significance for mean differences in each of the four numerical competence items between students using the multiplication formula (11) and a layer strategy (121) for the calculation of volume in four volume measurement tasks.

Table VII
Differences in method of (successful) calculation of volume and performance on the numerical competence tasks

	Measurement Task							
Method of calculation	Capacity of container (3x3x5)		Volume of separated block (3x4x5)		Volume of solid block (2x3x4)		Volume of separated block (3x3x4)	
	Multiplication of two numbers (axb=?)							
	N	Mean	N	Mean	N	Mean	N	Mean
11	7	5.85	9	6.00	15	6.00	12	6.00
121	58	5.98	51	5.98	35	5.91	48	5.93
	N.S.		N.S.		$p<0.083$		$p<0.08$	
	Multiplication of three numbers (axbxc=?)							
	N	Mean	N	Mean	N	Mean	N	Mean
11	7	5.85	9	6.00	15	5.93	12	5.91
121	58	5.55	51	5.50	35	5.34	48	5.43
	N.S.		$p<0.011$		$p<0.039$		$p<0.052$	
	Multiplication equations (axb=cx?)							
	N	Mean	N	Mean	N	Mean	N	Mean
11	7	6.00	9	6.00	15	5.73	12	5.75
121	58	5.25	51	5.43	35	5.11	48	5.25
	$p<0.000$		$p<0.003$		$p<0.090$		N.S.	
	Division of two numbers (a/b=?)							
	N	Mean	N	Mean	N	Mean	N	Mean
11	7	5.85	9	5.33	15	5.93	12	6.00
121	58	5.53	51	5.31	35	5.05	48	5.14
	N.S.		N.S.		$p<0.003$		$p<0.001$	

6.2. Age and gender differences

As one would expect overall older students (grade 6) in our sample performed better than younger students (grade 5) in all volume measurement tasks including the transformation task but these differences were not so distinct as to reach statistical significance except in the case of the conservation of displacement volume task.

More grade 6 students were identified as conservers of displacement volume (16 students) compared with grade 5 students (7 students) while more grade 5 students were identified as non-conservers (16 students) compared with grade 6 students (7 students) (chi-square value=7.95, df=3, $p<0.05$).

With regard to possible gender differences, overall boys were shown to perform slightly better than girls in all volume measurement tasks as well as the numerical competence tasks but the differences did not reach statistical significance except in the case of the transformation task. More boys (38 out of 44) of both grades completed both parts of the transformation task successfully compared with girls (30 out of 46) of both grades (chi-square=5.44, df=1, $p<0.05$).

Overall a weak trend was identified for boys to be more competent than girls as manifested through their performance in numerical operations, volume conservation and measurement tasks examined in this study. However, these differences are too small indicating that this trend is probably on the decline compared with findings of earlier studies in the 60's, 70's and 80's whereby gender differences in mathematics performance were more prominent.

7. DISCUSSION OF RESULTS

Children's responses to the volume measurement tasks clearly follow a pattern which reflects the topological appreciation of an object described by Piaget et. al. (1960) and fit very well within the categorisation of response level proposed by the SOLO Taxonomy theory.

At the lowest level of response (code 351, 352) children are bound by the visible aspects of the object - its external characteristics - and perceive them as an uncoordinated set of faces without structural organisation. This would correspond to a unistructural level of response for it takes into account only one aspect of the object - the visible one. Thus the object is perceived as two dimensional and not as taking up space. The space bound by the outside surfaces of the object is not accounted for.

At the next level (code 241, 242) children still produce a unistructural response but they attempt to count cubes rather than faces and most importantly they are trying to account for cubes in the middle of the construction (invisible cubes). In other words, children producing such responses show attempts to account for space beyond the visible boundary surfaces of the construction but still lack understanding of its structural complexity and thus fail to measure volume correctly.

The third level of response is clearly multistructural for here the structural organisation of the object is realised and two aspects of the construction are processed one after the other. That is, first the number of cubes in a layer or a column are calculated and then successive addition or multiplication by the number of layers is used leading to the total number of cubes in the construction.

At the top level, the response is relational for the three dimensions of the construction are co-ordinated and processed simultaneously rather than sequentially. The number of cubes in the construction is obtained by the multiplication of the three numbers corresponding to the three dimensions.

The subjects in our sample were found to operate at the concrete symbolic mode of functioning following the criteria proposed by Piaget and the SOLO Taxonomy theory

(Campbell, Watson, and Collis, 1992). Almost all children (except four) seemed to conserve interior volume and showed ability and competence in producing concrete quantitative responses not qualitative or intuitive as it would have been the case had they been operating in the ikonic mode of functioning. The tasks used, were themselves concrete for they involved real objects (physical tasks) or real representations of objects (pictured constructions).

Children's responses to different tasks presented in physical form involving measurement of capacity, volume of a separated block and volume of an undivided block, clearly show that these tasks become progressively more difficult.

The number of students who solved the capacity task correctly progressively decreases in the next task of the separated block and in the third task of the solid block only about half the students calculate volume correctly.

This decrease in correct answers probably reflects the degree of children's awareness of the structural complexity of each task.

In the capacity task children can become more easily aware of the structural organisation of unit cubes in terms of layers or columns. On the other hand it could be just a case of simply fill-in the box with individual unit cubes counting them one by one not necessarily realising the cube arrangement in terms of columns and layers, let alone using the multiplication formula.

In the following task of the rectangular block separated into unit cubes the structural organisation of the construction is more easy to grasp than in the case of the unseparated solid block, where no clues are provided pointing to the structural arrangement in terms of columns or layers.

Although the number of correct responses decreases as we move from the capacity task to the solid task the number of students using the multiplication formula steadily increases. This leads us to conclude that through these tasks some students became increasingly aware of the structural organisation of a rectangular construction in general resorting to a more abstract method of calculation (volume formula) while other students lose sight of the structural organisation of the construction as the visual clues disappear and resort to a lower response level strategy.

It could also be likely that for older students (grade 6) who have already been introduced to the use of the volume formula it is easier to grasp the structural complexity of the construction aided by past learning experiences. This however, does not seem to be the case since out of the 15 students who used the volume formula in the task with the unseparated solid block 7 attended grade 5 and 8 of them attended grade 6.

Further indications that realisation of the structural organisation of a construction leads to the correct calculation of its volume come from results obtained from the comparison between performance in identical tasks presented in physical and pictured form.

When the unsuccessful students in the pictured task were asked to physically construct the block using unit cubes their performance improved by 8.9 % of the total sample.

A further indication supporting the idea that differences in performance are related to learning experiences gained through experimenting and practising with concrete-physical tasks comes from a comparison between a physical task involving a separated block at the beginning of the testing sequence and an identical task presented in pictured form at the end of the testing sequence. The results show that the performance on the pictured task was no different from that in the physical task at the beginning of the testing sequence. Additionally there was a considerable increase in the number of students who used the multiplication formula at the beginning (physical task) from 9 students to 20 students in the pictured task at the end of the testing sequence. This can only be due to the learning experiences that must have taken place during the testing sequence possibly due to children's practising with the physical material presented in the different tasks, gaining on their understanding of the structural complexity of the construction.

The results in this study seem to agree with the view supported by the research findings of Ben-Haim, Lappan and Houng (1985) who concluded that students in grades 5 through to 8 have difficulty relating isometric type drawings to rectangular solids they represent and that children should be given the opportunity to make concrete representations with individual cubes. It is further suggested that concrete experiences with cube-building are helpful in improving students' performance.

Similarly Battista and Clements (1996) found that student's initial conception of rectangular arrays is as uncoordinated as a set of faces. Eventually students after becoming capable of co-ordinating views they see arrays as space filling and strive to restructure them as such. Furthermore, they argue in favour of including practical material in the school curriculum that would enable the students to gain mental structures through physical manipulation with certain rectangular objects before grasping of the multiplication formula is introduced with any reasonable expectation as to its meaningful use. It was concluded that for most students in their study, the traditional formula was unlikely to describe a personally constructed procedure for determining the number of cubes in a 3-D array.

Analysis of the responses to the task on conservation of displacement volume in the present study showed that there is a strong association between conservation of volume in all its aspects (conservation of true volume) and understanding of the structural complexity of the blocks in the measurement tasks, leading to correct calculation of volume for all conservers. Additionally among students in the group identified as conservers of displacement volume the multiplication formula was used more frequently than in the other groups, while among students identified as non-conservers of displacement volume the multiplication formula was hardly used at all.

It seems that like Piaget et. al. (1960) argues, having understood the relation between boundary lines and interior volume children now deepen their sense of conservation of volume and they understand that it is not the interior contained which is invariant but the space occupied in general. Children who conserve occupied and displaced volume can understand space more abstractly in terms of its metrical continuity extending their thinking beyond the mere shape or positioning of objects.

For conservers, volume (as an abstraction based on metrical continuity) can be more easily grasped in terms of the multiplicative relation between the three dimensions and the multiplication formula is readily applied.

Understanding, however, volume in terms of the multiplication formula does not necessarily mean that it will be used. A number of conservers of displacement volume in the present study, although they produced clear signs of understanding the formula still resorted to a layer strategy to calculate volume.

When students using the formula and students using a layer strategy were compared on the basis of their competence with different multiplication and division operation it became quite clear that students consistently using the formula were more proficient at multiplication and division than students resorting to a layer strategy for the calculation of volume. Thus, our results seem to agree with Hart's (1989) conclusions that the multiplication numbers bonds need to be known and that the multiplication of three numbers must be taught or revised as a necessary prerequisite to using the volume formula.

Overall the present study showed that children, attending grade 5 and 6 in Cypriot primary schools, operate on their overwhelming majority at the concrete symbolic mode of functioning and that their responses to different tasks on measurement and conservation of volume fit into the developmental model of response level proposed by the SOLO Taxonomy theory.

Some age differences were observed with older students performing slightly better, especially on conservation of displacement volume. Some gender differences were also observed identifying a tendency for boys to perform better than girls but this trend was rather weak and possibly on the decline.

8. CONCLUSIONS

The overwhelming evidence presented in this study leads us to conclude that there are specific skills necessary for children to develop before we can expect meaningful use of the multiplication formula.

First children need to practice with concrete tasks of increasing structural complexity through which they can acquire personally constructed views of the organisation of the three dimensional rectangular arrays made of individual cubes before engaging with pictorial representations of divided or undivided rectangular solids.

Second children have to master conservation and guided through transformation tasks come to a realisation of volume in terms of its metrical continuity doing away with distraction imposed by shape or positioning of objects.

Third children must become proficient with the numerical operations of multiplication and division.

Finally teaching practices must observe that the above conditions are met and that these individual skills are adequately developed through the regular practice of students with materials that would enable them to integrate these skills leading to the use of the multiplication formula rather than making the formula the starting point of teaching volume in late primary school.

The latter could lead to rotely use of the volume formula and to its mechanical use through the end of primary and well into the secondary school handicapping children's understanding not only in mathematics but also in various science subjects were use of the formula becomes increasingly necessary.

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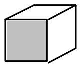
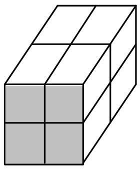
Question 1: Complete the equations:

3x3=--	3x3x4=--
4x3=--	4x3x4=--
5x4=--	3x4x5=--
4x4=--	4x4x4=--
15x2=--	3x2x8=--
6x3=--	6x5x2=--

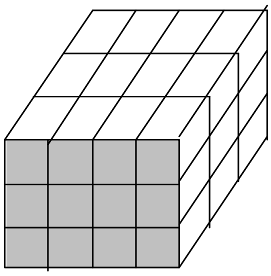
4x9=3x--	60÷5=--
3x20=6x--	48÷4=--
6x4=3x--	36÷4=--
16x4=--x8	24÷3=--
8x6=--x3	48÷6=--
12x5=--x6	363÷=--

<u>Question 2</u>: Block A is made by putting 8 small cubes like this  together.



A

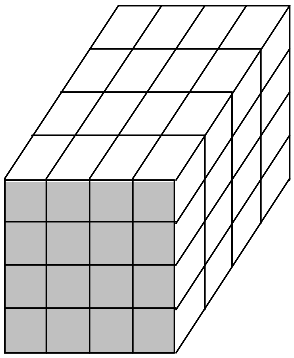
(a) How many cubes make block **B** (there are no gaps inside)?



B

Block **B** is made out of ________ cubes.

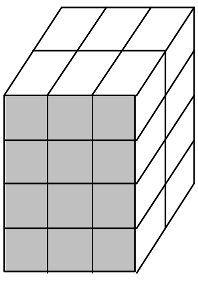
(b) How many cubes make block **C** (there are no gaps inside)?



C

Block **C** is made out of ___________cubes.

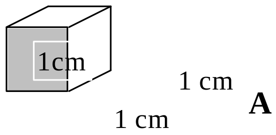
(c) How many cubes make block **D** (there are no gaps inside)?



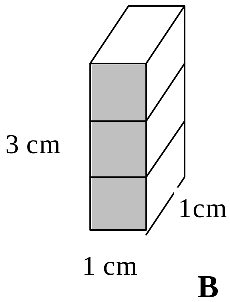
D

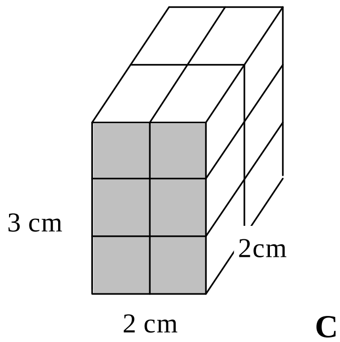
Block **D** is made out of ___________cubes.

The amount of room inside a block is called its volume.

The volume of block **A** measures 1 cubic centimetre: 

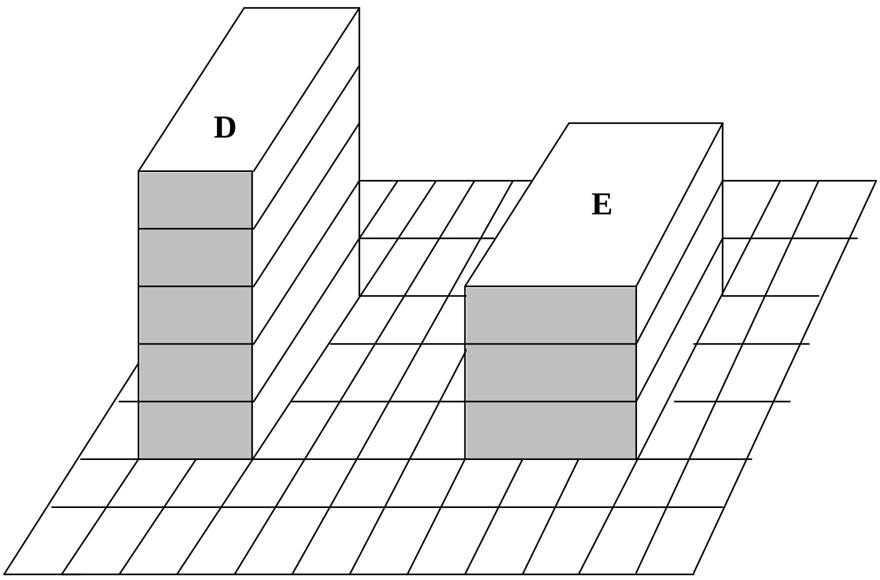
Question 3: Find the volume in cubic centimetres of each block **B** and **C**:



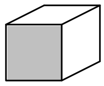


Volume of **B**= ____________ Volume of **C**=____________________

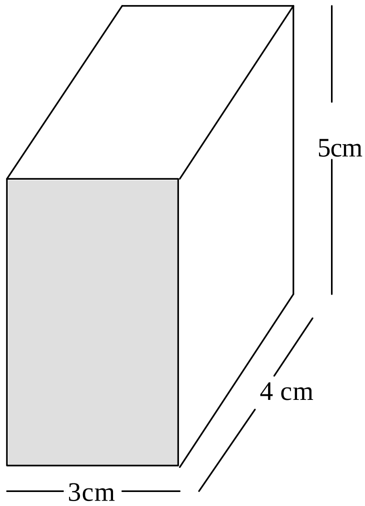
Question 4: Find the volume in cubic centimetres of each block **D** and **E**.



Volume of D=_______ Volume of E=_______

This is a cubic centimetre. 

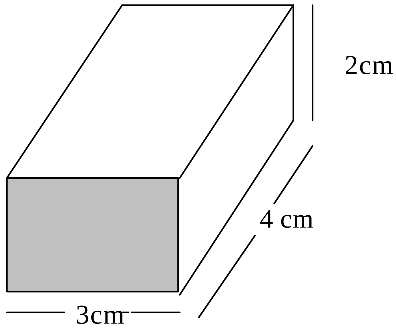
Question 5: This box is empty.



How many cubic centimetres can fit in the box?

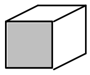
________cubic centimetres can fit in the box.

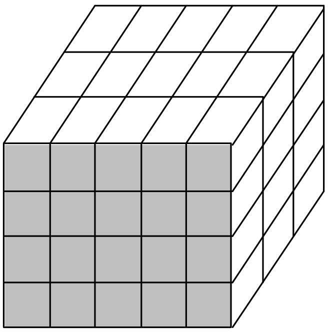
Question 6: This block is made of wood (there is no empty space inside it).



How many cubic centimetres do you need to build a block with exactly as much room as the block ?

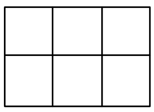
I need ______ cubic centimetres.

Question 7: This is a house made out of small bricks like this. 



We break the house and put all the bricks in this pile here:

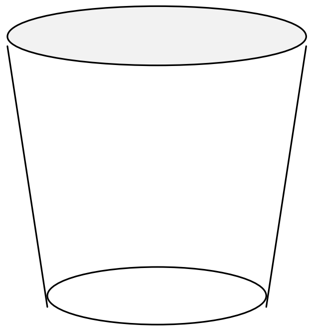
We are going to use all the bricks from the old house
to build a new house on this island here:



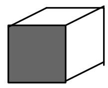
How many bricks high will the new house be from the ground?

The new house is going to be ________ bricks high from the ground.

<u>Question 8</u>: This glass is full of water.



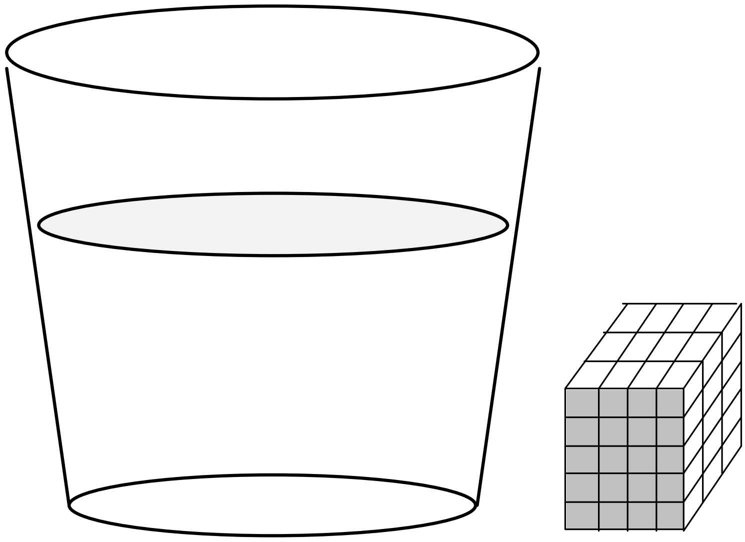
This cube is made of iron.



What will happen if I put the cube into the glass?

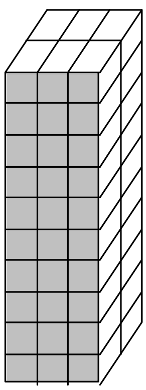
Explain your answer.

<u>Question 9</u>: This bowl is half full with water and block **G** is made out of unit iron cubes.



G

What will happen if we put block **G** in the water?
What will happen to the water?
Explain your answer.



H

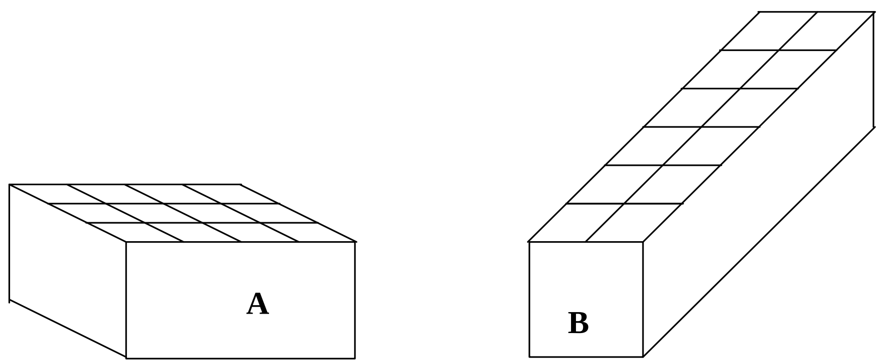
We break down the block **G** and use all the iron cubes to make this block **H**.

Does the block **H** take the same amount of room, more room or less room than block G? Explain your answer.

We are now going to put block **H** in the water. Will block **H** take equal amount of room, more or less amount of room than block G in the water? Explain your answer.

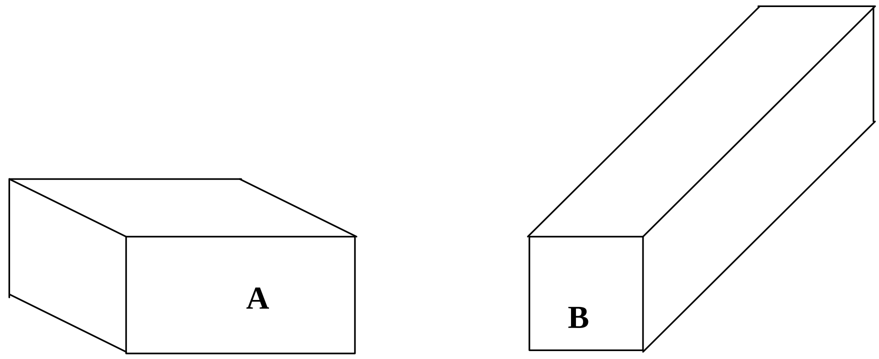
What will happen to the water, when we put block **H** in? Explain your answer.

Question 10: I have 24 cubes which will just fit into box **A**, leaving no spaces.



The same 24 cubes will just fit into box **B**, leaving no spaces.

Tick (ν) the statement which is true about the **volume of air-space**
in the two boxes when the cubes are taken out.



Box **A** has more air-space.----------------

Box **B** has more air-space.----------------

Box **A** and **B** have the same air-space.--------------

You cannot tell if one has more air-space or not.--------------

Explain your answer:--

