“Quaderni di Ricerca in Didattica”, n16, 2006.
G.R.I.M. (Department of Mathematics, University of Palermo, Italy)

Conditions of validation in a situation/problem: analysis of a case

Erika Kupková

ABSTRACT

In this work, we are designing a tool to cope with a following task: Find out, how pupils verify the correctness of their answer when solving a simple problem with specified conditions. First, we give the problem, then the analysis a priori and finally illustrate the utilization of this tool.

RESUMÉ

Le but de cet article est le développement d'un util didactique qui nous permettrait d‘examiner la queston suivante: Comment les élèves vériefient-ils si leur réponse est correcte, quand ils cherchons une solution à un problème simple - un problème avec des conditions définies d‘une manière exacte? Nous commençons avec une définition du problème étudié, nous présentons une analyse “a priori” de ce problème et finalement nous présentons l‘application de l‘util étudié.

INTRODUCTION

In the [EK99] we are documenting a situation, where pupils claim about two different answers to be both correct. However, in the given problem, only one answer was correct. To decide which one, it was enough to check whether the conditions given in the formulation of the problem are fulfilled. We were facing a problem : Why didn’t the pupils do this simple check, or if they did it, how could they still claim both answers to be good?

In this situation, we have chosen the methods of the Theory of Didactical Situations [GB98]. An illustration of using the methods of the Theory of Didactical Situations can be seen in [ST01] and [LF03].

THE METOLOGY USED

In our situation, we need to clarify an ambiguity experimentally found in other context. The theory of the situations and the tools of analysis connected to it allow in the phase of validation to clarify such situations.

Given a situation problem, through the analysis of a case one can analyze the different plans of the didactic situation: situation of action, of formulation and of validation. Most commonly the situation of learning also comes in evidence.

THE EXPERIMENTAL CONDITIONS

We have prepared "the test" (see below) and given it to pupils aged 12-13 years. The experimenter told the pupils about the aim of this experiment: "We are trying to find out, how pupils of your age do they reasoning. For us, the answer of every one of yours is very important. This time, there is no good answer or bad answer – just the answer, which allows us to see, how this or that pupil is thinking about the given problem. We are not going to give your work to your math teacher or anybody else. You do not need to sign your work with your real name – just write down the real class and then any pseudonym you like." After this introduction, the pupils were very cooperative. Each pupil has worked on his own solution.

FORMULATION OF THE PROBLEM

In a class of pupils aged 12-13 years, the following written text (we shall reference it as "the test") was given to each pupil:

At a school, pupils were solving the next problem:

Divide 24 postcards into two piles so there is by 6 postcards more on one pile as on the other.

When the mistress went over they results, she noticed that:
Majka's result is: on one pile 18 and on the second 6 postcards,
Palko's result is: on one pile 12 and on the second 6 postcards,
Andrej's result is: on one pile 15 and on the second 9 postcards.

Is the result of Majka correct? Why?
Is the result of Palko correct? Why?
Is the result of Andrej correct? Why?

FORMULATION OF THE HYPOTHESIS

Our hypothesis is: "To check if the given conditions are fulfilled, pupils use different strategies. In one, the pupil looks for a procedure which guarantees the fulfilling of the conditions. If the pupil believes the procedure is right, he doesn't feel the urge to check the result. In the second, the fulfilling of the conditions is guaranteed by checking the result. If the formulation of the problem supports it, one pupil can use both strategies and still the second strategy does not have any impact on the first one."

ANALYSIS APRIORI

Analysis of the requirements on the respondents

The next sentence from the test is the crucial one to determine the requirements: "Divide 24 postcards into two piles so there is by 6 postcards more on one pile as on the other." To solve this problem, knowledge on the level of 2.-3. grade of grammar school is sufficient. The formulation of the test is more demanding according to it's length and the requirement to answer the "Why?" questions. There is the demand to read with understanding and to formulate one's thought's on paper.

Analysis apriori of the didactical situations

First, we have to select the part of the problem, which we can afterwards use as the basic one. Depending on the problem, this can be a situation of action or a situation of reference. They have a common feature, they happen in the material milieu.

The objective situation is characterized by coming around in a material milieu.

Material milieu – the real material the subject dispose when solving the problem. In our case - the postcards. “Divide 24 postcards” means that in the situation we refer to, the postcards are real.

S5 – subject S5 – is a person who performs an elementary action in material milieu. An elementary action is an action everybody agrees to be uncomplicated. In our situation S5 is a person dividing 24 postcards into two piles. From this milieu he gets the feedback whether he fulfilled his task well – he can see whether the postcards are on two piles and neither of them is left somewhere else. He is solving the **problem Pb5** – “divide 24 postcards into two piles”. We suppose the children we are giving the test whom are able to imagine someone doing this activity.

The material milieu, subject S5 and problem Pb5 produce the first level of a didactical situation – the **objective situation** (situation objective).

The next level is the **situation of reference** (situation d’action). **Subject S4** is solving the **problem Pb4**: “Divide 24 postcards into two piles so there is by 6 postcards more on one pile as on the other.” Subject S4 is a person solving this problem using 24 postcards and profiting from the activity of subject S5.

The S4 and S5 subjects are implicit persons, they are not mentioned in the test.

The action of subject S4 includes more options. The most common are:

Procedure A1 : (On even number of postcards) The subject S4 divides the postcards into two equal piles, then takes x postcards away form one pile and adds them to the other. The number of the postcards taken away is x = 6 (incorrect) or 3. In both cases, two possibilities are distinguished. The first – S4 is looking for a feedback. The second – he doesn’t look for a feedback (that’s the control of his successfulness).

Procedure A2 : He makes the two piles at random and then corrects his solution using the feedback.

Procedure A3: First he places 6 postcards on the first pile and none on the second. Then he keeps on adding one card to both piles until the postcards are gone.

The pupil who is solving the test, doesn’t have any postcards on hand. But he can imagine he would solve the problem with real postcards. This image can help him in the abstract solution.

The pupils of young age use to help themselves depicting the real situation. A pupil draws rectangles or circles, which represent the postcards, crosses them out and draws them on another location. This pupil locates himself to the position of subject S4.

The next notation can be assigned to Procedure A1 and x = 6 :
24 : 2 = 12
12 – 6 = 6 12 + 6 = 18

And the next notation can be assigned to Procedure A1 and x = 3 :
24 : 2 = 12 6 : 2 = 3

12 – 3 = 9   12 + 3 = 15

To procedure A2 we can assign a notation, where couples of integers with sum 24 are chosen at random, and then are crossed out until the couple 15 a 9 is found.

To Procedure A3 the following notation can be assigned :

24 – 6 = 18
18 : 2 = 9
9 + 6 = 15

The third level is the **learning situation** (situation d'apprentisage). **Subject S3** is solving the **problem Pb3** : "Is the result of Majka correct? Why? Is the result of Palko correct? Why? Is the result of Andrej correct? Why?" The output of subject S3 is an answer on paper. The environment in which the S3 works are the informations he is getting on reading the test. While reading the test, S3 can imagine the situation of action, or he can position himself to the role of a teacher evaluating the solutions of his pupils. Or he can position himself successively to the roles of Majka, Palko and Andrej. This position has a lack of information – the pupil can't see the way Majka, Palko an Andrej got they results. The environment of S3 gives no feedback.

The activity of S3 is distinguished on three levels: **level of formulation, level of pragmatic proof and level of intellectual proof.**

In the written answer of S3 we will be looking for:

- the formulation of numeric result of the problem Pb4 (there is no demand for this in the test)
- the answer to the "Is the result of Majka correct? " question
- the answer to the " Why? " question.

In our case, the subject is on **the level of formulation**, if he either presents his result, possibly the calculation of the problem Pb4 or just gives the straight answer to the "Is the result of Majka correct? " question. His written reasoning is like "For I've got the same result".

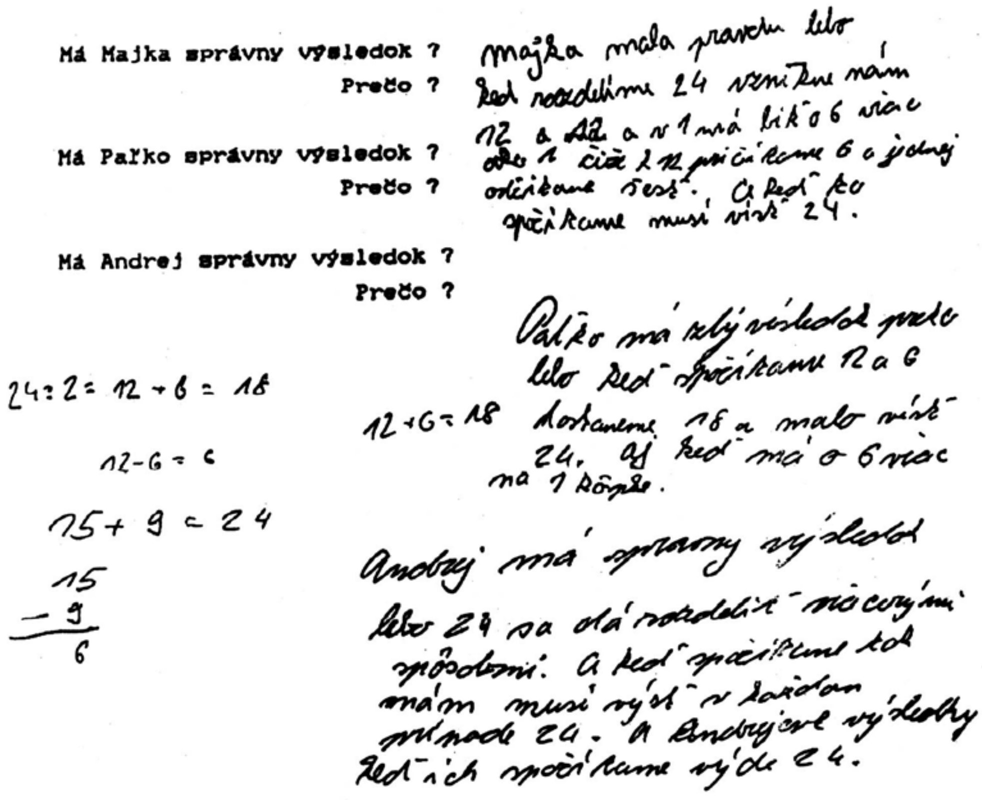
Our subject is on **the level of pragmatic proof** if he justifies the correctness or incorrectness of the answer by reasoning about fulfilling the conditions given in the Problem Pb4. His written reasoning is like "The result of Palko is incorrect because 6 plus 12 doesn't give 24" or "The result of Majka is incorrect because she has more by 12 and not by 6".

A pupil, who claims Majka has a good result and justifies it giving the A1 procedure with x = 6 and doesn't look for a feedback, can't be on the level of pragmatic proof. Even if for Andrej he justifies by fulfilling the conditions. Since in the case of Majka he was not looking for a pragmatic proof.

Our subject is on the **level of intellectual proof** if he presents the right solution of problem Pb4. Moreover he tries to find out where did Majka make a mistake when receiving the result of 18 and 6. Or he tries to find out the mistake of Palko when receiving the result of 12 and 6. The formulation of the test is not supporting the level of intellectual proof since the pragmatic proof is good enough to give the answers. In spite of this, there are pupils who even in this situation prefer the intellectual proof.

Using the analysis in a particular case

Let's take a solution of a pupil, which was presented in [CD6]. We are giving the original Slovak version and the corresponding translation.


Má Majka správny výsledok ?
Prečo ?
Má Palko správny výsledok ?
Prečo ?
Má Andrej správny výsledok ?
Prečo ?

Majka mala pravdu lebo keď rozdelíme 24 vznikne nám 12 a 12 a v 1 má byť o 6 viac tak k 1 čiže k 12 pričítame 6 a z jednej odčítame šesť. A keď to spočítame musí vyjsť 24.

Palko má zlý výsledok preto lebo keď spočítame 12 a 6 dostaneme 18 a malo vyjsť 24. aj keď má o 6 viac na 1 kôpke.

Andrej má správny výsledok lebo 24 sa dá rozdeliť viacerými spôsobmi. A keď spočítame tak nám musí vyjsť v každom prípade 24. A Andrejove výsledky keď ich spočítame vyjde 24.

24 : 2 = 12 + 6 = 18
12 – 6 = 6
12 + 6 = 18
15 + 9 = 24
15 – 9 = 6

According to the layout of this solution we can divide it into six parts. There are the text parts:

„Majka was right because when we divide 24 we get 12 and 12 and on 1 should be six more. So we add 6 to 12 and deduct 6 from one. And if we count up, we must get 24."

(T1)

„The result of Palko is bad because when we add 12 and 6 we get 18 and we should get 24. Even if he has by 6 more on one pile.

(T2)

„The result of Andrej is right because 24 can be divided by more ways. And if we count up we have to get 24 in every case. And if we count up the numbers of Andrej we get 24."

(T3)

Then there are the numeric calculations:

24 : 2 = 12 + 6 = 18
12 – 6 = 6

(N1)

12 + 6 = 18

(N2)

$$\begin{array}{r} 15 + 9 = 24 \\ 15 \\ -\ \ 9 \\ \hline 6 \end{array}$$

(N3)

Our hypothesis about the order the pupil was writing these parts is following: (N1), (T1), (N2), (T2), (N3), (T3). We would say with confidence that (N2) and (T2) are connected to Palko. The text part flows around the numerical and the numeric part is commented in the text part. Furthermore we can claim the parts (N3) and (T3) connected to Andrej. In the numeric part, the pupil is dealing with numbers 15 and 9, which in the test are present only for Andrej.

The part (N1) could be easily regarded as a part connected to Majka. However, by analyzing the whole written work of this pupil we came to the following conclusion: The part (N1) is the pupil's own solution of problem Pb4 and not a part of solving the answer about Majka. We were guided by the following facts: There is no other evidence about the pupil's own solution of problem Pb4. Part (N1) differs from (N2) and (N3) in giving the procedure and in the absence of conditions verification.

Our hypothesis is that the pupil first solved the problem himself and then went on by checking the results of Majka, Palko and Andrej. As the result of Majka was the same as his own, Majka get over the check "automatically". When argumenting about the solution of Majka, the pupil gave the description of the Procedure A1. Palko and Andrej had different results, so our pupil went to look for the error. He was looking for it by verifying whether the conditions given in the problem formulation are fulfilled.

When one tries to determine the level on which the S3 subject works, it seems to be the level of pragmatic proof. In every separated case the pupil argues by fulfilling or not fulfilling the conditions.

In part for Majka there are the words: "*on 1 should be six more*" , "*And if we count up, we must get 24.*"

In part for Palko: "*because when we add 12 and 6 we get 18 and we should get 24*".

In part for Andrej: "*And if we count up the numbers of Andrej we get 24.*"

But doing the analysis in detail, we get the following facts: In the case of Palko, the pupil is numerically verifying the condition of difference and this verification is part of argumentation: *"Even if he has by 6 more on one pile.*" In the case of Andrej, the pupil verifies the condition of difference in the numeric part and does not mention it in the text part. In case of Majka he does not verify the conditions. He only claims they must be satisfied. So the solution of this pupil is on the level of formulation. At the same time we have to admit that it is not a typical level of formulation solution, since taking just the solution for Palko, this is on the level of pragmatic proof.

There is one more thing we can note about the activity of subject S3. The formulation of the numeric result of problem Pb4 is explicitly not present. However the abstract inscription of the solution of Pb4 by procedure A1 is present. So the numerical result is implicitly present. We claim implicitly, since the result is nor highlighted either commented.

CONCLUSIONS

The didactical tool we have developed is a rather strong one. It helped us to recognize a pupil's solution supporting the hypothesis given in the introduction.

The following problems remain open for further research:

1. Let's call the strategy of a pupil following the process mentioned in our hypothesis as a "double strategy". How many pupils claiming about two different answers to be both correct do use this strategy?

2. Are there other strategies, different from the "double strategy", leading to the situation where a pupil claims about two different answers to be both correct?

LITERATURE

[HB03]Bereková H., Foldesiová L., Regecová M., Kremžárová L., Slávičková M., Trenčanský I., Vankúš.P, Zámožníková Z.: Slovník teórie didaktických situácií, II.časť, Zborník 5 Bratislavského seminára z teórie vyučovania matematiky, UK v Bratislave 2003, str. 113.- 122.
[GB98] Brousseau G.: Théorie des situations didactique, Grenoble, 1998
[LF03] Foldesiová, L.: Sequence analytical and vector geometry at teaching of solid geometry at secondary school, Quaderni di ricerca in didattica, N.13 – Palermo 2003, p. 33. – 42.
[MK98] Krállová, M.: Úlohy o pohybe, Zborník príspevkov na seminári z teórie vyučovania matematiky, UK v Bratislave 1998, str. 43.- 46.
[EK99] Kupková, E.: Matematický úsudok v reálnej situácii, Zborník Bratislavského seminára z teórie vyučovania matematiky, UK v Bratislave 1999, str. 86.-90.
[ST01] Spagnolo, F., Trenčanský, I.: Efficacite de l'enseignement du calcul vectoriel, Quaderni di ricerca in didattica, N.10 – Palermo 2001, p. 77. – 101.