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FROM THE CHALDEAN PEOPLES TO EINSTEIN BRIEF HISTORY OF TIME AND SPACE[1]

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Sommario: I concetti si spazio e di tempo, così essenziali nella costituzione della scienza moderna e contemporanea, vengono qui analizzati nel loro costituirsi, dapprima nella forma del mito, per poi gradualmente assumere i caratteri propri dei concetti scientifici. In particolare si analizza il processo attraverso cui spazio e tempo assumono caratteri di grandezze misurabili, prima separatamente per essere poi coordinati tra loro fino alla loro fusione concettuale nello *spazio-tempo* relativistico.
Emerge in modo significativo come la scienza possa progredire solo in un continuo intrecciarsi dei momenti formali, propriamente scientifici, con quelli pre-formali, intuitivi o simbolici, e con il procedere delle realizzazioni tecnologiche. Mito, scienza, filosofia e tecnica appaiono dunque come livelli necessariamente distinti ma non necessariamente indipendenti.

Summary: The concepts of space and time, so essential in the constitution of the modern and contemporary science, are analyzed here in their process of constitution, at first in the form of the myth, then gradually assuming the characters proper of the scientific concepts. Particularly the proceeding is analyzed through which space and time assume characters of measurable magnitudes, first separately, to be then coordinated one another until their conceptual fusion in the relativistic space-time.
It is shown in meaningful way that the science can progress only in a continuous to weave themselves of formal and properly scientific moments with those preformal (intuitive or symbolic) ones, and with the progress of the technological realizations. Myth, science, philosophy and technique appear therefore as necessarily separate but not necessarily independent levels.

1. *Introduction.*

It was 1905, exactly one century ago, when an unknown twenty-six year-old man which name was Albert Einstein, employee of the Brevets office of Berna, published the three famous articles[2] by which the Special Relativity theory was delineated.

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[1] Lecture given by Professor Renato Migliorato at March 9th 2005 within the Seminars organized by the Messina's Section of Mathesis on: "*The concepts of space and time in the evolution of the scientific thought*". The text of the lecture was elaborated in collaboration with Professor Giuseppe Gentile within a search on "*The evolution of the concept of space from Euclid to the modern age*" supported with local founds (P.R.A.) of the University of Messina. The final text had to be attributed to the authors in equal parties.

[2] dither papers of Einstein of 1905 are really four and precisely: [1] *On the motion of small particles suspended in liquids at rest required by the molecular-kinetic theory of heat*, Annalen der Physik, 17(1905), pp. 549-560.- [2] *On the electrodynamics of moving bodies*, Annalen der Physik, 17(1905), pp. 891-921.- [3] *Does the inertia of a body depend on its energy content?*, Annalen der Physik, 18(1905), pp. 639-41.- [4] *On a heuristic viewpoint concerning the production and transformation of light*, Annalen der Physik, 17(1905), pp. 132-148. The first one of these is however not connected to the theory of relativity.

Einstein, just because young and devoid of prejudices, had not uncertainties to set for the first time on the paper, in clear and explicit way, conclusions that were already mature and well could spring, for example, from the works and from the reflections of Henri Poincaré. He, however, as often it happens to the great masters of science, was convinced that the classical principles of the physics as the more consolidated bases of the mathematics, should not be abandoned, except in the case of absolute and proven impossibility to reconcile them with the empirical data, even by complex and artificial adjustments. In other terms Poincaré had clear awareness of the conventional character of concepts as those of space and of time; he had full knowledge of the difficulties met by the classical electromagnetic theory, possessed the technical and conceptual instruments for an approach able to radically change the bases of the physics. To make it, an action of courage needed however that can be effected only by who looks at the science with the surprised and perhaps a little naïve eyes of who is "*new of the trade*". It is always, after all, the same history that is cyclically repeated, also many times in the same century. The great Gauss had matured, for example, the awareness of the logical possibility of geometries different from the Euclidean one, but he neither had courage nor saw the utility of an explicit formulation of such geometries, so the worth was of two young mathematicians: János Bolyai (Hungarian) and Nikolai Lobachevsky (Ukrainian). Other example is that constituted by Cauchy and Galois, but we could to continue for a long time.
Having to introduce a cycle of seminars that, just in the centennial of the Einstein theory of relativity, wants to consider, also in didactic terms, the role of the fundamental physical and mathematical concepts in the culture of beginning millennium, we don't believe to be able to put aside from a panning, even brief, on the history of the concepts of space and time.
Why however we thought to depart (at least in the title) from the Chaldean peoples? First because such civilization sinks its origins to no less than four thousands years before Christ, and so it surely brings us to the most distant human thought. Nevertheless this element seems substantially to be frustrated by the fact that very few things we know of what and how the most ancient inhabitants of the Mesopotamia thought. Here then the second and perhaps main reason for our choice: the fact that this people had fame to cultivate the divination in extraordinary way. And there is no doubt that the art of divination has a lot to do with the concept of time. This for at least two reasons.
Firstly because to divine means to try to imagine the future, implying with this the first and fundamental constitutive element of the idea of time: the trisection past, present, future. There is no idea of time if not getting conscience of an expectation that can or cannot come true, in opposition to a memory that we can recall to liking, but not more to realize; from this, precisely, derives the idea of an unidirectional and irreversible temporal flow. Secondly the divination, intended as art of the prevision (and the modern science in this doesn't have anything different), can't to do other that to found it upon the search of regularities and cyclical repetitions. Therefore in first place those cosmic: the to taking turns of the day and night, of the lunar phases, of the seasons, etc… and, connected with these, all those cyclical phenomena that, as the climatic alternations and the vegetative cycles of the nature, condition our life on the earth.
Now, if all this can seem obvious, in reality possesses meanings and values on which we perhaps not reflected enough.
The fact is that we live in a civilization in which everything is articulated according to a precise and regular order, continually checked by technological tools, that somehow reassume in itself and almost materialize abstract and fleeing concepts as precisely those of space and time.
And above all the most fleeing between two, that of time, seems today to us so under control that do not allow us to see what conceptual abysses it hides.
To understand however the terms of the Nine hundred scientific revolution, means also to understand what deep epistemological breaking is come to determine in the meaning of words as *space*, *time* and, a little before, also *cause, effect, casualness, necessity,* etc…
Limiting us to the first two concepts, *space* and *time,* that constitute the fundamental object of this cycle, we will begin our brief history attending to what we will call the *conquest of the time*.

2. *The conquest of the time.*

Everything today is regulated according to precise temporal scanning. Every morning, to the awakening, my digital table's clock projects on the ceiling the bright image of the exact time, continually regulated by a radio signal coming from a plant situated in Germany. After stood up I turn on my computer and the exact time appears me once more on the right in low of the monitor. If the two hours don't coincide I immediately think that something doesn't work, because I exclude previously that the two hours, that one of my digital clock and that one of my computer, are referred to different things. I can be connected now by Internet with a site that gives the exact time and to make a comparison. Then the day continues being articulated by a great protagonist, my wristwatch that tells me when it is the time for my lesson. And once more if I don't find my students in the classroom I think that something special happened, but not certainly that my time is something different from that one of the students. And I can still turn on the television with the reasonable certainty that in few seconds the television news will begin. If then I want to communicate by my cellular telephone, then I compose a number, sure that thanks to the tuning on frequencies of order of MHz or GHz, (that is extremely small temporal scanning) operated by an intermediary manager, I can quietly converse with the desired person.

All of this makes us able to perceive the time as an objective and very precise something, that is measurable with more and more accurate approximation, that reach today the millionth of second. We can have some wrong sensations, to estimate that one hour only is spent when two of them are passed, but really the time is there, is objective and I have only to find the tools to correctly measure it!

But has it always been this way? Today it is difficult to imagine how the things were perceived by our progenitors, either those lived in Mesopotamia in 3000 B.C., or in the ancient Rome or also simply in Europe few centuries ago.

Yet a simple reflection would be enough to understand as the time is one of the most fleeing concepts that the human mind was able to conceive. Let us then suppose that we want to scan the time of an action, of an event, by means of a series of equal intervals. After all this is the first fundamental condition by which the time appears as a magnitude: that an interval of time can be compared with another. But what does it mean to say that two intervals of time are equal? Can we adapt the same experiences that we use for the lengths?

If I want to explain the meaning of the expression "these two lengths are equal" I have only to take a ruler that can be exactly superimposed to the first length and then to move it to verify if it is possible to superimpose it also to the second one. Certainly, to do this I have to assume axiomatically that the ruler is unchanged during the motion, thing *inter alia* corresponding to the perceptive experience. In each case any measurement presupposes the assumption of something as reference, to consider therefore, provisionally and in conventional way, as an absolute. But while I am moving my ruler, the two lengths keep on being there, I perceive both, and in case of uncertainty I can repeat the operation of measure.

To measure intervals of time, I can compare them only with other intervals of time, that is with the duration of something that happens exactly in contemporary. Therefore if I want to compare the duration of an event A, with the duration of a following event B, I need first to compare the duration of A with a specimen event C that happens contemporarily to A. But in this case when B happens neither A, nor C are there. Both have vanished, they belong to a past that we can evoke in the memory but we cannot make present any more. The irreversible annihilate itself of the time is what makes it perceivable to our conscience, because it allows us to separate past and future, but it is at the same time what makes it evanescent and elusive. Just for this the initial taking of conscience of the time and its irreversibility can't to occur but in the form of myth. And, in the Greek mythology, Cronus is really the god that devours his own children, that is the entity at the same time creator and devouring of every his creature.

But if every event is devoured and annulled by the same time we would like to measure, how is it possible to find a specimen event, an unit of measure, that can be compared now with an event of

today and afterward with an event that will happen tomorrow when the today there won't be any more? And here then the function of the circularity appears, the cyclical events that are repeated unchanged, perennial and unchangeable (or at least it is supposed).
This is the representation of the snake that bites the own tail, the myth of the eternal return. But it is also to the alternating of the day and night, to the repeating of lunar phases, to the regular succeeding of the seasons. The cyclicity of the events assumes then, with regard to the measurability of the time, the same role that the rigid movement has with regard the measurability of distances. But it is also the alternation of day and night, the repeating of the lunar phases, the regular sequence of the seasons. The cyclical event becomes our temporal ruler, under condition that the cycles can be considered all of equal duration. But this is not the biggest difficulty, because the conventionality of this assumption is not very different than the conventionality by which we suppose the length of the ruler invariable during the motion in the space. The fact is that while a ruler can be moved with continuity, and therefore it is not difficult to set one extremity of it to the beginning of the length to measure, the cyclical events that are spontaneously offered in nature follow some well precise scanning, and so we cannot for instance make to coincide the dawn of this day with the beginning of an event that will start at 11:25. Moreover a ruler can be broken for getting smaller objects usable they also as rulers, while a cyclical event breaking itself given place of course to smaller events, but generally not cyclical. So, for instance, the day, intended as complete cycle of 24 hours, can be divided in day and night, but these two entities are not in some way equivalent or comparable one another. Rather the perception itself that we have of it signals to us that their durations are different and varying with the seasons. If then we go to subsequently divide every of these two parts, the things become even more complex. For the diurnal part, for instance, we can mark on the ground the position of the shadow of a vertical cane in the two instants, that respectively follow and precede the dawn and the sunset. The spatial interval between these two positions can be then divided in equal parts (for instance twelve) that conventionally we can call hours. The difficulty is that in this way the duration of every hour will be sensitively different between night and day and between summer and winter. Nevertheless this was the system of measure used in the whole antiquity until the Roman age. The subdivision of a day in twenty-four hours derived in fact from the Mesopotamic civilization, the hours nevertheless were not equal, but twelve of them were assigned to the time of light, twelve to the night, with beginning of the new day (change of date) to every sunset. Obviously the diurnal hours were long at summer and short at winter, while the contrary happened for the night time hours, that among other things, failing the sun didn't have an evident visibility, and therefore a meaning, except for those peoples that were accustomed to the study of the stars. Here it appears absolutely clear the conventionality of the terms of comparison and therefore of the concept itself of measure. To say that the time was three after the dawn, meant that since the sun had risen, three twelfth of the time of light were spent, but nothing didn't point out in absolute terms. The same thing could be said for the measures of length, but in this case all the rules are comparable one another, and so at the end it is not difficult to refer to only one system. The matter is very more complex for the time, because, as we said, the cyclical events are already given and are not easily linkable one another. Then the solar year is not exactly constituted by an integer number of days, the alternation of the day and the night varies with the seasons, the lunar cyclicity itself have not a period exactly linkable with the other cyclical events.
The solution then can be to produce artificially some cyclical events which beginning can be put in correspondence with something other. This is the case of the hourglass and of the clocks. To measure, for instance, the time granted to some speakers, in order to don’t favour any of them in comparison to the others, the hourglass could be turned upside-down in the moment in which he had beginning his discourse. More unities of time could be gotten manually turning upside-down the hourglass in the exact moment in which the sand was finished.
The history of the clock and its improvements has then its starting. The water clock, in its simplest version, doesn't differ very much, as principle, from the hourglass. In this case however it is easier to further on divide the time, graduating the container in which the water, coming from a superior

container, slowly goes to collect it. In the third century B.C. this type of clock reached elevated levels of precision. To prevent that the flow of the liquid suffers variations, it was done so that in the superior container the level was maintained constant with the help of a device that allows new water to flow when the level had the tendency to go down. In such way the constancy of the pressure guaranteed a constant flow through a small hole[3]. In some cases, the water collected in the inferior container, rather than to be directly measured to sight, moved by its weight a device that operating an index, allowed to move it on a graduated quadrant.
The history of the mechanical clocks and their systems of regulation was longer and more complex until to the invention of the pendulum and the balance wheel. What however we want here to notice, is that, during the history, the time is not introduced as an unique and objective something, that was there and which measurement was only a technical problem of great or small precision. The fact is that there was not one only time but a multiplicity of times, conceptually different one another and therefore not commensurable. The time of the day of twelve hours between dawn and sunset was a well different something from the time of the hourglass or from the time of the water clock; other things yet were the annual cycle and the cosmic one measured on the motions of the stars. We stop here for the moment and we will resume the discourse when we will speak of the space-time coordination.

3. *The space between geometry and cosmology.*

As the time is not thinkable without the wait of the future in opposition with the memory of the past and without the perception of duration of the cyclical temporal scanning, equally we are not succeeded in conceiving the space only as container of the appreciable things and our bodily movements. Geometry is born therefore as art of the measurement. From this point of view, the real measurement of lengths appears, at least as principle, very little problem in comparison to that one of the time. The complications born however in relationship to two essential matters.

1. If the time seems to develop along a line of unidirectional flow (past-present), the space is perceived and experimented according to three dimensions, giving place to three classes of magnitudes (linear, of surface, of volume) so that magnitudes of the same class are comparable one another, magnitudes of different classes are never. Now if the comparison of rectilinear magnitudes, as we saw in the preceding section, doesn't introduce difficulty of principle, these begin to appear when one had to compare rectilinear magnitudes with curvilinear magnitudes or with magnitudes of surface and volume.
2. The problem still appears when we want to measure inaccessible magnitudes, and it becomes really arduous when these assume cosmic dimensions.

To deal all the connected problem linked to these two orders of matters, would mean to develop the whole history of the geometry and the cosmology, but this is away from our intentions. We will deal instead some central matters that will help us to make to emerge the most obscure and fleeing aspects of the notion of space and its connections with that one of the time.
As it is known, in passing from the Near East to classical Greece, the mathematical knowledge, already cultivated in the Mesopotamia and in Egypt, underwent a deep mutation, losing their character of "*art of the measurement and the calculation*", to gradually assume that one more idealized and abstract of the logical-deductive science.
The fundamental characters of the new science, invented by the Greek peoples, are those of the universality, the formal necessity of the propositions, the idealization of its objects. Which were the reasons for this sudden and very unlikely mutation of the thought, spontaneously happened only one time in the history of the humanity, and then absorbed and developed by other peoples, discovered

[3] We can legitimately thought that to the base of these technologies there was theoretical knowledge of hydrostatics of which the work of Archimedes constitutes the main testimony.

again and resumed in the Renaissance to give place to what today we call western civilization? Many different answers can be given, but for the moment we limit ourselves to gather the datum and to take note of it.
It needs to say on the other hand that, despite the high levels of abstraction and idealization, in the western ancient world we can't found a concept of geometric space that is separated by that one of the objects in it contained. The most accredited explanation is referred to the difficulty of the Greek thought to conceive the "*nothing*", the "*not to be*", that just for the fact of *not to be something*, could not be even thought. A conclusion, this, that is strongly linked to the verbal and logical character of the Greek thought, and therefore to the form itself of the language. The Indian thought appears instead less conditioned by such difficult and so it was able to give place, for instance, to the concepts of zero and empty space, that in posterior age was imported by the western civilization also.
It is still opportune to say however that also the classical Greek thought was not so compact and uniform as it can appear even for the dominant role that is attributed today to Plato and to Aristotle. Among the Pythagoric philosophers there were for instance different and conflicting opinions about the finiteness or not of the Cosmos (that is the entirely). Among the arguments maintaining the infinity, one was the following: if we think that the cosmos is limited, we imagine a man that is exactly at the limit of it; what does it happen if he tries to extend his arm beyond this limit? If there is anything, then nothing can impede him to extend the arm that therefore will be stay beyond the limit and this is not a limit anymore. If something prevents him instead from extending his arm, this something has to be beyond the limit that therefore is not a limit anymore[4].
Also the exclusion of the void was not so general. The atoms of Democritus, for instance, as those of Epicurus too, seem to move just in an empty space.
In every case the space, as nothing, can be said to be, not for itself, but only as filled or susceptible to be filled by something other, therefore Greek geometry doesn't concern directly the space but the form of things that fill it. And these forms are considered in abstract, that is independently from the objects which can be assigned. Rather, since any object of the world perceived by the senses can't possess the perfect forms of the geometry, these can exist only as ideas. And according to Plato, ideas are the aboriginal essences, the prototype of all the appreciable things. These are produced instead by a demiurge with image and similarity of the ideal, perfect, eternal and unchangeable ones[5].
According to Aristotle, instead, as we know, ideas don't exist *in sé*, but they are determinations of the single beings. Only the single beings (substances) exist *in sé*, and they are perceivable by the senses but they become intelligible only when we acknowledge the determinations of gender and species, when we understand the first cause for which *they are and can't not to be*.
What are then, in particular, the fundamental geometric entities? Surface is for Aristotle what limits a body, line what limits a surface, point what limits a line. They are, that is, the constitutive elements of the form of the bodies, which are constituted precisely of matter and form, but if a body cannot subsist without a form, this without a body can subsist in power. This means that given whatever form, it can be assumed by a body and being so been realized. So the form of the Moses of Michel-Ange, thought by his author, was in power because potentially feasible, and become then actual when it was impressed in the marble.
But only the finite forms can be realized, so according to Aristotle the infinity in act doesn't exist, but only the infinity in power exists. In other terms, to say that the numbers are infinite doesn't mean that they are all given, but only that however a number is given, then a greater one can always be given. And equally, to say that a straight line is infinite can mean, for us, only that a finished line can always be prolonged.

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[4] SIMPLICIUS, *Phys*, 467,26 with reference to Eudemus, *Phys*, Fr. 30. (See also M. TIMPANARO CARDINI: *Pitagorici. Testimonianze e frammenti*, Vol. II, pp. 348-349).
[5] PLATO: *Timaeus*.

4. *Dimension of the space and Platonic cosmology.*

Among the many problems of classical geometry, we choose to give a place of particular importance to the concept of dimension, just because it is one of the most remarkable matters in the context of the scientific revolution of the ‘900 and, as we will see, also one of the most original lines of the thought of Poincaré.
The distinction among lines, surfaces and solids is enough clear in the whole ancient geometry, and it is present in the definitions of point, line and surface that opens the editions reached to us of the *Elements* of Euclid[6]. While the point, defined as what doesn't have parts, appears deprived of dimension, the line defined as what has length but not width and thickness, is introduced as one-dimensional and the surface, having length and width but not thickness appears two-dimensional. A further specification is given by Aristotle that observes that the line is described by the movement of a point, the surface by the movement of a line, a solid by the movement of a surface. The impossibility of a great number of dimensions would be concretised in the fact that, according to Aristotle, the movement of a solid would not produce anything other.
The affirmation of the three-dimensionality of the world is present in Plato in a finer and indirect form, that requires a premise on the problem of the incommensurable magnitudes and on that one of the duplication of the cube.
It is known that the discovery of the incommensurable magnitudes, and particularly the incommensurability of the side and the diagonal of a square, was at the base of a deep crisis of the Pythagoric thought[7]. In fact, to base of all the existing, the Pythagoric philosophers set the presupposition of an indivisible element that constitutes all the things: the monad or unity. This perhaps appeared necessary because the opposite idea, that one of an unlimited divisibility, made somehow fleeing and elusive the base itself of the *being*. The proof that the side and the diagonal of a square aren‘t never multiple of a same unity, sets an insuperable difficulty for this conception. Nevertheless the progressive skidding of the Greek mathematical thought from the Arithmetic toward the Geometry, makes however acceptable the coexistence of the square and its diagonal, and in general of all the pairs of incommensurable magnitudes.
The matter of the diagonal of a square comes so to interlace with that purely geometric problem of the construction of multiple or sub-multiple magnitudes of a given magnitude. It is therefore a central matter inside our discourse because it constitutes a fundamental condition for the comparison among geometric magnitudes and therefore for the structure itself of the concept of space.
In the case of linear magnitudes, the problem of the comparison and measurement, as we saw, appears almost trivial and however it is deprived of those difficulties that are present in the quantification of the temporal magnitudes. In the case of plane or solid figures, instead, the problems are many and complex, until they appear insurmountable. This is for example the case of the squaring of the circle that becomes proverbial as synonymous of irresolvable problem. One of the essential passages for the comparison of figures is however the duplication of surfaces and volumes, as first step of the multiplication of a submultiple magnitude used as unit of measure.
So the duplication of a square doesn't introduce excessive difficulties and, as Plato himself clarifies in a passage of *Meno*, the double square (that is the square of double surface) of a given square is that one which side is equal to the diagonal of the first one. Now, although the diagonal is incommensurable with the side, and therefore problematic from the arithmetical (Pythagorean) point of view, it doesn't introduce geometric difficulty because is simply obtained connecting two points.
The situation is more different for the duplication of the cube. According to one of the legendary versions handed on[8], the problem would be been born by the request of the oracle that would have imposed to the inhabitants of Delos to double the altar of Apollo of cubic form. Not succeeding in

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6 The authenticity of the first seven definitions of the Elements of Euclide was contested by L. Russo in two papers respectively of 1992 and 1998 (see references). See also MIGLIORATO, GENTILE (2005) and MIGLIORATO: *La rivoluzione euclidea*…. Such definitions, nevertheless, draw their roots from a precedent pre-euclidean tradition.
7 For a more extensive exposure see R. MIGLIORATO, *Corso di epistemologia della matematica. Parte prima*.
8 Theon of Smyrna, *De Utilitate Matematicae*, p.2.

giving solution to the problem, the experts would addressed to Plato which would have blamed them affirming that the god certainly could not have intended to give the literal assignment of the construction of a double altar, but would have liked to attract their attention on the geometry, that they instead neglected.
A step forward, on this problem has been accomplished then by Hyppocrates of Chios, that brought back the problem to that to insert two proportional means between two given magnitudes, one double of the other. That can easily be understood, setting in modern language:

$$\frac{2}{x}=\frac{x}{y}=\frac{y}{1}.$$

It follows that $x^2 = 2y$, $x = y^2$ and so $y^3 = 2$, by which the solution easily follows and obviously it depends, always in our language, by an extraction of cubic root.
The problem made a step forward but it isn't resolved until we are able to build two magnitudes *x* and *y* proportional means between 1 and 2. Today moreover we know that this problem cannot be resolved by rule and compass just because it is a problem of third degree, while the rule and the compass allow to trace only lines of first and second order. Solutions of this problem are been found by methods that use mechanical tools to draw curves of order greater then two but, just for the use of mechanical tools, such methods would not be considered acceptable in the view of Plato.
At this point we have all the tools to understand the connection among the problems of duplication, the platonic cosmology and the dimension of the space. To such purpose we have to make some references still to *Timaeus*, the work in which Plato exposes, just by the words of this personage, his cosmological vision by the description of the creative action of a demiurge. After having said that the demiurge, in the creation of the cosmos, was not able whether to assume as model nothing but the only things that are perfect and incorruptible in itself, that is the ideas, he explain the reason why the constitutive elements of the appreciable world had to necessarily be four: earth, water, air, fire. Premised that it is necessary to have the earth and the fire, the first one as element of stability, the second one of movement, in order these two to stay together it needs that some proportional mean element acts as intermediary. Now, Timaeus says, if the world was plane only one proportional mean would be enough, but since it is necessary that the bodily things have thickness, two proportional means need: water and air. Which is the meaning of this passage? The reference to the problem of the duplication seems to us evident[9]. It is necessary to add to this point that the four elements are conceived by Plato as constituted by four of the five Platonic solids: the cube for the earth, the tetrahedron for the fire, the octahedron for the water, the icosahedron for the air. For each of these solids can be found in nature exemplary of different magnitudes that would stay the one another according to relationships that have beginning with the doubling and continue according to the laws of the Pythagoric musical harmony[10]. But how is it possible to double a solid? Here therefore the problem of the duplication of the cube. If we had to duplicate a square, it would be enough to insert one proportional mean between a given magnitude and a double one, having to duplicate the cube, it is necessary to insert, as we showed, two proportional means.

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[9] We can set the question in the following terms. The duplication of a square is equivalent to insert one proportional mean between two magnitudes, one double of the other. In fact by the equation $2/x = x/1$ immediately follows that $x = \sqrt{2}$, and this is precisely the measure of the side of a double square. Passing from two to three dimensions, the analogous problem (duplication of the cube) requires instead the insertion of two proportional means. It is possible that this discovery has been suggested to Hyppocrates of Chios just by the analogy with the case of the square.
[10] The relationship 2:1 is the one that, because it constitutes the interval of octave, is at the base of the musical Pythagoric sequence; this is set in turn in the Platonic cosmology to base of the cosmic order itself and of that "*harmony of the spheres*" of which we can find echo in the Dante's *Inferno*. It is only in this way that it is possible to understand that the problems of duplication (and therefore particularly of the duplication of the cube) assumes, also symbolically, values and meanings that allude to the deepest cosmological mysteries.
For a panoramic treatment of the relationships between arithmetic and harmony musical see for inst. A. SARRITZU: *Modelli matematici e armonia musicale. Una proposta per la didattica*, Proceedings of the Congress "Quali prospettive per la Matematica e la sua didattica", Piazza Armerina (EN) Italy, Sept. 16-19, 2004. (On line on the site of G.R.I.M. – University of Palermo, address: http://math.unipa.it/~grim/convreg1_gruppiprogramma_04.htm).

5. *From the geometry of the bodies to the absolute space.*

It seems therefore that Greek ancient thought didn't succeed in conceiving the space as a something in itself, a something that subsists independently from the things that can stay or not stay there inside. This difficulty doesn't seem instead to subsist anymore in the modern and contemporary age. But when and how the perceptive and cognitive change of the notion would be verified? The matter is not easy and interpretative disputes has given place.

An interpretation that offers a sure force of attraction is the one that by Panofsky in a famous essay on the meaning of the perspective [11]. He sustains that unlike the perspective used in the Hellenistic paintings, that one used in the Renaissance would presuppose a notion of space as entity in itself meaningful. In synthesis his reasoning concerns on the following points.

1. The perspective, how it comes to be delineate in the essay on the *Optics* of Euclid, it would certainly furnish a rigorous representation of the phenomenon of the vision, but this is possible only because the optic cone that has its vertex in the eye, is not supposed cut by a plan or picture, as it is instead done in the Renaissance's perspective. Among the presuppositions of the *Optics* of Euclid, there is that *the greatness under which the things are sights is given by the angle formed by the radiuses that include it*[12]. A correct representation of what the eye sees, therefore, could be had only (it is the conclusion of Panofsky) on a spherical surface with centre in the eye. If instead the cone of the visual rays is projected on a plan, we have inevitably that to equal visual angles don't correspond anymore equal linear magnitudes, and these will become as greater as they are much distant from the optic axle. We have, that is, that well known distortion that is manifested as in a more visible way as greater is the picture and is particularly evident in the photos made by a wide-angle lens.
2. The ancients, according to Panofsky, corrected this aberration by re-establishing in the picture the proportions of every figure[13]. In other words the painters should used the perspective construction in order to find the mutual positions and apparent dimensions of many figures, but after they should painted every figure maintaining the correct dimension of everyone of its. This was admissible since the space was conceived as having the only consistency coming from the mutual position among the objects.
3. On the contrary, Renaissance's artists accepted perspective aberrations if only don't modify nothing in the geometric construction. In other words they are inclined to pay the reckoning of figures that, mostly in the limits of the picture, have deformations and perceptible disproportions, in order to don't change what appears as a geometric "*truth*", understood by the reason even if not at all conformable to the senses. So, in the name of what one pays the reckoning of such a visible deformation? The answer of Panofsky is: in the name of an abstract geometric coherence. It is just in this abstract geometric coherence that can be summarized the essence of a new conceptual existence: the *geometric space*.

This statement was variously criticized and contested mostly on the ground of geometric considerations[14]. The fact that the picture cuts the cone of visual rays does not modify the same rays. In a perspective construction, an eye put in the viewpoint will continue to see things exactly as the picture does not is present, and so it cannot perceive the perspective aberrations. The arguments of Panofsky should have not any geometric fundament.

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[11] E. PANOFSKY, *La prospettiva come forma simbolica e altri scritti*, Milano 1961. Engl. ed.: *Perspective as symbolic form*, New York, 1997.
[12] EUCLID, *Optics*, Post. 5, 6, 7.
[13] The remarks of Panofsky refer mostly to mural pictures in Pompei and Ercolano also because meaningful examples of pictures contemporary to Euclid are not arrived to us.
[14] See for instance D. GIOSEFFI: *Perspectiva Artificialis: Per la storia della prospettiva. Spigolature e appunti*, Univ. Di Trieste, 1957.

Nevertheless these critics don't appear suitable if one thinks that perspective construction is only a fiction, a conventional way to represent spatial objects on a plane[15]. Really the pictorial representations never are looked by only one eye fixed in the viewpoint. More interesting it seems the profound meaning of the paper of Panofsky, meaning that can be more easily understood putting the attention to the citations to Cassirer contained in the text and already announced in the title (*Perspective as symbolic form*). By *symbolic forms* Cassirer means these "*forms*" of the intellect that allow to give a meaning to sensorial data and that, according to Kant, was *a priori* and therefore not modifiable. On the contrary, following the Neokantian philosopher, such forms are intellective structures that are constructed and evolve by a cultural way[16].

So which is the fundamental thesis of Panofsky? It is that the concept of space, as an entity *in sé*, should constituted since the Renaissance and it should be just a symbolic form in the sense defined by Cassirer. This point appears fundamental in order to understand the cultural role and the limits of that conception of absolute space on which is founded the classical Physics since Newton until the nineteen century.

Then we try to summarize on the ground of all we said until now. In the Greek antiquity it seems do not exists a concept of space, as an entity apart, even if there are many controversies about the finiteness or infinity of the "*Cosmos*", and about all it is existent. Following Plato (and after Aristotle too) the cosmos is a finite and *spherical* entity just because only the sphere is perfect[17], where the various parts constitute an harmonic unity and are linked following ratios repeating those ones of Pythagoras' musical scale. The motions of asters too must be necessarily circular since only the circles are *in sé* perfect lines.

In the Aristotelian cosmology (followed by the Ptolemaic one), although many meanings are overturned and the existence of geometric figures is only potential, it is not changed the substantial lack of an idea of space *in sé*. Only the individual substances exist, of which the space is a sort of premise, since every body occupies space. But without bodies only nothing remains and nothing does not exist.

Since the Renaissance it seems to became the idea of geometric space as a structured and logically coherent entity, that is as an absolute rationality to which perhaps the divine creating activity itself cannot evade[18]. Still on the threshold of eighteenth century, according to Gerolamo Saccheri, the Euclid's fifth postulate seems proved not because its negation leads at a logic contradiction, but since it "*disgusts*" the nature of the straight-line.

At last the "*absolute space*" reaches its approval in physical-mathematical terms with the Newtonian physics and further on it improves itself in the theory of *ether*, pre-supposing the classical electromagnetic theory.

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[15] In the plane representation of a three-dimensional space it is inevitable that a part of information is lost. The binocular vision consents, in normal conditions, to recover a part of the lost information, at least for that part of space no too far from us. In pictorial representation not only these information are not (geometrically) recoverable but also the binocular vision is so that the disposition on a unique plane is clearly perceived in such a way it reveals the "false". What furnish then, at the same time, the sense of three-dimensionality? The fact is that the perspective of space is perceived not by the eye but by the mind by means of a cultural conditioning that allow to recognize the represented objects. By means of such recognition, the mind can choice among many possible interpretations, so that it can reconstruct the missing information. It is not a chance that in particular situations in which objects are either unknown or with ambiguous forms, the perspective perception can appear strongly modified as, for instance, the false or ambiguous perspectives of Escher.

[16] E. CASSIRER, *Filosofia delle forme simboliche: 3. Fenomenologia della conoscenza*, Firenze, 1966.

[17] See PLATO, *Timaeus*.

[18] In XXVI canto of Inferno Guido da Montefeltro is inexorably forced to damnation by a strong logical principle: "*ch'assolver non si può chi non si pente, / né pentere e volere insieme puossi / per la contradizion che nol consente*" (DANTE, Inf. XXVI, 118-120). In XIII canto of Paradiso, San Thomas d'Aquino explains that when Salomon "…*chiese senno / acciò che re sufficiente fosse*", he didn't ask "…*se nocesse / con contingente mai necesse fenno*" or "*o se del mezzo cerchio far si puote / triangol sì ch'un retto non avesse*." (DANTE, Par. XIII, 95-102). In both cases it seems to be posed the question on a supposed (metaphysical) nature of logical and geometric truths.

6. *Toward absolute time and space.*

We saw that the possibility to measure intervals of time does not imply the existence neither of an absolute time nor of an absolute space. Nay about time the different system of measure at the beginning have not even the same meaning since the concept of period was not univocally determined. One hour, as one twelfth of the solar day, did not measure the period with the same meaning of hourglass and clocks. But also the subdivision of the year in months and days did have univocal meanings. The month, as civil subdivision, was a conventional instrument of measure that told collective temporal scanning, as festivities, celebrations, periods of political offices, and so on. Until Gregorian reform of the calendar, some differences between civil and solar year were present and they, since were accumulated in the long run, caused large shifting in seasons and therefore in natural cycles. On the other hand temporal scanning during the year was born in primitive agricultural civilizations, not by counting the days, but depending on work in the fields and then on vegetative cycles, and so it was more important to establish when the sun reached its minimum and was beginning increase (winter solstice). To make possible such calculation, since the most archaic civilizations, it was just the observation and the study of apparent motion of asters.
To this not univocity of time corresponded a not homogeneity of space, or better, of cosmos. According to Aristotelian convincement, followed by Christian theology, while sub-lunar world is constituted by corruptible substance and subject to worldly natural laws, the external part of the cosmos, beginning from the moon, should be constituted by perfect substance, not corruptible, not subject to worldly laws, following the circular (and so perfect), eternal and invariable motions.
The conquest of an univocal meaning in measuring the time was realized in centuries, slowly and step by step, on one hand by the reforms of the calendar (we remember that one of Julius Caesar and the Gregorian one), on the other by the progressive improving of clocks and by using the daily cycle of 24 hours. But to do an univocal sense to the measure of time does not means the conquest of an absolute time. First because the adoption of various reforms is not uniform and contemporary for all peoples, secondly because the diffusion of clocks takes place slowly and, until some centuries ago, few people have one. Besides in the country-side the time and the civil date is few useful: the time continued to be scanned by natural cycles, following the necessities of the work in the fields. Anyway clocks must be adjusted and this could happens only on astronomical ground, place by place, by the determination, for instance, of the instant in which the sun rises at the horizon in a certain day of the year. But how to coordinate the clocks situated in different places? About this we will speak later.
At the end of previous section we outlined the progress that since the Renaissance and until the *modern scientific revolution*, gave rise to the notion of absolute space. As we saw there are many passages and they involve primarily the conception of geometric space as an unitary and *in sé* coherent entity. The geometric space itself assumes a more resolute description in seventeenth century by the introduction of Cartesian analytic method: now any point of space not only is "*something*" rather than "*nothing*", but also, once it is fixed a system of reference, it has a name constituted by a triple a numbers, its coordinates. Nevertheless we remain in the geometric space, an immobile space without movement. The Newtonian gravitational theory, as we said, constitutes the decisive step toward the absolutization of cosmic space and this, in its turn, is prepared by the Copernican revolution encouraged by Galileo.
Once more it needs to do a brief remark in order to avoid commonplaces widely diffused in scientific environments and that are still present as residuals of a Positivistic conception of science. First there is a reason (and on this there is large concordance) why Copernicus, inventor of heliocentric system adopted by Galileo, is not considered the author of modern scientific revolution, that is attributed to Galileo. The fact is that after Copernicus his system, even if was appreciated mostly by painters of nautical papers, it did not produce (and fundamentally did want to produce) a change of general conception of cosmos and of science itself.
In order to understand it can be necessary a little digression. What means to say that a thing moves while another one stands still. The classical example of two trains in the station (if I stand still on a

train in a station, I have difficulty to understand which train is moving between mine and that one of the near tracks), or the more classical one of ships mentioned by Galileo, well illustrates the concept of relative motion. Now, in the case of celestial motions, all one can observe it is the progressive change of reciprocal positions, but no visual observation (with or without optical instruments) can allow to assign to one of observed bodies the privileged condition of *centre* of motion or of immobile body. It needs, as in case of systems of measure, decide to fix a reference. If we decide to fix our system of reference on the Earth, we obtain a geocentric cosmological system like the Ptolemaic one. If we fix it on the Sun we obtain a heliocentric system like the Copernican one. This principles of relativity was, as we can reasonably think, already present in Hellenistic cosmology of third century B.C. and constituted perhaps the base of heliocentric theory of Aristarchus of Samos mentioned, *inter alia*, by Archimedes that appears do not refuse it[19]. Nevertheless this relativistic conception of celestial motions will be surely rejected with the decadence of Alexandrinian school. The cosmological system set up by Ptolomeus in the second century A.D. did not make only an arbitrary choice, but it declared the centrality of the Earth in the cosmos as a metaphysical (and after theological) truth, in coherence whit the already mentioned Aristotelian idea of cosmos. The heliocentric theory of Copernicus does not have the aim to modify this cosmological view, in which sky and earth have different roles, nor it renounces to the assumption of the circular motion of celestial bodies and of eternity of these motions. What appreciated in such model is the greater descriptive facility, in mathematical terms, when the system of reference is fixed in the Sun instead of in the Earth.
It needs to say that also in the long process seeing Galileo in contrast to the Church and the Holy Office, Galileo should avoid the condemnation by accepting the hint of Cardinal Bellarmino, incline to bear the heliocentric thesis if this was declared as a simple *mathematical description*, without the pretence to remove the Earth from the centre of the cosmos. But it was just the cosmic centrality of the Earth, and not to fix an arbitrary system of coordinates, that appeared to Galileo as an obstacle to the development of a new science.
The principle of inertia itself, posing the bases to the modern Mechanics, does not born directly from the observations of phenomenal data, as a simplistic view tends often to strengthen. No one experiment made by usual means can suggest nor rigorously confirm the idea that a body, abandoned to itself moves indefinitely with uniform velocity. The experience suggests exactly the opposite, that is every moved body (clearly on the Earth) sooner or later stops. However even if we don't see the stop we cannot make an infinite experiment. The inertia appears as essential if one like remove the limit between Sky and Earth, in order to affirm that Universe entire is subject to same laws and that these are eternal, unchangeable and always valid. In fact one cannot explain the continue motions of celestial bodies, and more Galileo, without the principle of inertia, should had some difficulties to face the strongest critics against him: if Earth moves with a speed for us dizzy, why we don't realize it? And mainly why an object, left in the air, falls on the vertical and it does not remain behind in respect to the Earth?
Therefore in Galileo it is present a metaphysical position that substitutes another metaphysical position. The statement that "*all is governed by a natural law, always and everywhere valid*" is a necessary presupposition to found a science greatly prolific and producing results. We can consider it as a simple conventional presupposition, useful to produce solutions of problems, but all this fact does not means that to its bases there is a metaphysics in contrast to another previous metaphysics less prolific and less productive. We would like to remark that the "*natural law*" implicitly postulated by Galileo has all characteristics of the Parmenidean *Being*: it is unitary, indivisible, unchangeable and

[19] Notoriously the thesis is credited that heliocentrism of Aristarchus should be rejected by his contemporaries since judged "*impious*". But it needs to say that such thesis comes only from a correction made by Harold Cherniss to a passage of the unique known text of "*De facie quae in orbe lunae apparet*" by Plutarchus. Cherniss in fact, on the ground perhaps of philological emendations dated back to the XVII century (that is in the period just following the condemnation of Galileo), thinks that in respect to the original text one must change a subject and a predicate (Aristarchus - Cleantes). Without this emendations the text, in the version handed down, should assume a more different meaning and the thesis of a condemnation of impiety against Aristarchus should be without any base (See L. RUSSO, S.M. MEDAGLIA).

eternal. From this point of view we can affirm that in a certain sense the modern science appears as a updated reformulation of the Parmenidean conception of *Being*.

7. *The space-time coordinating.*

By the idea of an unlimited universe, governed by always valid, eternal and unchangeable laws, an unitary conception of space and time is recomposed. But if the unity is reached on a conceptual level, many problems remain open when from the general and metaphysical statements one needs to go toward the more specific field of scientific elaborations. In fact we can have an absolute idea of space and time, but if we must confront with the phenomena, then these ideas must give protocols of measures in terms of observation. The first step is that one to be able to make space-time measure on the Earth's surface. At first sight all seems easy. But if I wish to measure the time, after a subdivision of the day un 24 hours, I have only to set the watch in a way that it finish a complete cycle between two successive risings of a determined star at the same point of the horizon. Apart more strictly technical questions, the problem appears resolved in that concerns the measure of the period of the day and its subdivision. The problem is that if I wish a measure of absolute and objective time, the period is not enough; I must to fix an origin too, a zero of the scale of time, valid for all people. Now the instant in which any aster (the Sun or a star) appears on the horizon, depends on the place and, more precisely, on the meridian in which I stay; therefore, by measuring time in this way, the zero-instant will be different if I stay in Rome, New York or Sydney. All could be on right if it can be possible to find a system of corresponding in order to convert exactly the time here noticed with the time there noticed. Today we are accustomed to operations of this kind when we know the longitude between two points of which we wish calculate the corresponding time. The problem is so reduced to that one of determine the longitude.
Now we well know that the problem to calculate the latitude was solved since the antiquity by measuring the apparent height of an aster on the horizon. That one of the longitude could be solved looking at the position of the sun exactly at midday. But which midday? In order to establish it we need to know the time of a prefixed meridian and adopting it as zero-meridian. The procedure could be the following: to start from Paris with a clock synchronized on the local time, to make a complete round of the planet and, point by point, to observe in which instant (according to the Paris' time) the sun appears on the horizon. But here some complications born: how one can be certain that the clock, transported along the path, continues to scan the time in a constant way? The assumption that phenomena happen by same laws in every point of space is indisputably necessary, but it is not sufficient. Even by admitting that in two points of earth's surface two equal clocks scan time by the same rhythm, nobody can assure us that the same is true for a still clock, or better if it moves *solidal to* the earth's surface, while the other one moves *in respect to* it.
This is a problem that began more urgent with the beginning of transoceanic navigations since the XVI century and it will reaches a meaningful answer only during the XVII century by the studies on electricity and the consequent invention of the telegraph. In fact the possibility to send electric signals by cave with velocity comparable with that one of light allowed an "*almost perfect*" timing of clocks and the consequent calculation of longitudes in every point of the planet.
The problem we have exposed seem to be merely technical, that is concerning the technological media for measuring physical magnitudes already defined and the improving of such instruments in order to have more and more precise measures. This impression is strengthen by the fact that the efforts made to give an answer to these problems were not directed to the mere scientific knowledge, but were mostly linked to practical demands with economical interests at great level. At the beginning it was navigation. So, by the invention of the steam-engine and the extension of railway's networks, the problem to coordinate the times among different stations, the many trains passing on same tracks and the exigency to establish connections, turned to intransgressible and primary needs, connected to great interests. On the other hand, without these interests it should be difficult to have

financial resources and necessary political will at its own disposal. In fact it was necessary not only to invest on researches, but also surmount political, cultural, nationalistic obstacles to define standard references and systems of measure everywhere valid.
The impression that one can receive from all problems we dealt is that they are concerning of practical, technological or simply conventional problems (as international units of measure or the choice of zero-meridian). The magnitudes we are measuring (space and time) should remain anyway entities *in sé* previously defined.
But this is not true. In fact the risk is to consider purely metaphysical entities, that is lacking in those characters of objectivity necessary in the science. That means that concepts of space and time can assume a character of absolute magnitudes, in scientific and not purely metaphysical sense, only if one is able to define for them an univocal system of measuring. This is an enormously relevant point to ponder beyond the subject of this paper, just because it reveals that the mental processes to increase the knowledge never can be compressed inside the pure schemes in which they are presented on one hand by science, on the other by philosophy, finally by practical and technical activities. The inextricable interlacing of these three dimensions appear here, we believe, enough evident; even if this cannot and don't means to have conceptual confusion of three levels that, at least on a theoretical level, is suitable to clearly distinguish.
Let come back to space, time and their reciprocal coordinating. The possibility to transmit electric (and today electromagnetic too) signals that can synchronize (at least on principle) all clocks on planet, permitted to trace an univocal space-time map. This means that it is possible, at the same time, to fix the coordinates in every point (clearly on the earth's surface) in respect to a system of reference constituted by equator (zero-parallel) and by Greenwich's meridian (zero-meridian) and to establish for every point a precise correspondence between local time and zero-meridian time.
Summarizing, it happened that, at the beginning, space was only a place occupied by bodies and a positional relation among themselves. It becomes, first by the geometric perspective and so by system of coordinates, a relational system having a logic coherence; an abstract geometric entity to which the physical world cannot but be subjected; and so domain of a inevitable logical necessity.
Then it becomes *cosmic space* when, since Galileo, it is declared uniformity and homogeneity in respect to eternal and unchangeable laws that summarize and translate in a system of equations the unity of the *Being* that refers to the original Parmenidean conception. But this concept of unitary, homogenous and unchangeable space, together with the time to which must be necessarily coordinate, remains only a metaphysical statement until it produces controllable effects in the phenomenical world in which we act, that daily confront us and to which we entrust the meanings of our existence.
Then on the one hand there is the place of creative activity and pure rationality, of metaphysics and philosophical thought that produces images and representations, meanings and values of the world; on the other there is the practical activity, in the world of phenomena and daily necessities, where the representations and the values collide with what we call "*reality*". Among these we can think science as the only one instrument able to link the creative moment of philosophical thought to the activity and concrete experience of life. The instrument by which the intellectual production of concepts and images of the world, exceeding the pure metaphysical enunciation, becomes something that assumes meaning in the concrete work. So, a concept lost its metaphysical character and assumes scientific meaning when criteria and protocols are established in order to compare it with data of phenomenical experience, having the possibility to forecast new phenomena and to produce technologies.
It is what we saw with space-time. But till a certain point.

8. *Space and time: absolute or relative?*

We are at the last section of our discussion. Why have we concluded the previous section by affirming "till a certain point"? Let's we reflect on it happened when it was established an unitary space-time connection on planetary scale. From a substantial viewpoint, all this is equivalent to fix a criterion of "simultaneity" between two events happening in different places. Nevertheless this criterion is limited and can be attacked under many points of view. The principal one comes from the fact that the transmission of electric impulse (but the same thing holds if one uses electromagnetic impulses) can be considered simultaneous for all phenomena usually involving our daily life. But an electric impulse, as well as light itself, in fact travels by a non zero velocity, and so if we wish to examine electromagnetic phenomena, so involving velocities comparable with our signal of synchronization, the concept of simultaneity is irreparably compromised. What is the meaning of time until we cannot be able to establish a criterion of "*simultaneity*", valid also for the new class of phenomena? How can we to preserve it to that metaphysical character that should prevent an effective confront with data of phenomenical world?

The years between XIX and XX century see, for different reasons, on one hand the crisis of fundaments of mathematics, on the other an increasing difficulty to explain, by models of classical physics, a series of phenomena concerning light and, more in general, the propagation of electromagnetic fields[20].

For what concerns the crisis of fundaments of mathematics, here we can only remember the birth of non-Euclidean geometries and the problems posed by the developments of infinitesimal calculus and of set theory, postponing the discussion of these themes to specific moments.

But we wish to remember that already in this historic period some positions are matured preluding to a profound reflection of fundaments not only in mathematics, but also in physics. In this regard it seems particularly interesting the position assumed just in those years by the mathematician and physicist (but also mining engineering) Henri Poincaré. He formulated an original conventionalistic position of which we make, of course, only some sketches, referring for more to his essays that was published respectively in 1882 and 1905[21]. In the second one, in particular, he gives an explication of concepts of continuity and space by processes of abstraction coming from experience and concrete operations of measure. Among the convictions he expresses in concluding the essay, two seem particularly meaningful. The first one, relative to the notion of space, tends to substitute the Kantian idea of "*a priori forms*" by that one of "*intellective structures*" which origin could be searched partly in sensorial experiences[22], partly in cultural stratifications[23]. From this point of view we can

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[20] This topic was treated in a successive seminar, within the activities of Mathesis, by Prof. Liliana Restuccia.

[21] "La science e l'Hypothese" and "La valeur de la science" respectively, both cited in bibliography.

[22] He expresses also the convincement that if the human sensorial apparatus should be different, perhaps we will have a perception of space with a number of dimension different from three.

[23] The cultural component in the conception of space show itself mostly, in terms discussed, by the forms of Euclidean geometry. The character of conventionality of these aspects reveals itself mostly by the judgement of full legitimacy of non-Euclidean geometry, but not only. For instance Poincaré, trying to explain the concept of the one dimensional continuous, distinguishes first the physical continuous from the mathematical continuous. The first one, strictly linked to the experience, born from the material operation to confront homogenous magnitudes one another. This operation clearly has some limits since, by physical characteristic themselves of sensorial apparatus and/or instrument of measure, there is a minimum value (let d be) for the possibility to evaluate the difference between two magnitudes under which it is impossible to distinguish this difference. So, for instance, if A is smaller than B and the difference $d = B - A$ is just the minimum valuable difference, if there is a third magnitude C between A and B, it will be impossible to distinguish if from the first two. An observer would like to compare C and A will conclude they are equal; if then he will compare C and B he will conclude that these too are equal but, comparing directly A and B, he will observe a difference. For the physical continuous, which concept comes from operations of measure and confront, the transitive property does not hold. This datum, constituting a strong obstacle to the rational domain of phenomenical world, is exceeded by the passage from the empiric notion of physical continuous to that one of abstract mathematical continuous, in which the transitivity is conventionally postulated. This partly clarifies in which sense the gnosiological conception of Poincaré assumes together aspects coming from empiricism and conventionalism.

say that the thought of Poincaré on spatial notions anticipates both Neokantian positions of Cassirer and psychological theory of Jean Piagét.

The second convincement expressed by Poincaré, in the same essay, is relative to the criteria of validity of a scientific theory. Here also every resort to not only metaphysical concepts of truth and reality, but also (once more) every idea *a priori* is refused. The *maintaining* position towards classical physics is motivated by affirming that a scientific theory should be defended and conserved by using all possible repairing, until some cracks open in a way that one must necessarily abandon it in favour of a new theory. Here also we find a strong component of conventionality that, for some aspects, can refer to those concepts of normal science and scientific revolution that many years later will be theorized by Thomas Kuhn.

This fully justifies, we believe, the initial statement that the theory of Einstein was already inscribed in a certain way in the scientific-cultural panorama of the moment. And perhaps it is true for all scientific revolutions. When problems, ideas, possible solutions pile up till they constitute the bases of a radically new view, it needs only that a scientist, having a "*young*" mind, has the courage to say the first « *the king is naked!*»[24].

The problems that physics was deal between the end of XIX and the beginning of XX century were born by attempting to reduce all phenomena relative to the propagation of light and those ones at subatomic scale to Maxwell's equations. For the first time one had to deal with magnitudes not comparable to those ones that happen in the scale of our sensorial experience. What comes out is that involved magnitudes escape classical definitions and especially the hypothesis of an absolute space (as the Newtonian one) and absolute time was inadequate. The Lorentz's transformations, firstly thought as adjusting of classical theories in order to reconcile them with experimental data[25], did not solve the fundamental problem, that was more profound and involving the possibility to define physical magnitudes in metaphysical way, but not translatable in precise procedural strategies if empiric data can be pointed out.

For what concern the difficulties caused by the concepts of space and time, these are connected with the meaning itself of "*simultaneity*" of two events, a concept that risks to become metaphysical if there is any possibility of experimental control. The telegraph, permitting to synchronize two clocks at any distance, allowed to unify the scale of times, but this was well when the matter was the control for instance of a trajectory of a projectile or other phenomena at macroscopic scale and having velocities not comparable with that one of light. In these cases, in fact, the time employed by electric signal for synchronizing is so little that it can be disregarded: to send and to receive a signal can be considered simultaneous, and more the procedure itself to synchronizing can be assumed as a good definition of simultaneity. But all this has no value when one wish to study phenomena, as the electromagnetic ones, travelling with the same velocity (or with comparable velocity) of the signal used to synchronize.

Let we suppose to send a luminous synchronizing signal from a point A to a point B and suppose it is reflected from B to A. Let t_A be the instant in which the signal leaves A, t_B the instant in which the signal reaches B and it is reflected, t_C the instant in which it returns at the point A. If $t = t_C - t_A$ is the interval between the instants t_A and t_C, can we consider simultaneous two events that happens respectively in B at the instant t_B and in A at the instant $t_A + t/2$? The answer is positive only if we suppose that light covers outward and back journey in the same time. In order to understand this assumption, let we come back for a moment to the decision we took to consider as *absolute* the length of the regulus-specimen to measure lengths and the periodicity of an event-specimen to measure times: only at this point the definition of velocity takes sense. We are in condition to must take as an

[24] In this connection it is exemplar the case of Gauss that didn't publish as first his conclusions on the question of parallel lines (see section 1) because, as he said in a own letter, was afraid of «strida dei beoti». Analogous conclusions were published firstly by the young mathematician Ukrainian Lobachevsky. He in fact preceded the other young mathematician Bolyai that, even if he wrote firstly his work, didn't publish by following the opinion of Gauss.

[25] With his memoir of 1892 (H. A. LORENTZ, *La théorie électromagnétique de Maxwell et son application aux corps mouvants*) Lorentz tried to save the theory of ether after the experiments made by Michelson and Morley.

absolute the velocity of light and therefore bringing up for discussion again and so making relative the other involved magnitudes. Anyway we cannot renounce to a conventional assumption of an absolute, but nobody forbids to assume this one or the other one if the choice appears more useful and functional. From now on all that follow is consequential, also when it seems in contrast with more consolidated common opinions.

9. *Conclusion*

At the end of this paper, it seems to us to can propose some reflections that, beyond treated specific questions, refer to the meaning itself of science, its potentiality and its limits.

First we note the mistake often committed when one opposes the concepts of "*absolute*" and "*relative*" putting them in relation respectively with classical physics (Newtonian mechanics) and Einstein's relativistic theories. On the contrary we hope that, from what we tried to expose, could emerge with enough clarity that every physical theory, or better every scientific paradigm, has its own "*absolute*", its own undefined and indefinable universe of reference, a nucleus *dogmatically* admitted and supposed without any problematicity. It could be the constancy of the velocity of light in void or its isotropy, the constancy of oscillations of an atom of Cesius 133, the concept of absolute space or absolute time, or the existence itself of a natural law to which all is subject and that is always and everywhere the same.

It is an non-eliminable metaphysical nucleus, that we find at the base of every scientific discourse and without which no theory, in its formal abstractness, could assume for us some meanings. It is this nucleus that feeds also on daily experiences, values and human needs, projects and hopes, practical and technological knowledge (even if it is nothing of this), so it is this nucleus that give us the intelligible bricks of our view of the world, and that allow the heuristic processes of hypothesis and discovery.

Why then we use so much care and then we sponge out every trace of metaphysical enunciation inside scientific discourse? Is it perhaps not true that also in the present paper we have denounced the risk that, without an effective procedure to determine the simultaneity, the time should remain a merely metaphysical concept?

The fact is that science, as it developed inside western thought, can proceed successfully only if it is able to distinguish the formal moment (characterized by technical languages and rigorous procedures) and the pre-formal moment of the creation and of the heuristics. There is nothing new in all this fact: already Archimedes, in his work "*Methodus*", confides to Eratosthenes that his results, before to be rigorously demonstrated by the exhaustion method (generally accepted by scientists), was discovered by a mechanical method, more heuristically efficacious, but believed scientifically questionable.

Positivism, with its indomitable fight against every form of metaphysical thought, was not able to cancel the pre-formal moment of science, but it made it invisible, or better has conditioned the eyes and the consciences in such a way that we can see without look it. We believe that this to make to don't see the metaphysical and pre-formal base in science, is equally negative as to confuse the two levels. In both cases it results a distort view on to do science, on its results, on its applications, on the values that are transmitted. And all this in a moment in which the science appears as keystone of our future and of that one of Earth itself.

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