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FORMALISM AND INTUITION IN MATHEMATICS: THE ROLE OF THE PROBLEM

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ABSTRACT

A main characteristic of the intuitive – inductive philosophy of mathematics is the attention given to the problem – solving processes, in contrast to the formalistic – productive philosophy where emphasis is given to the content. Therefore a crucial question is what is actually the role that problem plays for the learning of mathematics.
The aim of the present paper is to give an answer to the above question. For this a review of the evolution of the problem – solving process in mathematical education is attempted – from the time that Polya presented his first ideas on the subject until today - in contrast to other existing views giving emphasis to other factors of the learning process like the acquisition of the proper schemas, the automation of rules, etc and our personal conclusions and beliefs are stated.

1. INTRODUCTION

From the origin of mathematics there exist two extreme philosophies about its orientation (presentation, teaching, research, etc) : The formalistic – productive, where emphasis is given to the content and the intuitive – inductive, where the attention is turned to problem solving processes.
The axiomatic foundation of Geometry in Euclid's "Elements", the most well known in the world mathematical classic, is a representative example of the formalistic philosophy. An analogous example for the intuitive philosophy is the less known to the West World Oriental counterpart "Jiu Zhang Suan Shu" (Nine Chapters on Mathematics); cf. Ma Li (2005). Although very different in form and structure from Euclid's "Elements', it has served as the foundation of traditional Oriental mathematics and it has been used as a mathematics text book for centuries in China and most of the other countries of Eastern Asia.
Its title has been translated in English in various ways. Although "mathematics" seems to be a more accurate translation of "Suan Shu" than mathematical art, it seems that mathematics in the East is indeed more of an art as compared to mathematics in the West as a science.
Very many centuries later, during the 19th and the beginning of the 20th century, the well known paradoxes found in the Set Theory was the main reason of an intense dispute among the followers of the two philosophies, which however was extended much deeper into the mathematical thought.
A major early proponent of formalism was Hilbert, whose program was a complete and consistent axiomatization of all of mathematics, while Cantor's Set Theory, a great goal in the history of mathematics, was an extension of the formalistic thought. Hilbert's goals on creating a system of mathematics that is\ both complete and consistent was later dealt a fatal blow by the second of Goebel's incompleteness theorems.
Other formalists, such as Carnap, Tarski, Curry etc,, considered mathematics to be the investigation of formal axiom systems.
On the other hand Brouwer, who rejected the usefulness of formalized logic of any sort for mathematics, was a major force behind ituitionism, as well as Kronecker, the Cantor's

teacher, who used to say that "The natural numbers come from God, everything else is a man's work", Weyl, etc
Examples of how the "mathematics pendulum" swung from one extreme to the other over the span of about a century, include the evolution from the mathematics of Bourbaki to the reawaking of experimental mathematics, from the complete banishment of the "eye" in the theoretical hard sciences to the computer graphics as an integral part of the process of thinking, research and discovery, and also the paradoxical evolution from the invention of "pathological monsters", such as Peano's curve or Cantor's set – which Poincare said should be cast away to a mathematical zoo never to be visited again – to the birth of a new Geometry, Madelbrot's Fractal Geometry of Nature (1983). To Madelbrot's surprise and to everyone else's, it turns out that these strange objects, coined fractals in 1975, are not mathematical anomalies but rather the very patterns of nature's chaos.
As a consequence of the "mathematics pendulum" swing, dramatic changes also happened in the area of mathematical education during the last fifty years. First the result of the post – war effort that mathematics as a teaching subject should be brought into harmony with mathematics as a science, as it has been developed since the last quarter of the 19^{th} century with an increasing gap between school mathematics and modern higher level mathematics, was the introduction, during the 60's, of the "New Mathematics" in the curricula of studies.
But it did not take many years to realize that the new curricula did not function satisfactorily all the way through, from primary school to university, even if the problems varied with the level (e.g. see Kline, 1973).
Thus, and after the rather vague "wave" of the "back to the basics", considerable emphasis has been placed during the 80's on the use of the problem as a tool and motive to teach and understand better mathematics.
The aim of the present paper is to examine the role of the problem for the learning of mathematics and state our conclusions and personal beliefs. For this it is useful first to make a review of the evolution of the problem - solving process in mathematical education - from the time that G. Polya (1945) presented his first ideas on the subject until today- in contrast to other existing views, that give emphasis to other factors of the learning process, such as the acquisition of the proper schemas, the automation of rules, etc.

2. PROBLEM – SOLVING IN MATHEMATICAL EDUCATION: A REVIEW

The learning of mathematics through the use of the problem–solving processes is highly based on the idea of **rediscovery**. Polya (1963), claiming that every new knowledge in mathematics can be obtained by considering a suitably chosen problem and using our previous knowledge, suggests that rediscovery is the main tool for the materialization of the Piagetian perspective of **active learning**. He distinguishes three **consecutive phases** in the whole process: Exploration, formalization and assimilation. The **best motivation**, i.e. the best way with which the teacher creates the suitable learning situation, is the third, but not less important, of his famous **axioms of learning**.
Polya (1945, 1954, 1962/65) laid also the foundation for exploration in heuristics, since he was the first person who described the problem-solving strategies in such a way that they could be taught. We recall that a **heuristic** is said to be a general suggestion, or technique, that helps problem solvers to understand or to solve a given problem.
Polya also offered his **rules of preference**, which is an approximation to put the given heuristics in some order for better management; e.g. the less difficult precedes the more difficult, an item having more points in common to the problem precedes an item having less such points ,etc.
Very many researchers in the area of mathematical education such as Lucas, Goldberg, Cantowski, Putt, etc worked on Polya's ideas and attempted to show that heuristics can help students to solve problems. They reached to the conclusion that the basic approach should be to teach them to solve their problems in the way that experts do.

Hatfield (1978) distinguished three types of teaching in the problem – solving process: Teaching **for** the problem – solving, teaching **around** the problem – solving and teaching **inside** of the problem – solving.
The first type, to which emphasis is given in the educational textbooks, turns the attention to the acquisition of the proper mathematical knowledge (notions, theorems and skills), which is useful in problem- solving.
The second type is centred on the teacher, who offers good models of behaviour, or leads the students to correct processes towards problem – solving.
The third type includes the presentation of the new mathematical content through the problem –solving and it is the type of teaching that Polya encourages (Wickelgren, 1974).
Much of the motivation of the emphasis that had been placed on problem – solving and heuristics during the 80's seems to derive from observations that students who have learnt a new principle are frequently unable to use it intelligently to solve problems. The conclusion obtained was that they lack suitable general problem – solving strategies.
Schoenfeld (1980, section 1) advises the solver to try to find the **cues** in the statement of the problems (i.e. characteristic words or phrases), which could help him to use the suitable heuristics to solve them. For example the word "unique" could suggest the use of the method of obtaining an "absurd conclusion", the phrase "for all positive integers" could suggest the application of an inductive argument etc.
However knowing how to use a strategy is not enough; the solver must know when it is appropriate. According to Schoenfeld (1980, section 4) we can think of a heuristic as a "key" to unlock a problem. There are a large number of such "keys" and a given problem is usually "openable" by only a few of them. Therefore a strategy for selecting the right "key" may possibly needed. Such a strategy is usually called a **global** heuristic; e.g. to solve a complicated problem it often helps to examine and solve an analogous simpler problem and then explicit your results.
Using a global heuristic you have to specify it according to the form of the given problem; e.g. using the above mentioned strategy of the "analogous problem" for the case of a complex problem with many variables you may consider first an analogous problem with fewer variables, using it for a geometric problem in space you may consider first the corresponding problem in the plane etc.
The **expert performance model** of Schoenfeld in problem - solving (Schoenfeld 1980, section 5) is actually an improved version of Polya's basic model. It consists of five stages and its real goal is that for each one of these stages a list of the possible heuristics, that could be used in order to get through, is given.
Thus in the first state of the **analysis** of the problem (understanding the statement, simplifying and reformulating the problem), the heuristics that could be possibly used are: (i) Draw a diagram, if at all possible, (ii) Examine special cases, (iii) Try to simplify the problem by exploiting symmetry, or "without loss of generality" arguments.
The next stage is the **design** of the solution, which is in a sense a "master control" (structuring the argument, hierarchical decomposition from global to specific).
The stage of **exploration** (looking for essentially equivalent problems and, if necessary, for slightly or broadly modified problems) is actually the heuristic "heart" of the whole process.
Notice that, being at the stage of design and having some minor difficulties, the solver may transfer to exploration and then go back to design to continue the process. On the contrary having major difficulties, after his transfer from design to exploration, he may return to analysis looking for a more accessible related problem, or for more (slighted at the first glance) information. Then he returns to design in order to continue the process. The same "circle" may be repeated several times.
The **implementation** of the solution (step by step execution, local verification) needs little comment, it could be the last step in the problem's solution (see below).
On the other hand **verification** (specific and general tests) needs some attention. At a local level one can locate silly mistakes, while at a global level a review of the solution can yield alternative methods, can show connections to other subject matter and very possibly clarify a useful technique, which can be used in one's global problem – solving approach.

Frequently an inexperienced solver does not realize that the tentative solution, found at the stage of implementation, in order to be acceptable needs to pass through all the necessary tests (e.g. it must conform to reasonable estimates or predictions, it must be substantiated by special cases, it must be reduced to known results etc). In other words the solver in this case considers implementation as the last step of the problem - solving process not approaching the state of verification.

Determining the level of students' problem – solving abilities and the effectiveness of instructional programs in developing these abilities requires measurement and several efforts have been made towards this direction.

Malone et all (1980) used the Rasch approach for this purpose, i.e. a probabilistic model for solving measurement problems (Wright 1977), while Schoen and Oehmke (1980) applied what they calledn the " Iowa Problem – Solving Test" (IPST).

Schoenfeld (1982) introduced three easily graded measures (A "Plausible Approach" analysis of fully solved questions – Students' qualitative assessments of their problem solving – Heuristic fluency and transfer) and used them to demonstrate the impact that a month long intensive problem – solving course can have on students' performance.

Voskoglou and Perdikaris (1991, 1993, 1994) introduced a stochastic model for measuring the ability of a group of solvers and gave several examples to illustrate it. The finite Markov chain that they used in their model has as states the corresponding steps of Schoenfeld's expert performance model for problem – solving presented above.

Stillman and Galbraith (1998) reporting on an intensive study of problem solving activity of female students at the senior secondary level, they found that more time was spent in general on orientation and execution activities (exploration and implementation of the solution), with little time being spent on organization and verification activities (design and verification of the solution).

It has been observed in general that many students' difficulties may be due to their being comparatively inexperienced in problem - solving. Such novices tend to perform poorly compared to experts, as may be expected. This seems to be due to novices possessing a much smaller and more poorly structured knowledge base, making difficult to them to know which information is relevant, what type of problem are dealing with and which techniques and procedures to apply, while experts generally have the experience and knowledge to do this successfully (Sternberg 1997).

Related research on analogical mapping (Gick and Holyoak 1980, 1983, Needham and Begg 1991, Voskoglou 2003 etc) shows that students cannot easily be relied upon to link analogous situations. Thus care needs to be exercised in building problem banks of analogous questions.

Conclusively further research remains to be done on how experts solve problems and the relations, if any, between the processes employed by the expert solver and by the novice. This could provide insights on links among the several stages of the problem - solving process.

A very important component of problem – solving is the process of **Mathematical modelling**, that deal's with the solution of a special type of problems generated from corresponding situations of the real world (e.g. see Voskoglou 2006 (ii)). Mathematical modelling appears today as a dynamic tool for the teaching of mathematics, because it connects mathematics with our everyday life and gives to the students the possibility to understand the usefulness of it in practice (e.g. see Voskoglou 2006 (i)).

Finally the important role that the rational use of the new technologies could play for a further development of the students' problem – solving abilities must be mentioned. In fact the animation of the figures and mathematical representations - provided by suitable PC programs, videos, etc - increases the imagination of the students and helps them to find easier the solution of the corresponding problems. The role of the mathematical theory after such a process is not to convince, but to explain (Sarrazy, 2006).

3. PROBLEM – SOLVING, TRANSFER AND SCHEMAS OF KNOWLEDGE

Owen and Sweller (1989) question the wisdom of recent moves to allocate time in mathematics teaching for instruction in the use of general problem – solving strategies, because they doubt that such instruction will help to overcome problems appearing in the **transfer of knowledge** (i.e. the suitable use of already existing knowledge in order to obtain new knowledge).

According to them transfer failure is more likely to be the result of a lack of an appropriate schema, or of insufficient automation of rules. They imply the attention allocated to general problem – solving strategies would be more appropriately diverted to instruction concerned with domain– specific knowledge and practice with worked examples and goal – modified problems.

We recall that according to Anderson (1984) a **schema** is understood to be an abstract knowledge structure that summarizes information about many particular cases and the relations among them. Making this definition more specific Owen and Sweller accept that schema is a cognitive structure that specifies both the category to which a problem belongs as well as the most appropriate moves for the solution of problems of this category.

We also may explain that the term goal – modified problems means problems where the goal – state is not specified and therefore is unknown; e.g. requiring the calculation of all the unknown variables of the problem instead of one, as it usually happens to the classical "transformation problems" (Greeno, 1978). This type of problems prevent the use from the solver of the technique of "means ends analysis" (Anderson, 1985) , which involves attempting to reduce the differences between each problem state encountered and the goal state , and normally with a well specified goal involves working backwards from the goal to the givens. According to Owen and Sweller this technique interferes with schema acquisition, because it imposes a heavy cognitive load to the solver.

Lawson (1990) believes that Owen's and Sweller's view, that the evidence on the efficiency of the instruction of problem – solving strategies in mathematics curricula is very sparse, derives from the stance they take on the nature of them and on transfer.

It is important, he says, to distinguish among three different types of general problem-solving strategies:

Task orientation strategies - or beliefs according to Schoenfeld (1985)- which influence the disposition state of the student about the task ,

Executive strategies, being concerned with goal setting, monitoring allocation of attention and selection of more specific processing operations, and

Domain – specific strategies, which include heuristics such as means-ends analysis and other procedures developed by the individual for organizing and transforming knowledge (e.g. trying for simple cases, creating a table, drawing diagrams, looking for patterns or developing general rules etc).

Also, according to Lawson, transfer needs to be viewed as a complex chain of processing rather, than been treated as an afterthought learning resulting from generalization (Gelzheiser, 1984). The successful transfer does not depend only upon the awareness of problem relations, schema induction and automation of problem operators (Cooper and Sweller 1987), but also involves in its high-road form a mindful abstraction of the generic features of the content (Salomon and Perkins 1989), a chain of processing that is quite different from the spontaneous, automatic extension of learning, which is refereed as low-road transfer.

It is suggested that a strong candidate for an abstracting and awareness stimulating mechanism is a monitoring strategy. The similarity in the structure of the old and new problems is established through the operation of an executive strategy that initiates analysis of the new problem structure and comparison of the products of that analysis with already established structures or schemas.

All the above provide a good reason to continue the study of the role of heuristics in mathematical problem – solving and for the attention to these strategies in mathematics teaching. And concluding Lawson claims that Owen's and Sweller's view is well placed only with respect to the amount of time and effort which is recently devoted to general problem – solving strategies in classroom mathematics lessons. There is indeed a danger that problem solving could become a fashion and turn into fads, as innovations in education have the nasty habit to do.

4. CONCLUSIONS AND PERSONAL BELIEFS

I shall start with some comments on the ideas of Owen and Sweller about problem – solving.
According to their definition a schema specifies the category to which a problem belongs as well as the most appropriate *moves* for the solution of the problems of this category. But which are these *moves*? They are not the proper heuristics helping towards the understanding and solution of the problems? If yes, these heuristics must belong to the corresponding schema!
Even Marshall (1995), the introducer of the current schema theory, present schemas as the vehicles for problem solving, that can simplify and reconstruct a problem in order to make it more accessible to the solver.
I also strongly disagree with their view that the technique of "means ends analysis" interferes with schema acquisition, because it imposes a heavy cognitive load to the solver. I believe that one, in order to learn mathematics, **must learn to think mathematically** and this can be succeeded only through his personal efforts and mistakes. The practice with worked examples and the automation of rules help, but they are not enough!
The process of learning (not only mathematics, but a subject in general) has been strictly related by many researchers with problem –solving. Thus according to Voss (1987) learning basically consists of successive problem – solving activities, in which the input information is represented of existing knowledge, with the solution occurring when the input is appropriately interpreted. The process involves the following stages: Representation of the input data, interpretation, generalization and categorization.
The representation of the stimulus input is relied upon the individual's ability to use contents of his memory in order to find information that will facilitate a solution development. Learning consists of developing an appropriate number of interpretations and generalizing them to a variety of situations. When the knowledge becomes substantial, much of the process involves categorization, i.e. the input information is interpreted in terms of the classes of the existing knowledge. Thus the individual becomes able to relate new information to his knowledge structures (schemas).
According to my personal opinion Lawson's view about problem – solving is much more realistic than that of Owen and Sweller. The teaching of heuristics however need not to constitute a separate subject in mathematics curricula; it must be materialized from the teacher in practice at any time and level by the solution of the appropriate problems, or the proof of the appropriate theorems.
My strong belief is that, at the school level, Euclidean Geometry gives very many such opportunities to the teacher, since it is the mathematical subject that fits better than any other to the spiritual maturity of the children of this age (certain and solid, non absurd notions, they can "see" what they are doing). Therefore recent attempts in several countries to try to minimize the teaching of Euclidean Geometry at school under the excuse of adding "modern" material in mathematics curricula (e.g. Analytic Geometry, Differential and Integral Calculus etc) is - not only according to my opinion, but according to the opinions of many other researchers and educators - a big pedagogical mistake.
In particular this is unacceptable to happen in my country, Greece, with such a brilliant tradition on the subject from the ancient years (Euclid, Thales, Apollonius, Pythagoras, Archimedes, etc), but unfortunately nowadays we have reached to the point where our students don't study at all the Geometry of Space during the last three years of the secondary education (Lyceum)!

5. Epilogue.

According to Verstappen (1988) in mathematics education there is a continuous oscillation between the two extreme philosophies that we have mentioned in our introduction. A similar perception has been supported by Davis and Hersh (1981).

Verstappen believes that the period of this oscillation is of about 50 years, which is also crossed by Galbraith (1988), who uses a diagram, due to Shirley, representing a parallel process between the alterations of the economical conditions and the changes appearing in the mathematical education systems of the developed west countries.
The above estimation, if it is true, means that approximately every 50 years substantial changes happen in mathematical education! The consequences of this conclusion are many and important, but here we shall restrict our attention only to those which are related to our subject.
The failure of the introduction and the end of the period of the “New mathematics” in the school education means that now the above oscillation is moving towards the intuitive-productive philosophy. It seems that the perceptions of this movement are expressed through the “wave” of “Problem – solving, Mathematical Modelling and Applications”, which is supported by the new technology (introduction of informatics in mathematics education etc).
Thus, and regardless of personal beliefs and options, we ought to prepare the conditions under which the mathematical education will receive and assimilate gently and creatively the advancing changes, getting the maximum possible profit from them.
In Chinese philosophy the Yin and Yang represent all the opposite principles (Ma Li, 2005). It is important however to pay attention to the fact that these two aspects complement and supplement each other with one containing some part of the other than opposing each other.
Each of the several philosophies of mathematics has its own importance and advantages, but what we actually need is to find a proper balance among them.

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