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Report concerning the Ph.D. Thesis

"*Embodiment and a-didactical situation in the teaching-learning of the perpendicular straight-line concept.*",

submitted by Giannamaria Manno.

The main idea that make this work particularly interesting, is the search of connections between the Ssituations theory of G. Brousseau and the Embodiment theory that involves neuro-sciences and finds his theoretical and philosophical bases on the metaphor theory of Lakoff and Numez, (with a particular reference to G. Lakoff, R. Numez, 2000, Where Mathematics comes from).

The first one is an important tool through which a teacher can control and so drive the processes of learning by given pupils in a concrete didactic situation. The second, that instead doesn't constitute a perspective of concrete teaching (embodiment does not refers to physical manipulation of tangible objects, or to the virtual manipulation of graphical images and objects instantiated through technology) allows to understand the mental mechanisms through which the mathematical concepts are build by the mind beginning from the bodily experience and so from the native sensory-motor schemes that are referred directly to it.

After to drawn, in a brief introduction, the history of his thesis and a synthetic national and international research context, the author expose, in the first chapter, the theoretical square of the didactical situation theory and its mine concepts, starting from the triangle teacher-knowledge-pupil to examine particularly the dynamic of an a-didactical situation. So she resume the main obstacles in the theory of situation (Didactical and epistemological obstacles, cognitive conflicts).

In the second chapter the conflict between the mathematical language and the every day language is descript.

In the chapter III we enter in the original nucleus of this search with the building, experimentation and analysis of an a-didactical situation related to the concept of perpendicular straight lines. The aim of the research was to find out different notions or ideas of the concept of perpendicularity that students have, whether if they are spontaneous or coming from teachers. A question is *Do students have an inner model of perpendicularity between lines regarding whom the notion of perpendicularity is the same as uprightness (vertical)?*.

The test, that after accurate a-priori analysis was submitted to 5-7 age children, given interesting results that coherently with the hypothesis, shows difficulty and obstacles. Moreover some problems are pointed out that can be subject of other searches.

But the analysis do not finished here because, as we said, the author propose the search of liaisons with the embodiment theory, to explains the learning process. This approach is very interesting, to my opinion, because it connects un important tool for the study and the analysis of the didactic processes, as the situation theory, to an epistemological point of view that I hold notably meaningful and promising.

So the chapter IV is dedicate to shown the general basis of the embodiment theory. The following questions are mainly considered: 1) What are the mathematical ideas on the side of cognitive sciences? Which cognitive procedure do they use? 2) Given some inborn, which is the cognitive mechanism that magnifies this capability to generate new complex mathematical ideas? 3) What are the mathematical ideas that we can get from our experiences? 4) What mathematical ideas are metaphorical and what others are conceptual blends?

To analyze such problems in this theoretical framework the embodiment is handstand in two senses. In a weak sense every concept has to be influenced by our neural structure, every thought comes out from a neural process, learning too is possible due to a neural mechanism. In a strong sense, cognitive science has to clear up how is possible to think in a non-figurative way, how is possible to understand concepts that are not expressed by senses.

To this porpoise, the author examine different passages of the theory, as the rule of the culture and so the relation between culture and embodied mathematics, the performing of the conceptual metaphor and so the passage from the Image Schemas to the Spatial Relation Concepts. The structure of the conceptual Metaphor is also analysed.

Finally as conclusion of this chapter the previously described experience is considered at light of the embodiment theory showing as two different metaphors perform the correct concepts of per-

pendicular and vertical, so the confusion of this in one only metaphor, and not a misunderstand, is the cause of the considered didactic obstacle. This is coherent with the situation theory.

The V chapter is entirely devoted to neuronal approach. He comes therefore to the conclusion that the possibility for the brain to uses different reference systems, according to what it has to do and what kind of information it gets, can perhaps explain our skill toward geometry intended as the ability to change our mental point of view on different objects explain.

I I think that the work of Mss Manno, conducted with rigor and in systematic way, constitutes an interesting contribution to the search in didactics of mathematics, pointing out new relationships among different addresses of study and possible hypotheses to explore by further searches. Therefore my judgment is surely positive.

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