Semiotic and hermeneutic can help us to interpret teaching/learning?

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1.0 Introduction

In the intervention of Bruno D'amore[2] in the meeting in Amman has been put in evidence an use of semiotic to interpret the semiotic changement in teaching and learning.

In one of my intervention I analysed the relationship among semiotic, history and communication of mathematics[3].

In this paper I'll find to analyse the different instruments to interpret teaching and learning in the class.

The expression "Research in Didactic" put in evidence the preposition "in" to show a rule of research autonomy from other paradigm like psychology , pedagogy, mathematics, linguistic, for example. So, didactic is an autonomy rule with his specific language and its specific methods of research as has been more than once debated.

A lot of papers in the reviews in disciplinary Education are very good opinions of the authors of the articles without possibility to reproduce the experience or as in the case of Research-Actions or good description of experience from the teacher's point of view.

Starting from the study of "didactic situations" they can introduce the competence of discipline like epistemology, history, psychology.

The questions when they speak about research in didactic are:

- This kind of Research is reproducible and up to a point?
- To observe the phenomena of teaching/learning require the analyse of a lot of variable. Till when can we analyse it, describe, interpret ?
- What kind of relationship is there between didactic and teaching?

2.0 The position of r, and the researcher teachers.

We will find to see the positions, of the different subjects of "Research in Didactics". The following schema shows the relationship between Didactics Situation-Teacher-Pupil-Know (Savoir, Sapere), and the other linguistic planes.

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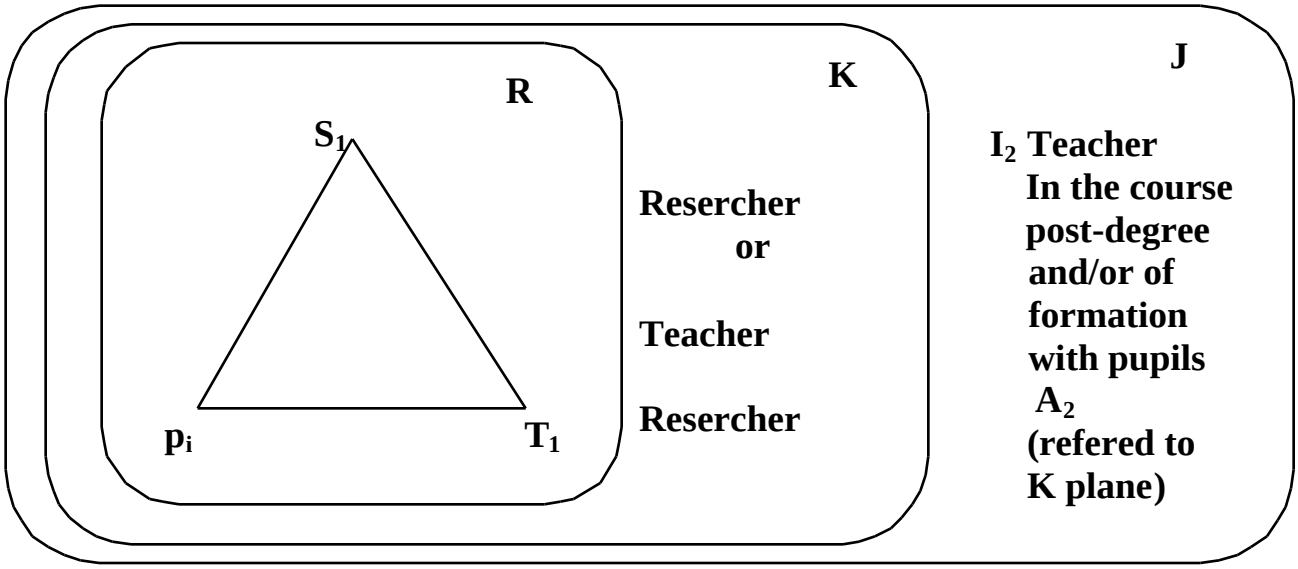
[2] B. D'Amore, Problems of Representing Concepts in the Learning of Mathematics, Proceeding, Ammann, 2000.

[3] [3] F. Spagnolo, The role of history in research in mathematics education, Proceeding, Amman, 2000.



In the plane **R** we have the starting situation $\mathbf{T_1}$-$\mathbf{S_1}$-$\mathbf{a_i}$ (Teacher, Savoir (Know, Sapere), pupil in a particular "didactic situation"[4]).

In the plane **K** the researcher or teacher researcher analyses the situation of the plane **R**. This represent his concrete. For example, the a- priori analyse of a didactic situation is in this phase.

Now, In the plane **J** is our landmark. The lessons of "didactics of mathematics", need to be organised for the teacher researcher (pupils $\mathbf{I_2}$). It's important to consider the teacher researcher, because the future professional of the teaching of the mathematics, at least once in his training face the problems of research, concerning to the communication of mathematics. This will agree later to establish an useful relationship with reading and the interpretation of results of research in didactics. When he will do his job as a teacher as a professional good at doing his job.

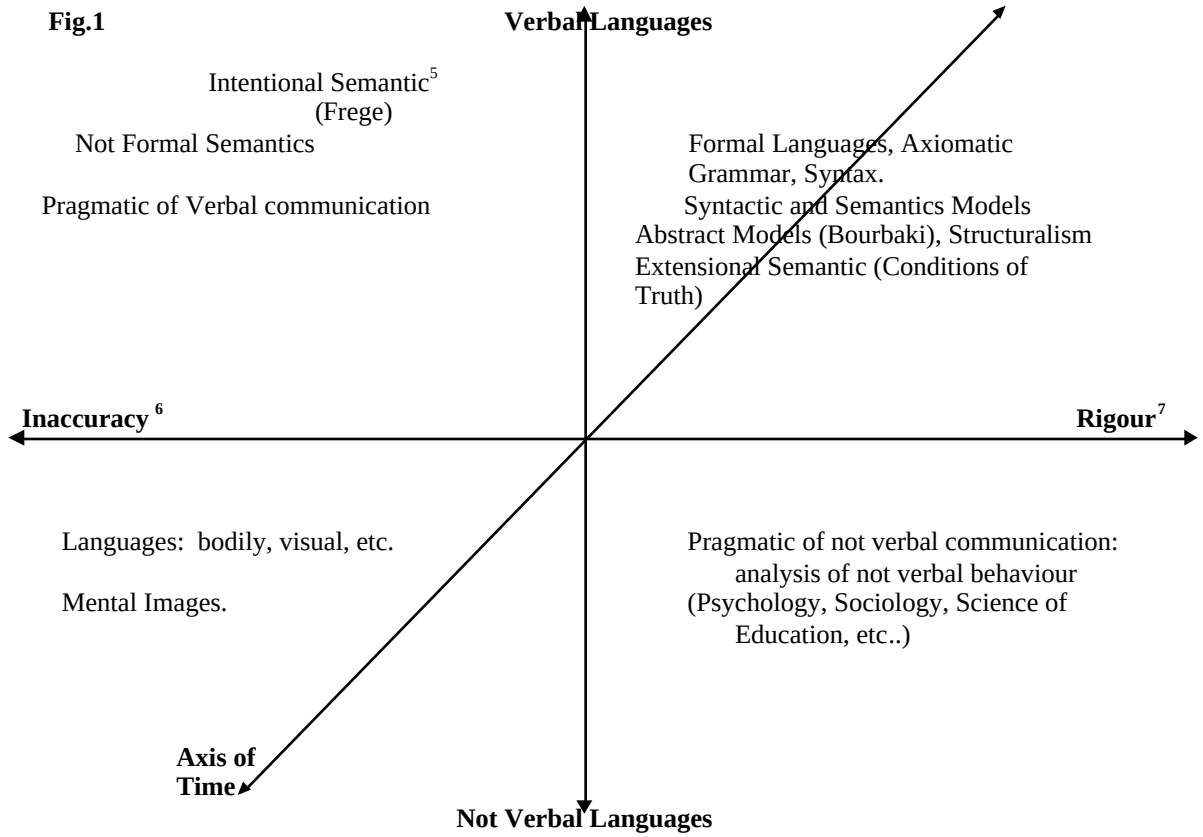
3.0 The semiotic and hermeneutic to interpret the communication of mathematics.

A model of reference for an **experimental epistemology** (Research in Mathematics Education) the schema consists on the plane *Verbal Languages - Non Verbal Languagse and Rigour - Inaccuracy*. Perpendicular to this plane there is the *time* which consent to review the semiotic phenomena (metalogic) in a dynamic vision.

The a-priori analysis, in the theory of didactic situations, can consent, according to the level of deeping of the analysis, a "prediction" of the temporal evolution of the didactics phenomena .

[4] Guy Brousseau, Théorie des situations didactiques, La pensée Sauvage, Grenoble, 1998.
Guy Brousseau, "Theory of Didactical situations in mathematics". 1970-1990" (304 pages), 1997, Kluwer Accademic Publischers. (Traduction M. COOPER, N. BALACHEFF, Rosamund SUTHERLAND et Virginia WARFIELD)

Fig.1



- In the **first** space *Verbal Language-Rigour*, we can note three thought regarding the systematisation of mathematics languages, which means that Logicism, Formailsm, Structuralism. As this space is seen in he dimension **time** we have the historic-epistemological evolution of the mathematics languages according to the interpretation shown from the different thought. The analysis that in this contest is made is that of "knowledge" (Savoir, Sapere) established and codified in a precise historic period and in the same time the evolution of the knowledge in the history.

- In the **second** space *Verbal Languages - Inaccuracy.* We find the semantic no formalized and pragmatic of Verbal Communication. Also in this case the dimension time consent us to analyse

[5]In the semantic of possible world of Kripte *The intention* is the same thing that the Sinn was in frege mind. The intention of a word isn't defined like a function which can give us the extension. If we consider the phrases " the formica drives the car" and "dog flies" a conventional semantic can say that they are impossible objects, an intentional semantic establish that their intention in the real world establish that it is the same, so, nothing. But, their extension isn't anything in other possible worlds where the two terms have the same extension. In the moment in which they need to recognise contradictions, the intentional semantic creates problem because has an useless extension in all the possible worlds. The formalisation of intentional logic drive to expression like *functions of functions*, each of them is functions of functions. The order of calculus establishes the comprehension which we can have. The intentional logic, and so, the intentional semantic inspiring to a ecological realism (Putnam), substitute to the possible worlds of the intentional logic (Montague) the "situations".

[6]the term inaccuracy can be integrated with Vagueness, Ambiguity, Indecidibility. The notion of "ambiguity" isn't only bounded to the presence (with) of more than one meaning, but it can be also associated to the same message or also to the ambiguity of formal-logic languages (Wittgenstein sustained these lacking of sense but not senseless). The ambiguity is a propriety of every language.

[7]In the sense of Tarski and Gödel: "...in relation to particular formal system".

hystorically the evolution of the Semantics and of instrument regarding the Prgamtic of verbal communication (analysis of text, hermeneutic, etc..).

- In the **third** space, *Inaccuracy - Not Verbal Languages*, we find, for example, the visual languages, bodily[8], languages the imagines of mind. In this space the dimension **time** it's difficult to put on an historic-epistemological also if in the history of mathematics the imagines of mind have had an important rule in the ordering as regards the learner and his history.

- In the **fourth** space, *Not Verbal Languages - Rigour*, we find the pragmatic of not verbal communication: analyses of not verbal behaviours. In the history of mathematics an important example is given from the "School of Italian Algebraically Geometry " in the last century, while they were looking for organising a grammatical concerning of the imagines of mind to solve geometrical problems. In this space we insert Psychology, Sociology, Science of Education especially as regard the study of the behaviours and the verbal languages we will indicate the 2 space. In the end, this matters are between the second and the fourth space.

- The **third** and **fourth** space in the time dimension they can evoke the History of Mathematics seen in the art dimension.

- In the reorganising process of the mathematics languages are 2°, 3°, 4°, space which tend to the first. This process can be explained easily by the assertion that all the formal languages are build and ordered thanks to the natural tongue. The model obtained to Rotnam and Peirce foresees for the mathematics Subject a tripartial dimension: a person, a subject, an agent "the person makes mathematics in informal way, the subject answered making a word with the imagination, and signing to a sosia agent the task to do a lot of imagined actions; The agent is a idealised model by himself who makes the commanding like an automa operating only with signs without meaning. In the passage from person to subject we can forget the indicators in the passage from subject to agent we can forget sign and meaning[9]. This tripartial model of the subject doing mathematics can be used also for the subject who wants to learn mathematics? The passage from the semantic fields to syntax hasn't been described very well or, sometimes, has been skimmed over. Perhaps we can find some controllable and reproducible model in the Theory of Situations for returning at "sense". Nowadays, In the m community there are Agent lot of declared a and few People.
  This scheme can be read in a **situation of didactic communication** and know the dimension time from the following points of view:

- From the point of view of " knowledge"(Savoir, Sapere) itinerary historic-epistemological. The analyse of extensional Semantic: evolution historic-cultural. The analyse of the intentional semantic: epistemology of the possible worlds.
- From the point of view of the pupils: Evolution of Conceptions (in the semiotic sense, in reference the 4 spaces) according to a contest (Didactic Situation). The intentional semantic. intervenes in the research of the referent and in the research of conditions of truth .
- From the point of view of the teacher: evolution of his own epistemology and control of the e relative to the relationship student-knowledge.

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[8] As reference to relationship between non verbal languages, perception of movement and mathematics behaviours:
- ß Lakoff G. & R. Núñez, Where Mathematcs Cames From: How the Embodied Mind Brings Mathematics into Being, New York: Basic Books.
- ß Alain Berthoz, Il senso del movimento, McGraw-Hill Libri Italia, Milano, 1998. (Le sens du mouvement, Edition Odile Jacob, 1997).

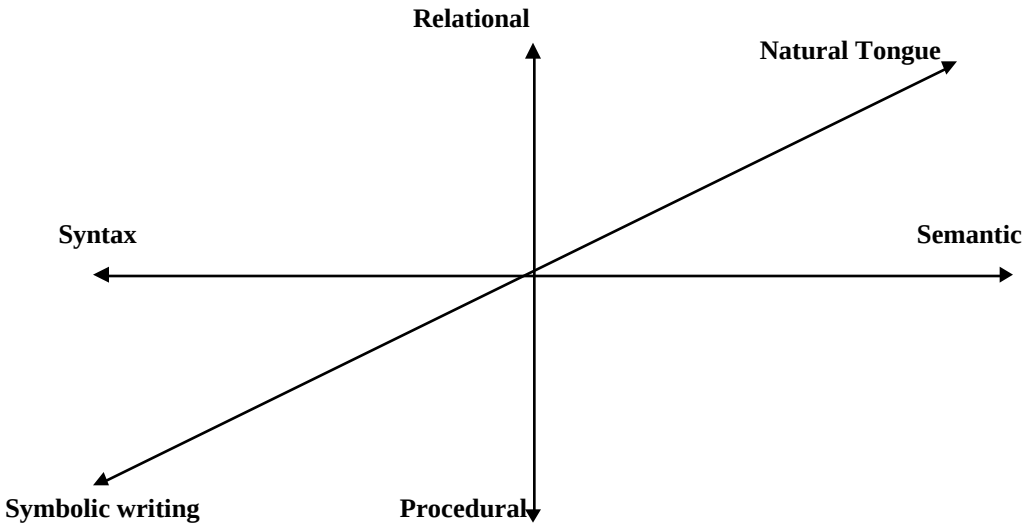
[9] G. Lolli, *Capire la matematica*, ed. Il Mulino, 1996, Bologna.

- From the point of view of the researcher: evolution of the didactic situation. The epistemological obstacles insert in this prospective. The intentional semantic intervenes in the moment in which they analyse the waited behaviours in an a-priori analyse in a didactic situation (Significant Conditions).

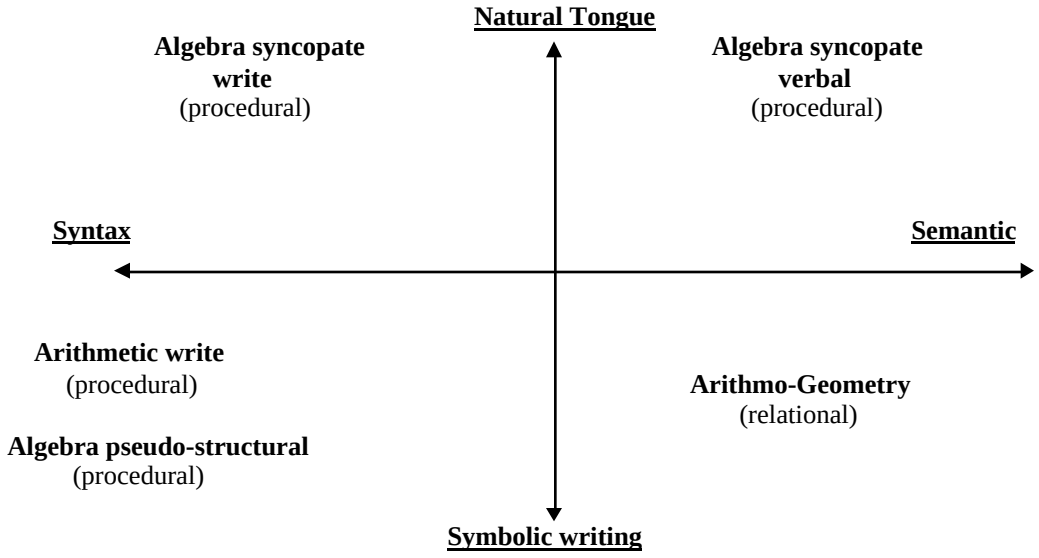
It seems very interesting to us refer on a model regarding the algebraically language[10] which consider three dimension of the algebraically language:

- axis x: Natural Tongue-symbol writing
- axis y: Semantic syntax
- axis z: relational-procedural

The three dimensions put in evidence the setting in which they unwind the processes of algebraically mind. The following schema puts in evidence the relations between the dimension:



Extracting from the last schema only two axis's we obtain the following relational plane which consent to analyse better some aspect of the algebraically language according to the famous classification of the historic-critical of the mathematics: Rhetoric Algebra, Syncopated Algebra, Symbolic Algebra .



[10] Ferdinando Arzarello - Luciana Bazzini - Giampaolo Chiappini, *L'algebra come strumento di pensiero (Analisi teorica e considerazioni didattiche),* Quaderno n.6 Progetto strategico C.N.R. Tecnologie e Innovazioni didattiche, Pavia, 1993.

These two schemes that we can draw out from the schemas of the Fig. 1 and especially regard the first two spaces. The analysis of the pragmatic of the communication of algebraically mind is made from a description of didactic situations regarding some cases chosen exactly. The analyse concerns an use of triangle of Frege Sense (Sinn) - Denotation (Bedeutung) - of a expression (Zeichen) applied on algebraically language.

The intentional semantic the changes of conception are interpreted through transformation of triangle of Frege.

4.0 Conclusion

The presentation of this schema creates some questions regarding the problem of the distinction between structural logic which is typical of an use of semiotic as a interpretative instrument of linguistic phenomena in static vision (research of invariants) and a dialectic logic that consider the historic evolution of the semiotic system. In this contest the use is the second type because try to analyse and interpret the didactics phenomena in their diachronic and synchronic evolution. The problem of the adaptation to the setting has an important place both neurophysiology[11] and cultural.

This interpretation of mathematics languages which we see in their semantic-syntactic-semantic evolution through the system of referent of Semiotic give a bigger possibility to interpret thr "didactics phenomena" in all the possible dimension from the formal languages to the not verbal languages (analysis of behaviours) and all considering the historic evolution of the mathematics languages the evolution which has different plane of reading from the recover of meaning to "senses" rebuilt syntactical of languages.

Can we consider this costructivist approach (Piaget, Bishop, Brousseau) or Pragmatic Logicist (Lolli)?

Nowadays the problem isn't refer, at least momentary, to only one theoretic interpretation of mathematics and communication of mathematics perhaps. We will need to be more useful to use, according to the situation, systems of theoretic different referent trying the generalisation without loving it very much. In the twentieth century the Semiotic has had and has still today like objective the generalisation. Can all the phenomena of the mathematics languages and be interpret by Semiotic?

Remain opened, for example, the problem if the genetic phenomena and neurophysiology are matter for the specialist of semiotic. The Umberto Eco's[12] answer is negative, on the contrary, they are matter for the specialist of semiotic the infomational theory of genetic and neurophysiology. At this point the neurophysiology phenomena must be face utilising the paradigm of research of experimental sciences. And what will be the relationship between the theory and paradigms?

The field of the hermeneutic can be useful to interpret the phenomena of teaching and learning.

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[11] Stanislas Dehaene, Il Pallino della matematica, Mondadori, Milano, 2000. (Traduction of La Bosse des maths, 1997).
Dehaene S.- Changeux J.P., Development of elementary numerical abilities: a neuronal model, Journal of cognitive Neuroscience, 5, 1993, pp.390-407.
Changeux J.P., L'homme neuronal, Fayard, Paris, 1983. (L'uomo neuronale, Milano, Feltrinelli, 1990).
Changeux J.P.-Connes A., Matière à pensée, Odile Jacob, Paris, 1989. (Pensiero e Materia, Torino, Boringhieri, 1991).

[12] Eco U., Trattato di semiotica generale, Milano, Bompiani, 1993.

4.1Relations beetwen Research and teaching.

The objectives of Research in Didactic can be shown in three big lines:

- ß Microdidactic: Experimental theoretical models to understand conceptions and/or obstacles.
- ß Situations a-didactic: qualitative analyse, protocol of single interviews, double, audio -video recorder of classroom-situation. The quantitative analyse of the objective proofs regarding the situation a-didactic. Always followed to a precise a-priori analysis.
- ß Macrodidactic: Statistic instruments for the macro observation. Questionnaire, tests, objective proofs regarding the situation a-didactic. Always followed to a precise a-priori analysis. Curricula.

As regard the microdidactic we can consider it like a basically research, which on fundamenta, to understand each other. What are the conceptions of pupils regards to determinate mathematics contents, what are the misconceptions, what are the obstacles of didactic nature, and/or epistemological. So, it's natural the use of this results to define, by the teacher, the didactics situations. Generally, they are the acquisition from the teacher at the end of his career, having accumulated a lot experience regarding the communication of his matter. So, The young teacher can go over again the study of communications of his matter in less time, utilizing the base research.

The building of situation a-didactics[13] is made by the teacher who knows, in the manipolation of knowledge, of all the researchers of Microdidactics.

The study of Macrodidactics is more wellknown. The definition of curricula, analyses of profit test on big trade, etc. This kind of researcher has been always made, but, has been founded on dignified "opinion" of the researcher of investigated spaces. The difference is that these three moments needs to a correlated study.

[13] F. Spagnolo, Insegnare le Matematiche nella scuola secondaria, La Nuova Italia, 1998, Firenze.